Natalia Kłosińska

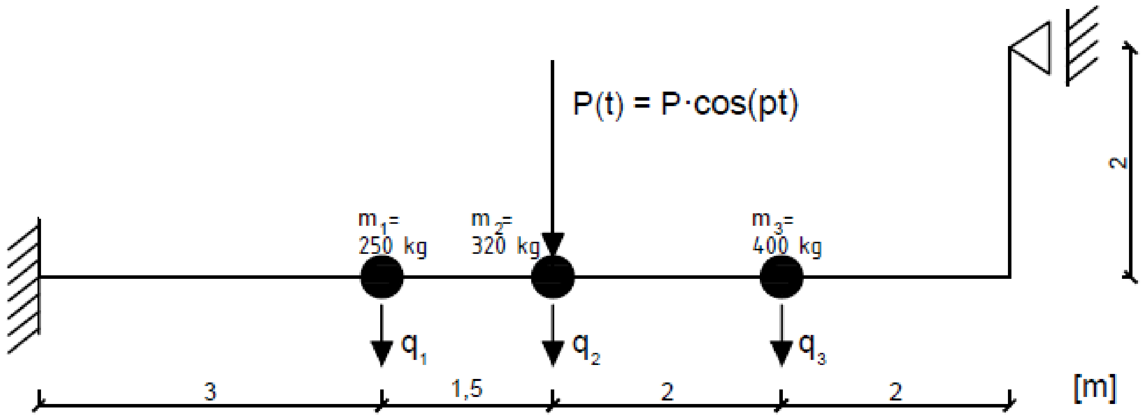
Mechanika budowli

Politechnika Poznańska

Budownictwo 2013/2014

IV semestr, grupa B3

# DYNAMIKA – UJĘCIE KLASYCZNE




Dane:

Przekrój: I 220 $I = 3060\ cm^4$ $W_x = 278\ m^3$

E = 205 GPa

$EI = 6273\ kNm^2 = 6273000\ Nm^2$

P = 25,5 kN = 25500 N

$p = 10{,}5\ Hz = 65{,}97\ \frac{rad}{s}$

SSD = 3

1. Wyprowadzenie wzorów do obliczeń:

$$q_i = -A_i \cdot \sin(\omega t)$$
$$\ddot{q}_i = -A_i \cdot \omega^2 \cdot \sin(\omega t)$$

$$\begin{cases} q_1 = \delta_{11}(-m_1\ddot{q}_1) + \delta_{12}(-m_2\ddot{q}_2) + \delta_{13}(-m_3\ddot{q}_3) \\ q_2 = \delta_{21}(-m_1\ddot{q}_1) + \delta_{22}(-m_2\ddot{q}_2) + \delta_{23}(-m_3\ddot{q}_3) \\ q_3 = \delta_{31}(-m_1\ddot{q}_1) + \delta_{32}(-m_2\ddot{q}_2) + \delta_{33}(-m_3\ddot{q}_3) \end{cases}$$

$$\begin{cases} A_1\cos(\omega t) = \delta_{11}m_1\omega^2 A_1\cos(\omega t) + \delta_{12}m_2\omega^2 A_2\cos(\omega t) + \delta_{13}m_3\omega^2 A_3\cos(\omega t) \\ A_2\cos(\omega t) = \delta_{21}m_1\omega^2 A_1\cos(\omega t) + \delta_{22}m_2\omega^2 A_2\cos(\omega t) + \delta_{23}m_3\omega^2 A_3\cos(\omega t) \\ A_3\cos(\omega t) = \delta_{31}m_1\omega^2 A_1\cos(\omega t) + \delta_{32}m_2\omega^2 A_2\cos(\omega t) + \delta_{33}m_3\omega^2 A_3\cos(\omega t) \end{cases}$$

$$\begin{cases} A_1(1-\delta_{11}m_1\omega^2) - A_2\delta_{12}m_2\omega^2 - A_3\delta_{13}m_3\omega^2 = 0 \\ -A_1\delta_{21}m_1\omega^2 + A_2(1-\delta_{22}m_2\omega^2) - A_3\delta_{23}m_3\omega^2 = 0 \\ -A_1\delta_{31}m_1\omega^2 - A_2\delta_{32}m_2\omega^2 + A_3(1-\delta_{33}m_3\omega^2) = 0 \end{cases}$$

2. Obliczenie częstości i postaci drgań własnych:

$$\lambda = \frac{m\omega^2}{EI_o}$$

$$\omega = \sqrt{\frac{\lambda EI_o}{m}}$$

m = 100 kg

$$\delta_{ik} = \sum \int \frac{M_i M_k}{EI} dx$$

$$\delta_{ik} = \frac{1}{EI_o}\delta'_{ik}$$

$$\frac{m_1\,\omega^2}{EI_o} = 2{,}5\,\frac{m\omega^2}{EI_o} = 2{,}5\,\lambda$$

$$\frac{m_2\,\omega^2}{EI_o} = 3{,}2\,\frac{m\omega^2}{EI_o} = 3{,}2\,\lambda$$

$$\frac{m_3\,\omega^2}{EI_o} = 4\,\frac{m\omega^2}{EI_o} = 4\,\lambda$$

$$\begin{cases} A_1(1-2{,}5\delta'_{11}\lambda) - A_2 3{,}2\delta'_{12}\lambda - A_3 4\delta'_{13}\lambda = 0 \\ -A_1 2{,}5\delta'_{21}\lambda + A_2(1-3{,}2\delta'_{22}\lambda) - A_3 4\delta'_{23}\lambda = 0 \\ -A_1 2{,}5\delta'_{31}\lambda - A_2 3{,}2\delta'_{32}\lambda + A_3(1-4\delta'_{33}\lambda) = 0 \end{cases}$$

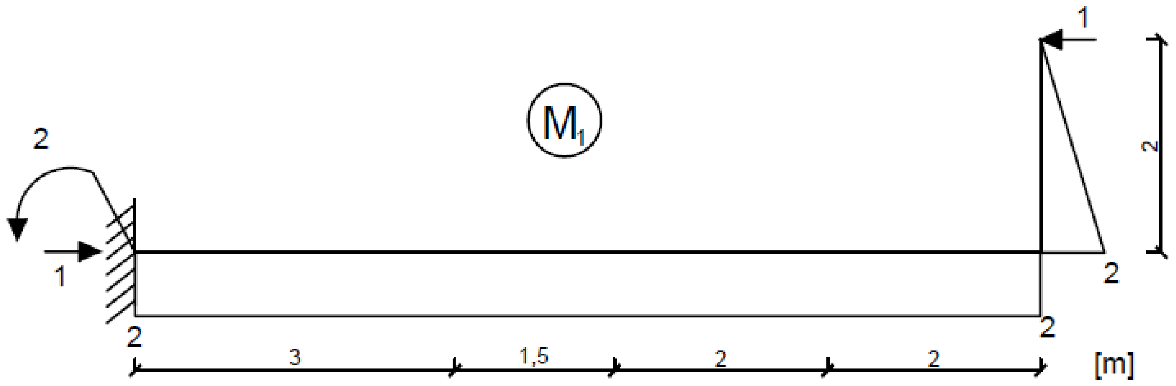
$$\begin{vmatrix} 1-2{,}5\delta'_{11}\lambda & -3{,}2\delta'_{12}\lambda & -4\delta'_{13}\lambda \\ -2{,}5\delta'_{21}\lambda & 1-3{,}2\delta'_{22}\lambda & -4\delta'_{23}\lambda \\ -2{,}5\delta'_{31}\lambda & -3{,}2\delta'_{32}\lambda & 1-4\delta'_{33}\lambda \end{vmatrix} = 0$$

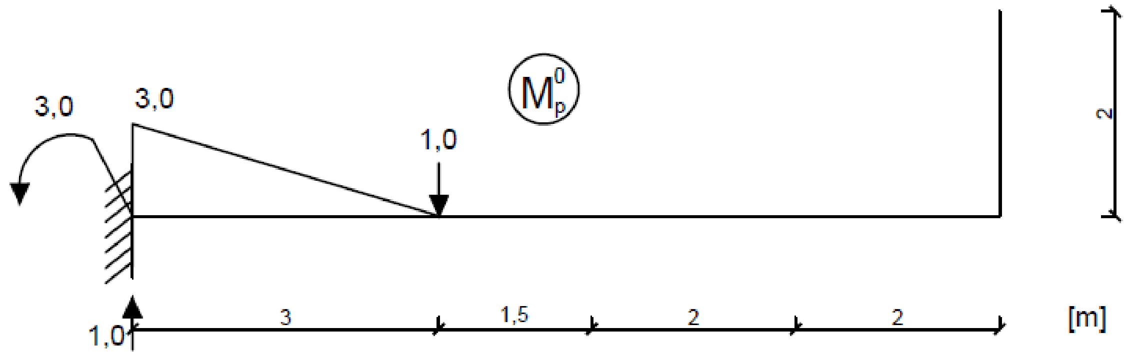
Wyznaczenie wykresu od siły P=1,0 po kierunku $q_1$ metodą sił:

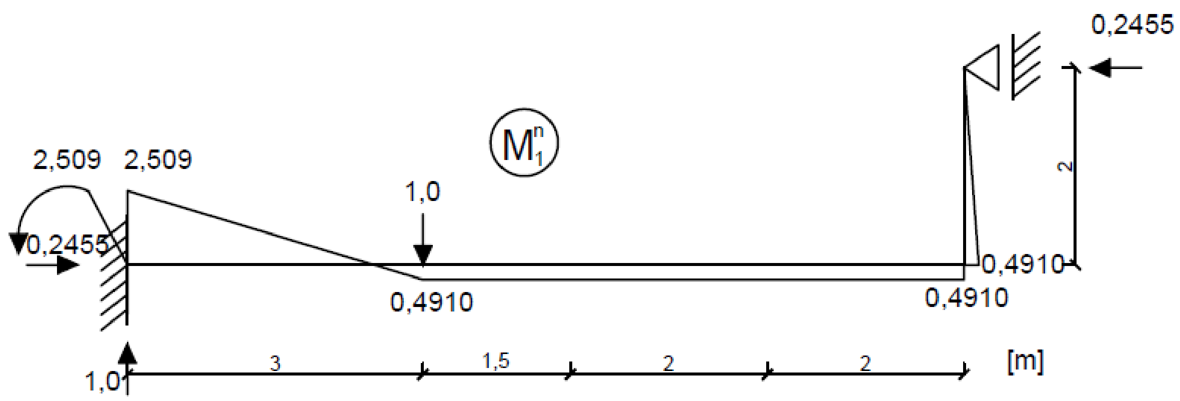
$\delta_{11} = \frac{1}{EI}\ (2\ \cdot 8{,}5\ \cdot 2 + 2 \cdot 2\ \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 2 = \frac{110}{3}\ \frac{1}{EI}$

$\delta_{1p} = \frac{1}{EI}\left(-3\ \cdot \frac{1}{2} \cdot 3\ \cdot 2\right) = -9\ \frac{1}{EI}$

$X_1 = -\frac{\delta_{1p}}{\delta_{11}} = 9\ \cdot \frac{3}{110} = 0{,}2455$



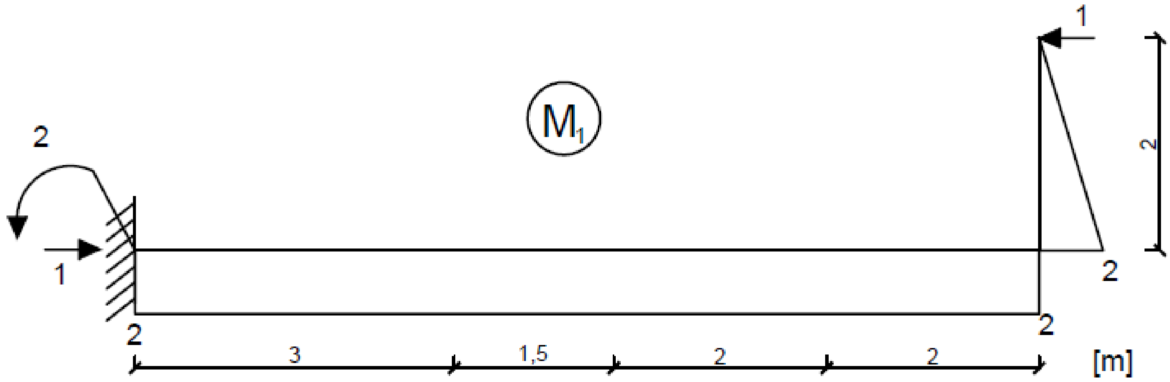



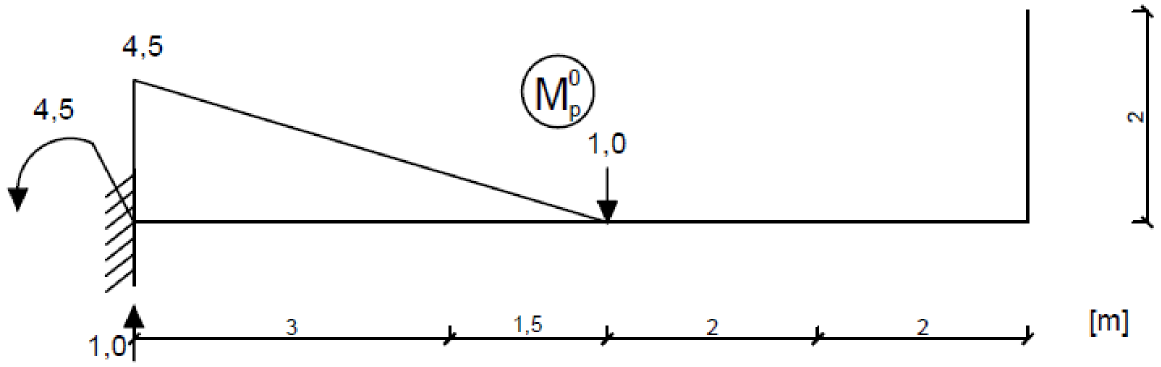
Wyznaczenie wykresu od siły P=1,0 po kierunku $q_2$ metodą sił:

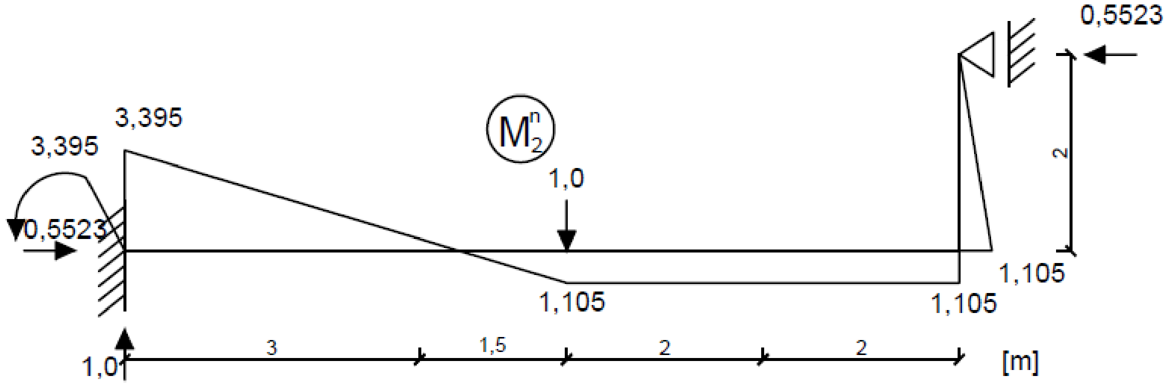
$$\delta_{11} = \frac{1}{EI}\ (2\ \cdot 8{,}5\ \cdot 2 + 2 \cdot 2\ \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 2 = \frac{110}{3}\ \frac{1}{EI}$$

$$\delta_{1p} = \frac{1}{EI}\left(-4{,}5\ \cdot \frac{1}{2} \cdot 4{,}5\ \cdot 2\right) = -20{,}25\ \frac{1}{EI}$$

$$X_2 = -\frac{\delta_{1p}}{\delta_{11}} = 20{,}25\ \cdot \frac{3}{110} = 0{,}5523$$



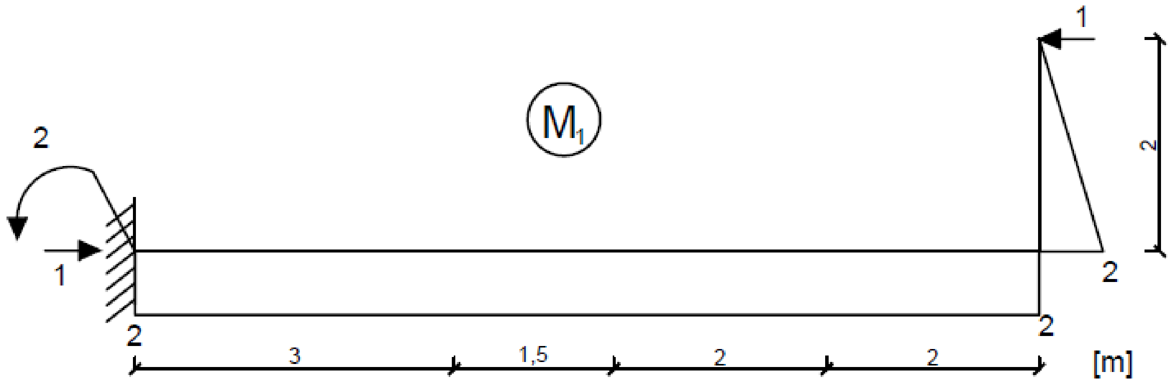



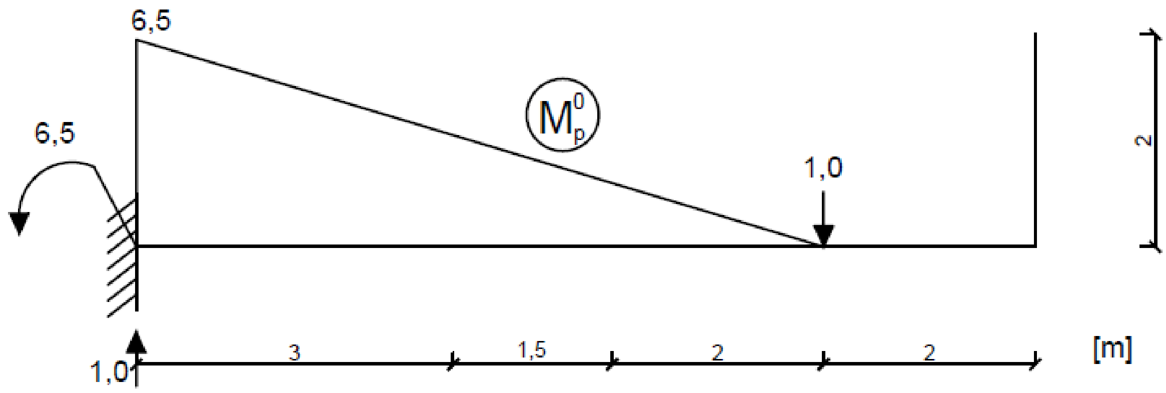
Wyznaczenie wykresu od Wyznaczenie wykresu od siły P=1,0 po kierunku $q_3$

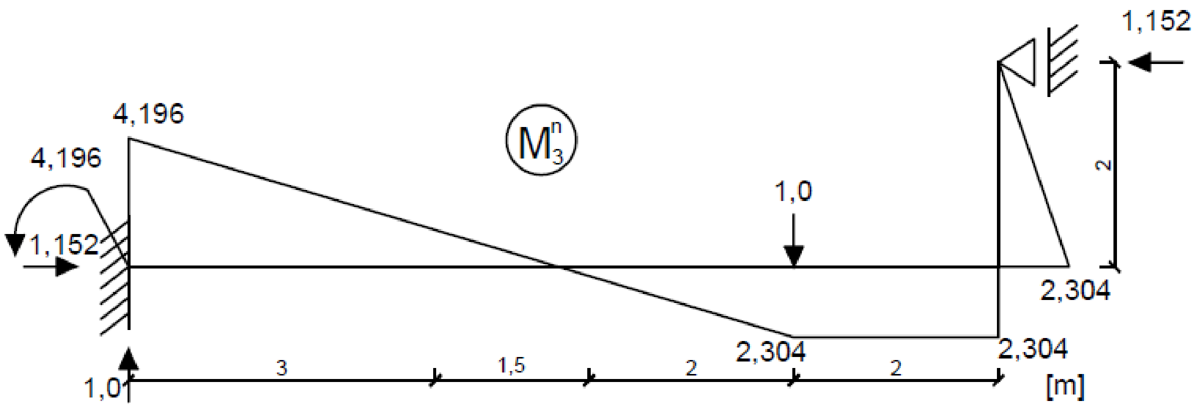
$\delta_{11} = \frac{1}{EI}\ (2\ \cdot 8{,}5\ \cdot 2 + 2 \cdot 2\ \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 2 = \frac{110}{3}\ \frac{1}{EI}$

$\delta_{1p} = \frac{1}{EI}\left(-6{,}5\ \cdot \frac{1}{2} \cdot 6{,}5\ \cdot 2\right) = -42{,}25\ \frac{1}{EI}$

$X_3 = -\frac{\delta_{1p}}{\delta_{11}} = 42{,}25\ \cdot \frac{3}{110} = 1{,}152$



Współczynniki $\delta_{ik}'$ z układu równań ruchu:

$$\delta_{ik}' = \sum \int M_i M_k dx$$

$\delta'_{11} = \int M_1^0 \cdot M_1^n \; dx = 2{,}51 \cdot 3 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 3 \; - \; 0{,}49 \cdot 3 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 3 \; = 6{,}795$

$\delta'_{12} = \int M_1^0 \cdot M_2^n \; dx = 3{,}396 \cdot 3 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 3 \; + \; 0{,}396 \cdot 3 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 3 \; = 10{,}782$

$\delta'_{22} = \int M_2^0 \cdot M_2^n \, dx = 3{,}396 \cdot 4{,}5 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 4{,}5 \; - \; 1{,}104 \cdot 4{,}5 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 4{,}5 \; = 19{,}197$

$\delta'_{23} = \int M_2^0 \cdot M_3^n \, dx = 4{,}196 \cdot 4{,}5 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 4{,}5 \; - \; 0{,}304 \cdot 4{,}5 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 4{,}5 \; = 27{,}297$

$\delta'_{33} = \int M_3^0 \cdot M_3^n \, dx = 4{,}196 \cdot 6{,}5 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 6{,}5 \; - \; 2{,}304 \cdot 6{,}5 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 6{,}5 \; = 42{,}87$

$\delta'_{13} = \int M_1^0 \cdot M_3^n \, dx = 4{,}196 \cdot 3 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 3 + \; 1{,}196 \cdot 3 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 3 \; = 14{,}382$

$$\begin{vmatrix} 1-16{,}988\lambda & -34{,}502\lambda & -57{,}528\lambda \\ -26{,}955\lambda & 1-61{,}430\lambda & -109{,}188\lambda \\ -35{,}955\lambda & -87{,}350\lambda & 1-171{,}48\lambda \end{vmatrix} = 0$$

$(1 - 16{,}988\lambda)(1 - 61{,}430\,\lambda)(1 - 171{,}48\,\lambda) - 135451\,\lambda^3 - 135450\,\lambda^3 - (1 - 61{,}430\,\lambda)2068{,}419\,\lambda^2 - (1-16{,}988\,\lambda)9537{,}572\,\lambda^2 - (1-171{,}48\,\lambda)930{,}001\,\lambda^2 = 0$

$(1 - 78{,}418\,\lambda + 1043{,}573\,\lambda^2)(1 - 171{,}48\,\lambda) - 135451\,\lambda^3 - 135450\,\lambda^3 - 2068{,}419\,\lambda^2 + 127063\,\lambda^3 - 9537{,}572\lambda^2 + 162024\,\lambda^3 - 930{,}001\,\lambda^2 + 159477\,\lambda^3 = 0$

$1 - 78{,}418\,\lambda + 1043{,}573\,\lambda^2 - 171{,}481\,\lambda + 13447{,}119\,\lambda^2 - 178951{,}898\,\lambda^3 - 135451\,\lambda^3 - 135450\,\lambda^3 - 2068{,}419\,\lambda^2 + 127063\,\lambda^3 - 9537{,}572\lambda^2 + 162024\,\lambda^3 - 930{,}001\,\lambda^2 + 159477\,\lambda^3 = 0$

$1 - 249{,}899\,\lambda + 1954{,}7\,\lambda^2 - 1288{,}898\,\lambda^3 = 0$

$\lambda_1 = 0{,}004135$

$\lambda_2 = 0{,}136353$

$\lambda_3 = 1{,}376079$

$$\omega = \sqrt{\frac{\lambda EI}{m}}$$

$$\omega_3 = \sqrt{\frac{1{,}376079 \cdot 6273000}{100}} = 293{,}805 \; \frac{rad}{s}$$

$$\omega_2 = \sqrt{\frac{0{,}136353 \cdot 6273000}{100}} = 92{,}485 \; \frac{rad}{s}$$

$$\omega_1 = \sqrt{\frac{0{,}004135 \cdot 6273000}{100}} = 16{,}106 \; \frac{rad}{s}$$

$$\begin{cases} A_1(1-16{,}988\lambda)-\ A_2 34{,}502\lambda-\ \ A_3 57{,}528\lambda=0 \\ -A_1 26{,}955\lambda\ +A_2(1-61{,}430\lambda)-\ \ A_3 109{,}188\lambda=0 \\ -A_1 35{,}955\lambda-\ -A_2 87{,}350\lambda+\ A_3(1-171{,}48\lambda)=0 \end{cases}$$

$$\begin{cases} A_1(1-16{,}988\lambda)-\ A_2 34{,}502\lambda-\ \ A_3 57{,}528\lambda=0 \\ -A_1 26{,}955\lambda\ +A_2(1-61{,}430\lambda)-\ \ A_3 109{,}188\lambda=0 \end{cases}$$

I postać drgań własnych:

$\lambda_1 = 0{,}004135$

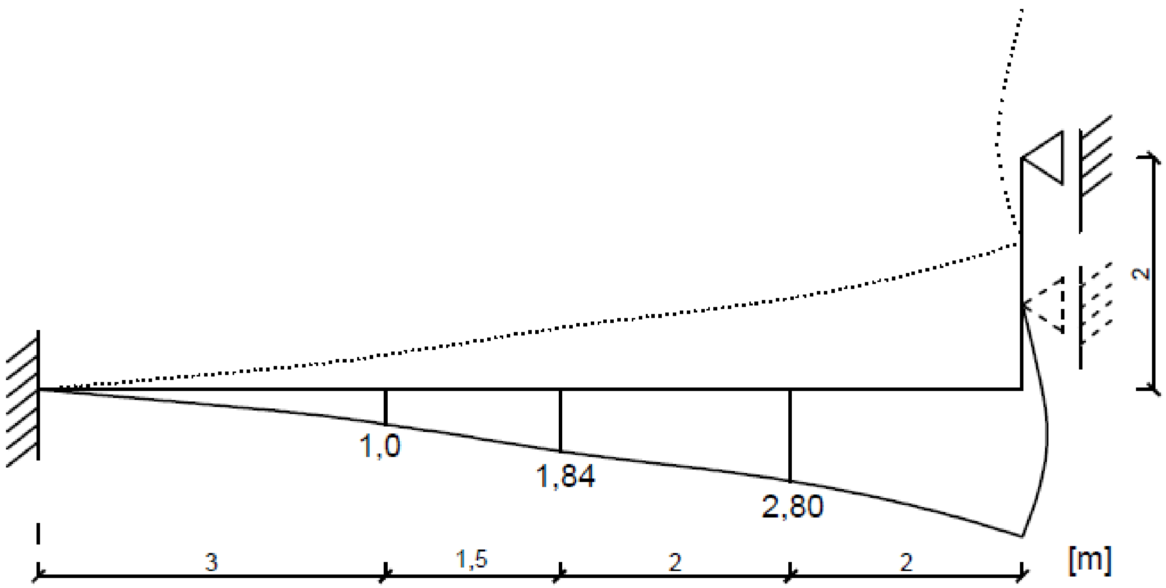
$A_1 = 1{,}0$

$$\begin{cases} 0{,}930-\ A_2 0{,}143-\ \ A_3 0{,}238=0 \\ -0{,}111+A_2 0{,}746-\ \ A_3 0{,}451=0 \end{cases}$$

$A_1 = 1{,}0$

$A_2 = 1{,}842$

$A_3 = 2{,}801$



II postać drgań własnych:

$\lambda_2$=0,136353

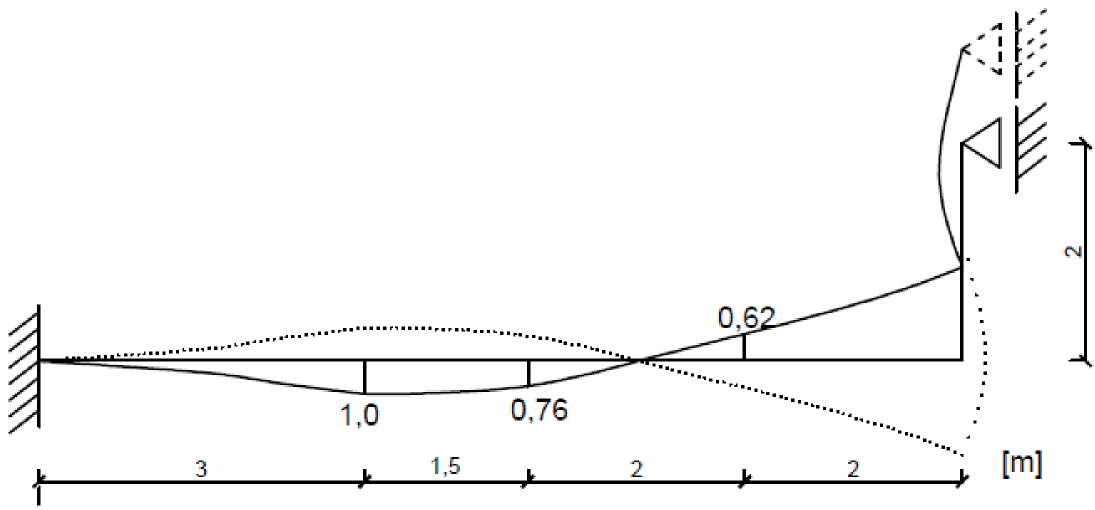
$A_1$ = 1,0

$$\begin{cases} -1{,}316 - A_2 4{,}704 - A_3 7{,}844 = 0 \\ -3{,}675 - A_2 7{,}376 - A_3 14{,}888 = 0 \end{cases}$$

$A_1$ = 1,0

$A_2$ = 0,7584

$A_3$ = - 0,6226



III postać drgań własnych:

$\lambda_3$ = 1,376079

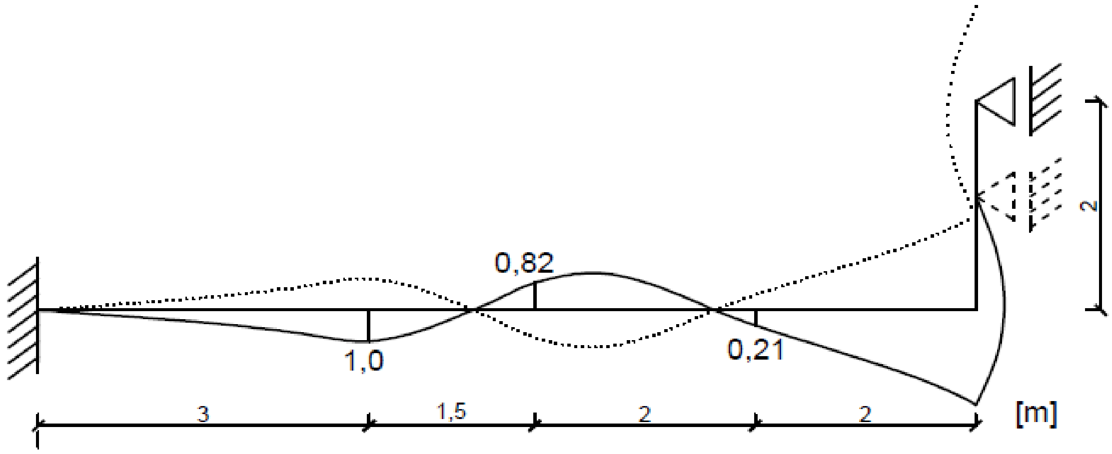
$A_1$ = 1,0

$$\begin{cases} -22,377 - A_2 47,477 - A_3 79,163 = 0 \\ -37,092 - A_2 83,533 - A_3 150,251 = 0 \end{cases}$$

$A_1$ = 1,0

$A_2$ = - 0,8178

$A_3$ = 0,2078




Sprawdzenie ortogonalności:

$\sum A_i^{II} \cdot m_i \cdot A_i^{I}$ = 1 · 2,5 · 1 + 0,7584· 3,2 · 1,842 + (- 0,6226) · 4 · 2,801 = -0,005 ≈0

$\sum A_i^{III} \cdot m_i \cdot A_i^{I}$ = 1 · 2,5 · 1 + (- 0,8178) · 3,2 · 1,842 + 0,2078 · 4 · 2,801 = 0,008 ≈0

$\sum A_i^{III} \cdot m_i \cdot A_i^{II}$ = 1 · 2,5 · 1 + (- 0,8178) · 3,2 · 0,7584 + 0,2078 · 4 · (- 0,6226) = -0,002≈ 0

3. Drgania wymuszone:

$$\begin{cases} A_1(1-2{,}5\delta_{11}\,p^2m) - A_2 3{,}2\delta_{12}\,p^2m - A_3 4\delta_{13}\,p^2m = P\delta_{12} \\ -A_1 2{,}5\delta_{21}\,p^2m + A_2(1-3{,}2\delta_{22}\,p^2m) - A_3 4\delta_{23}\,p^2m = P\delta_{22} \\ -A_1 2{,}5\delta_{31}\,p^2m - A_2 3{,}2\delta_{32}\,p^2m + A_3(1-4\delta_{33}\,p^2m) = P\delta_{32} \end{cases}$$

$$\frac{p^2}{EI_o} = \frac{65{,}97^2}{6273000}$$

$$\begin{cases} -A_1 0{,}1785 - A_2 2{,}3937 - A_3 3{,}9911 = 0{,}0438 \\ -A_1 1{,}8701 - A_2 3{,}2619 - A_3 7{,}5752 = 0{,}0780 \\ -A_1 2{,}4945 - A_2 6{,}0601 - A_3 10{,}8968 = 0{,}1743 \end{cases}$$

$A_1$ = - 0,039343 m

$A_2$ = - 0,051022 m

$A_3$ = 0,021386 m

4. Obwiednia momentów dynamicznych:

$B_1 = -m_1\ddot{q}_1 = -\mathrm{m}_1(-\mathrm{A}_1\mathrm{p}^2\sin(\mathrm{pt})) = m_1(\mathrm{A}_1\mathrm{p}^2\sin(\mathrm{pt})) = 250 \cdot (-0{,}039343) \cdot 65{,}97^2 \cdot \sin(pt)$

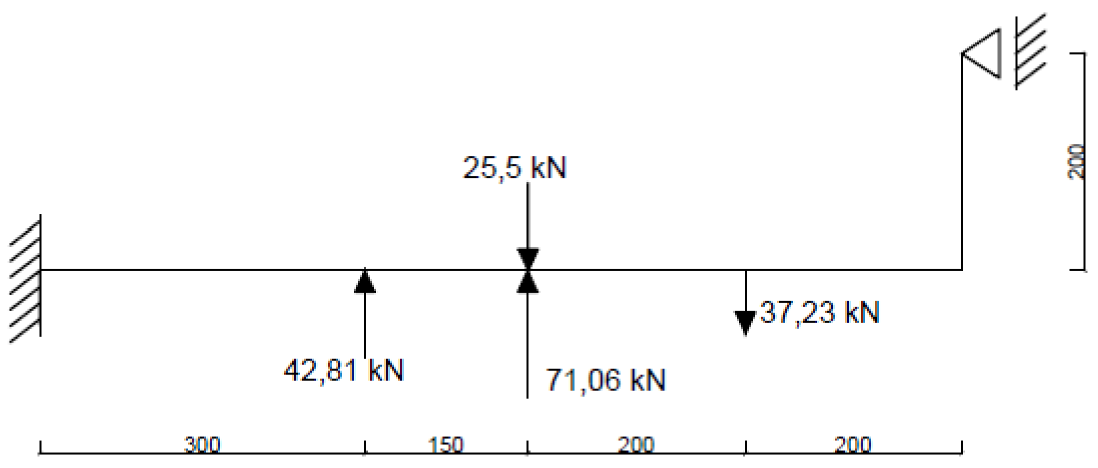
$dla\ \sin(pt) = 1$ $B_1$ = - 42805,58 N = - 42,81 kN
$dla\ \sin(pt) = -1$ $B_1$ = 42805,58 N = 42,81 kN

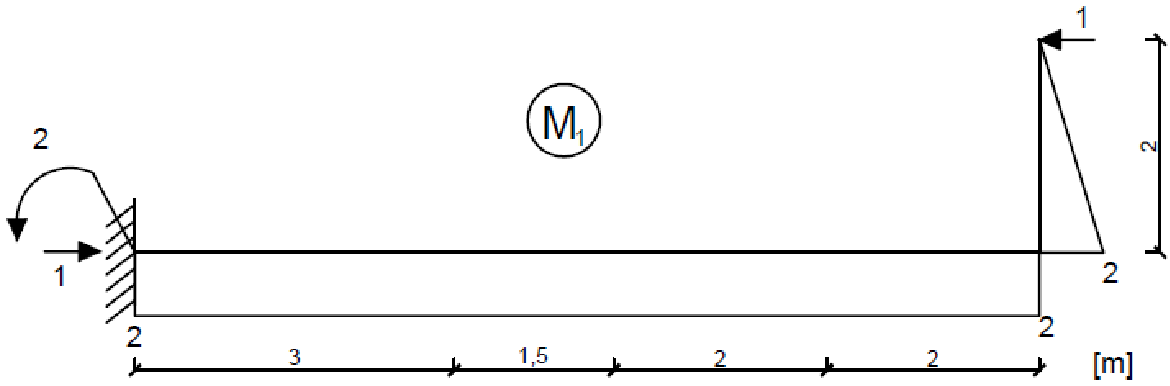
$B_2 = -m_2\ddot{q}_2 = -\mathrm{m}_2(-\mathrm{A}_2\mathrm{p}^2\sin(\mathrm{pt})) = m_2(\mathrm{A}_2\mathrm{p}^2\sin(\mathrm{pt})) = 320 \cdot (-0{,}051022) \cdot 65{,}97^2 \cdot \sin(pt)$

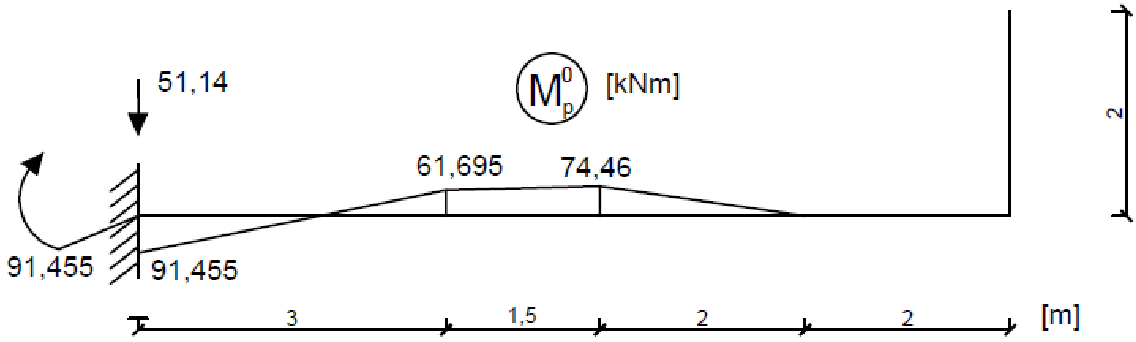
$dla\ \sin(pt) = 1$ $B_2$ = - 71055,95 N = - 71,06 kN
$dla\ \sin(pt) = -1$ $B_2$ = 71055,95 N = 71,06 kN

$B_3 = -m_3\ddot{q}_3 = -\mathrm{m}_3(-\mathrm{A}_3\mathrm{p}^2\sin(\mathrm{pt})) = m_3(\mathrm{A}_3\mathrm{p}^2\sin(\mathrm{pt})) = 400 \cdot 0{,}021386 \cdot 65{,}97^2 \cdot \sin(pt)$

$dla\ \sin(pt) = 1$ $B_3$ = 37229,10 N = 37,23 kN
$dla\ \sin(pt) = -1$ $B_3$ = - 37229,10 N = - 37,23 kN



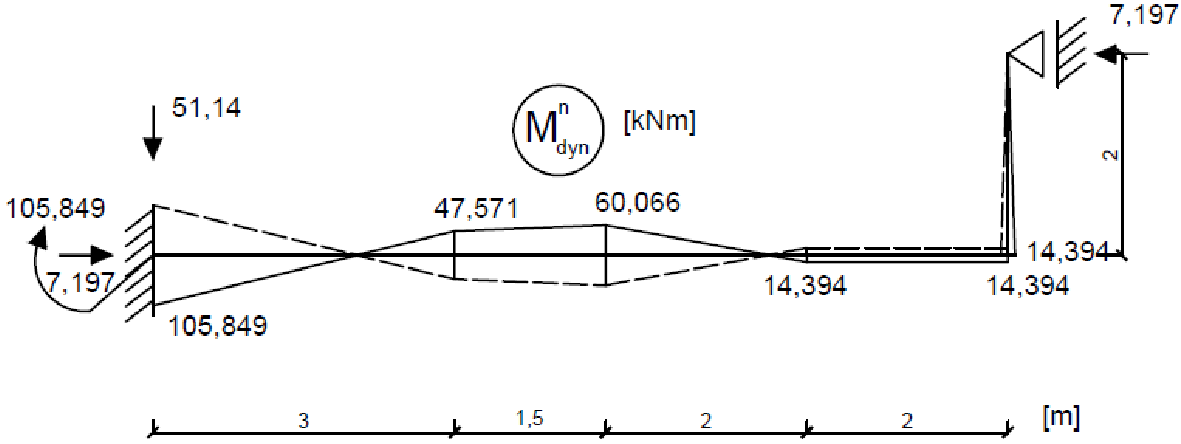


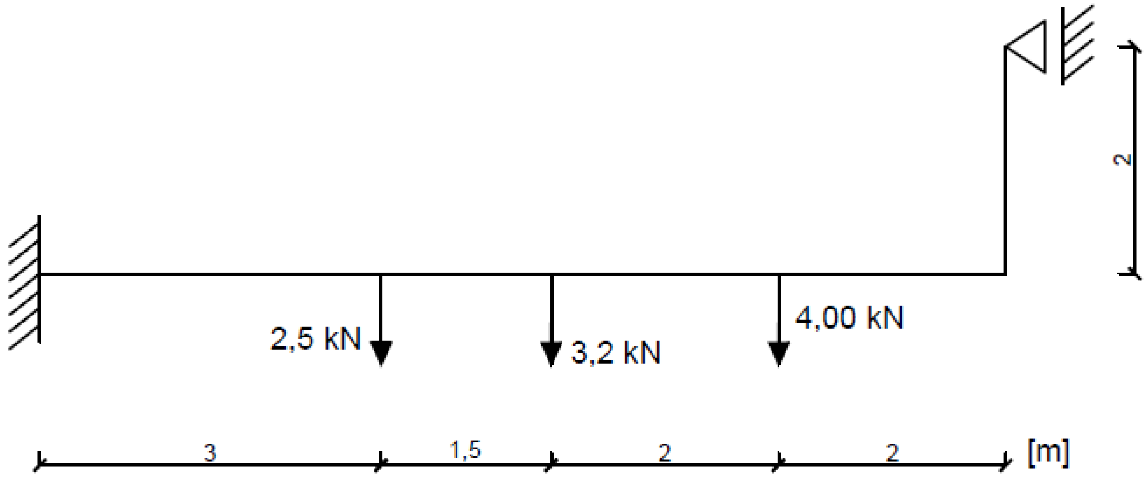

$$\delta_{11} = \frac{1}{EI}\left(2 \cdot 8{,}5 \cdot 2 + 2 \cdot 2 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 2 = \frac{110}{3}\,\frac{1}{EI}\right.$$

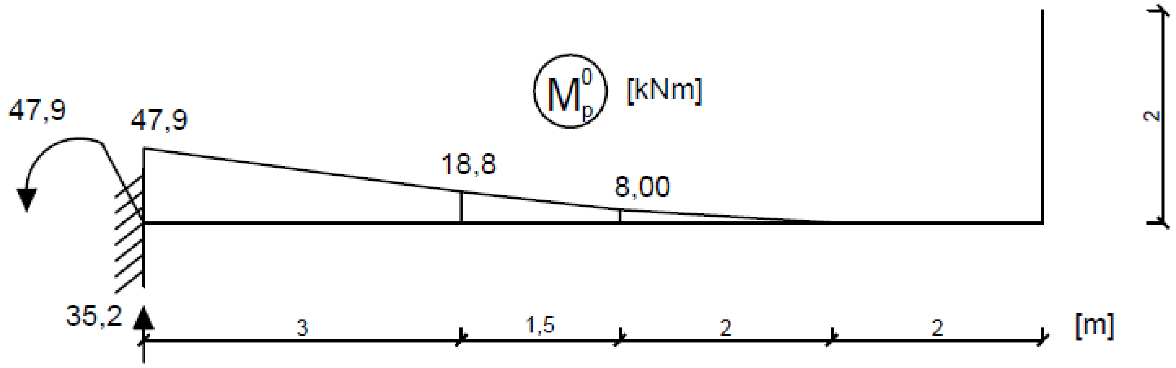
$$\delta_{1p} = \frac{1}{EI}\left(91{,}455 \cdot \frac{1}{2} \cdot 3 - 61{,}695 \cdot \frac{1}{2} \cdot 3 - 61{,}695 \cdot \frac{1}{2} \cdot 1{,}5 - 74{,}46 \cdot (1{,}5 + 2) \cdot \frac{1}{2}\right) \cdot 2\,] = -263{,}873\,\frac{1}{EI}$$

$$X = -\frac{\delta_{1p}}{\delta_{11}} = 263{,}873 \cdot \frac{3}{110} = 7{,}197\ kN$$



5. Rozkład momentów zginających of obciążenia statycznego (ciężar własny mas, ciężar własny prętów pominięto):

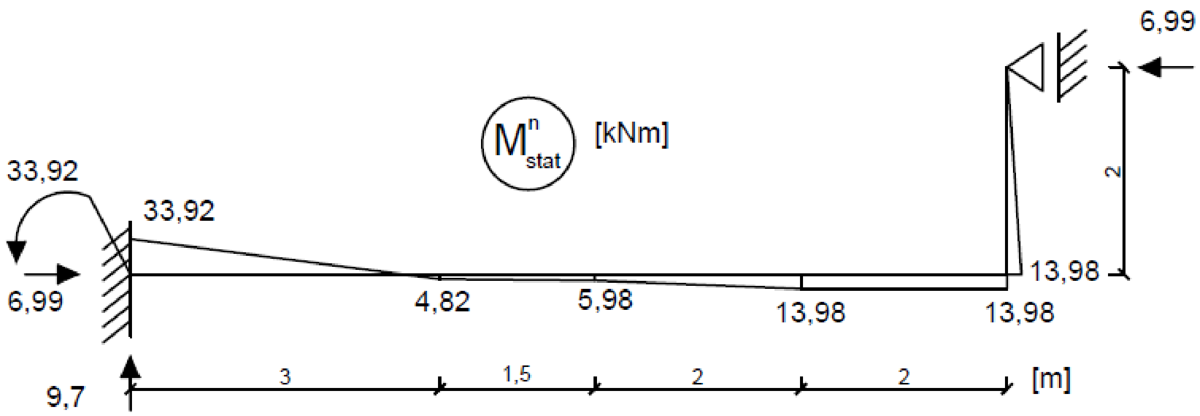



$\delta_{11} = \frac{1}{EI}\ (2\ \cdot 8{,}5\ \cdot 2 + 2 \cdot 2\ \cdot\ \frac{1}{2}\ \cdot\ \frac{2}{3}\ \cdot 2 = \frac{110}{3}\ \frac{1}{EI}$

$\delta_{1p} = \frac{1}{EI}\ [-\left(47{,}9\ \cdot\ \frac{1}{2}\ \cdot 3 + 18{,}8\ \cdot\ \frac{1}{2}\ \cdot 3 +\ 18{,}8\ \cdot\ \frac{1}{2}\ \cdot 1{,}5 + 8{,}0\ \cdot (1{,}5 + 2) \cdot\ \frac{1}{2}\right)]\ \cdot 2 =$
$-256{,}3\ \frac{1}{EI}$

$X = -\ \frac{\delta_{1p}}{\delta_{11}} =\ 256{,}3\ \ \cdot\ \frac{3}{110} = 6{,}99\ \mathrm{kN}$



6. Maksymalne naprężenia normalne:

$$\sigma = 1,2 \cdot \frac{M_{stat}}{W} + 5,0\ \frac{M_{dyn}}{W} = 1,2 \cdot \frac{3392}{278} + 5,0 \cdot \frac{10584,9}{278} = 205,12\ \frac{kN}{cm^2}$$

$$\sigma_{dop} = 21,5\ \frac{kN}{cm^2} \quad < \quad \sigma = 205,12\ \frac{kN}{cm^2}$$

Przekrój nie spełnia warunku nośności.