POLITECHNIKA POZNAŃSKA
WYDZIAŁ BUDOWNICTWA I INŻYNIERII ŚRODOWISKA
INSTYTUT KONSTRUKCJI *BUDOWLANYCH*
ZAKŁAD MECHANIKI BUDOWLI

Projekt nr 4

Dynamika – ujęcie klasyczne

Konrad Kaczmarek
Albert Kawałek
Grupa B3
Semestr IV
Rok akademicki 2013/2014

Dla podanego układu należy:
1. *Obliczyć częstości i postacie drgań własnych*
2. *Obliczyć amplitudy drgań wymuszonych, siły bezwładności, obwiednię dynamicznych momentów zginających i sprawdzić maksymalne naprężenia normalne uwzględniając także obciążenie statyczne ciężarami mas.*

$m_1 = 325\ kg$

$m_2 = 400\ kg$

$m_3 = 560\ kg$

$P - \text{amplituda siły wymuszającej} \rightarrow P = 26{,}5\ kN$

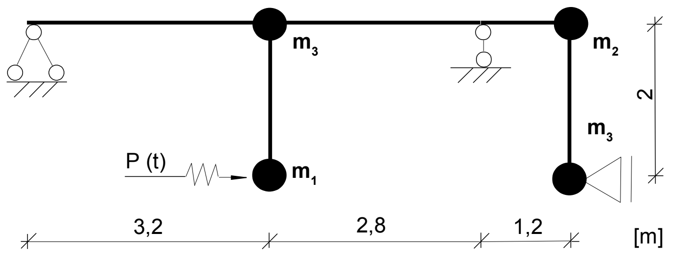
$p - \text{częstotliwość siły wymuszającej} \rightarrow p = 20{,}6\ Hz$

I220

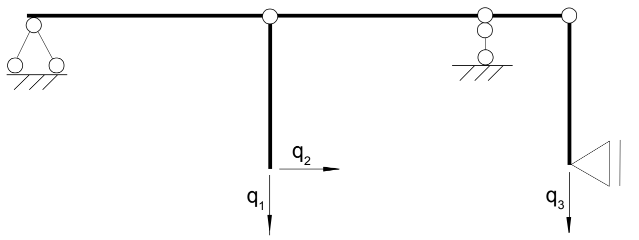
$I_y = 3060\ cm^4$

$W_y = 278\ cm^3$

$EI = 6273\ kNm^2 = 6273000\ Nm^2$

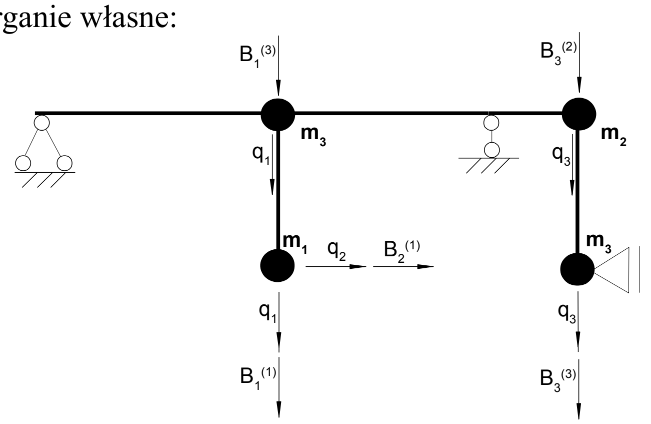


Łańcuch kinematyczny:



$SSD = 3$

Drganie własne:



$$B_1^{(3)} = -m_3 \cdot \ddot{q}_1$$
$$B_1^{(1)} = -m_1 \cdot \ddot{q}_1$$
$$B_2^{(1)} = -m_1 \cdot \ddot{q}_2$$
$$B_3^{(2)} = -m_2 \cdot \ddot{q}_3$$
$$B_3^{(3)} = -m_3 \cdot \ddot{q}_3$$

$$B_1 = B_1^{(3)} + B_1^{(1)}$$
$$B_2 = B_2^{(1)}$$
$$B_3 = B_3^{(2)} + B_3^{(3)}$$
$$B_1 = -(m_3 + m_1) \cdot \ddot{q}_1$$
$$B_2 = -m_1 \cdot \ddot{q}_2$$
$$B_3 = -(m_2 + m_3) \cdot \ddot{q}_3$$

$$q_1 = \delta_{11} \cdot B_1^{(3)} + \delta_{11} \cdot B_1^{(1)} + \delta_{12} \cdot B_2^{(1)} + \delta_{13} \cdot B_3^{(2)} + \delta_{13} \cdot B_3^{(3)}$$
$$q_2 = \delta_{21} \cdot B_1^{(3)} + \delta_{21} \cdot B_1^{(1)} + \delta_{22} \cdot B_2^{(1)} + \delta_{23} \cdot B_3^{(2)} + \delta_{23} \cdot B_3^{(3)}$$
$$q_3 = \delta_{31} \cdot B_1^{(3)} + \delta_{31} \cdot B_1^{(1)} + \delta_{32} \cdot B_2^{(1)} + \delta_{33} \cdot B_3^{(2)} + \delta_{33} \cdot B_3^{(3)}$$

$$q_1 = \delta_{11} \cdot B_1 + \delta_{12} \cdot B_2 + \delta_{13} \cdot B_3$$
$$q_2 = \delta_{21} \cdot B_1 + \delta_{22} \cdot B_2 + \delta_{23} \cdot B_3$$
$$q_3 = \delta_{31} \cdot B_1 + \delta_{32} \cdot B_2 + \delta_{33} \cdot B_3$$

$$q_1+\delta_{11}\cdot(m_3+m_1)\cdot\ddot{q}_1+\delta_{12}\cdot m_1\cdot\ddot{q}_2+\delta_{13}\cdot(m_2+m_3)\cdot\ddot{q}_3=0$$
$$q_2+\delta_{21}\cdot(m_3+m_1)\cdot\ddot{q}_1+\delta_{22}\cdot m_1\cdot\ddot{q}_2+\delta_{23}\cdot(m_2+m_3)\cdot\ddot{q}_3=0$$
$$q_3+\delta_{31}\cdot(m_3+m_1)\cdot\ddot{q}_1+\delta_{32}\cdot m_1\cdot\ddot{q}_2+\delta_{33}\cdot(m_2+m_3)\cdot\ddot{q}_3=0$$

$$q_1=A_1\cdot\cos\omega t \quad\rightarrow\quad \ddot{q}_1=-A_1\omega^2\cos\omega t$$
$$q_2=A_2\cdot\cos\omega t \quad\rightarrow\quad \ddot{q}_2=-A_2\omega^2\cos\omega t$$
$$q_3=A_3\cdot\cos\omega t \quad\rightarrow\quad \ddot{q}_3=-A_3\omega^2\cos\omega t$$

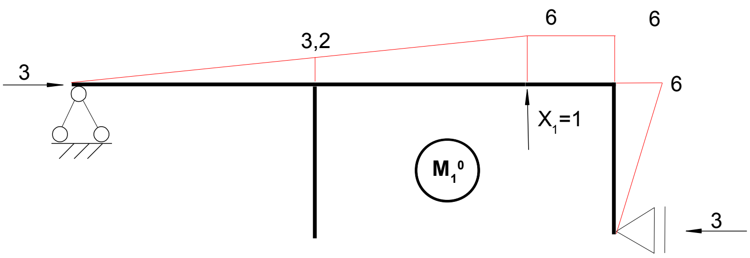
$$A_1\cos\omega t-\delta_{11}(m_3+m_1)A_1\omega^2\cos\omega t-\delta_{12}m_1A_2\omega^2\cos\omega t-\delta_{13}(m_2+m_3)A_3\omega^2\cos\omega t=0$$
$$A_2\cos\omega t-\delta_{21}(m_3+m_1)\cdot A_1\omega^2\cos\omega t-\delta_{22}m_1A_2\omega^2\cos\omega t-\delta_{23}(m_2+m_3)A_3\omega^2\cos\omega t=0$$
$$A_3\cos\omega t-\delta_{31}(m_3+m_1)\cdot A_1\omega^2\cos\omega t-\delta_{32}m_1A_2\omega^2\cos\omega t-\delta_{33}(m_2+m_3)A_3\omega^2\cos\omega t=0$$

$$A_1(1-\delta_{11}(m_3+m_1)\omega^2)-A_2\delta_{12}m_1\omega^2-A_3\delta_{13}(m_2+m_3)\omega^2=0$$
$$-A_1\delta_{21}(m_3+m_1)\omega^2+A_2(1-\delta_{22}m_1\omega^2)-A_3\delta_{23}(m_2+m_3)\omega^2=0$$
$$-A_1\delta_{31}(m_3+m_1)\omega^2-A_2\delta_{32}m_1\omega^2+A_3(1-\delta_{33}(m_2+m_3)\omega^2)=0$$

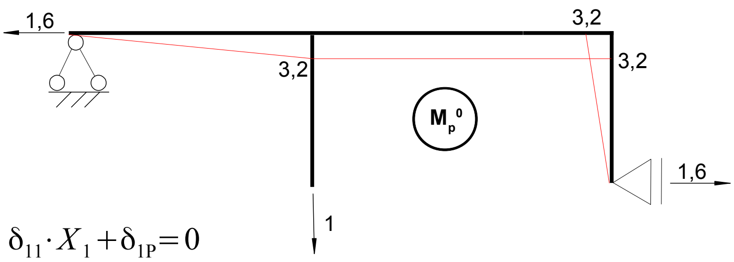
Wyznaczenie rozkładu momentów zginających M_i metodą sił:

$$\delta_{ik}=\sum\int\frac{M_i\cdot M_k}{EI}dx\text{, gdzie } M_i, M_k-\text{wykresy momentów zginających od siły jednostkowej po kierunku ,,}i\text{'' i ,,}k\text{''.}$$

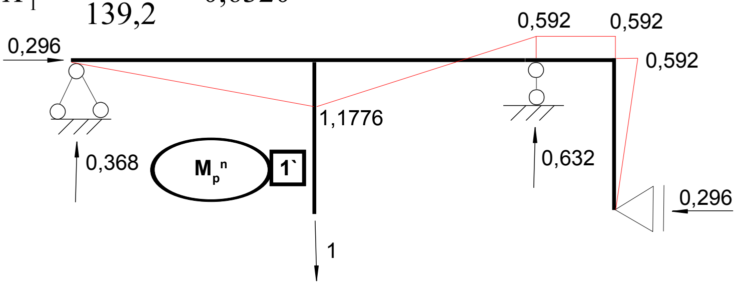
Stan X1:



Siła jednostkowa po kierunku q_1:

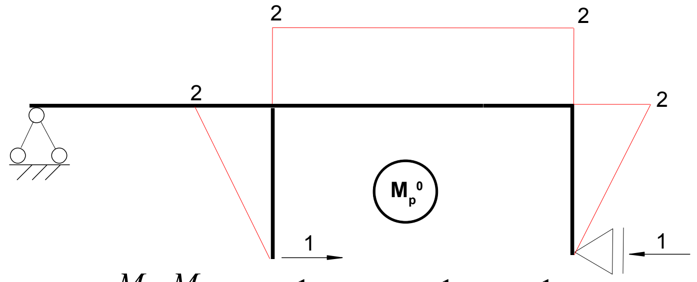


$$\delta_{11}\cdot X_1+\delta_{1P}=0$$
$$X_1=-\frac{\delta_{1P}}{\delta_{11}}$$
$$\delta_{11}=\sum\int\frac{M_1^2}{EI}dx=\frac{1}{EI}\cdot(\frac{1}{2}\cdot6\cdot6\cdot\frac{2}{3}\cdot6+6\cdot1{,}2\cdot6+\frac{1}{2}\cdot2\cdot6\cdot\frac{2}{3}\cdot6)=\frac{139{,}2}{EI}$$
$$\delta_{1P}=\sum\int\frac{M_1\cdot M_P}{EI}dx=\frac{1}{EI}\cdot(-\frac{1}{2}\cdot3{,}2\cdot3{,}2\cdot\frac{2}{3}\cdot3{,}2-3{,}2\cdot2{,}8\cdot(\frac{1}{2}\cdot3{,}2+\frac{1}{2}\cdot6)-6\cdot1{,}2\cdot3{,}2-\frac{1}{2}\cdot2\cdot6\cdot\frac{2}{3}\cdot3{,}2)=-\frac{87{,}9787}{EI}$$
$$X_1=\frac{87{,}9787}{139{,}2}=0{,}6320$$



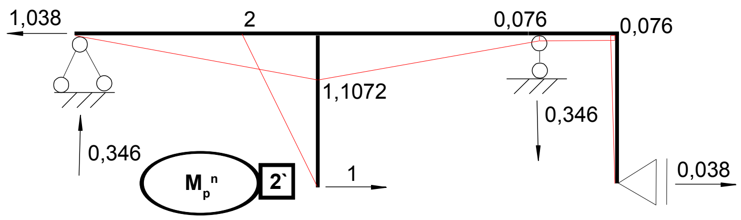
Siła jednostkowa po kierunku q_2:

$$\delta_{11}=\frac{139,2}{EI}$$

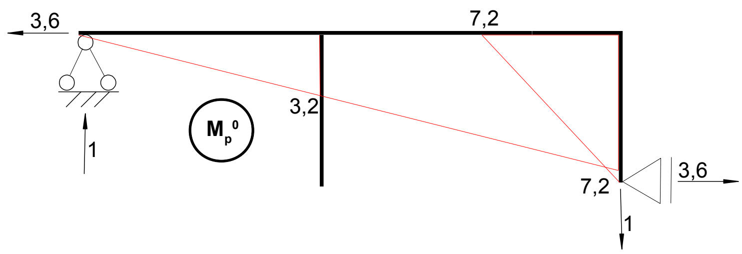


$$\delta_{1P}=\sum\int\frac{M_1\cdot M_P}{EI}dx=\frac{1}{EI}\cdot(2\cdot 2,8\cdot(\frac{1}{2}\cdot 3,2+\frac{1}{2}\cdot 6)+2\cdot 1,2\cdot 6+\frac{1}{2}\cdot 6\cdot 2\cdot\frac{2}{3}\cdot 2)=\frac{48,16}{EI}$$

$$X_1=-\frac{\delta_{1P}}{\delta_{11}}=-\frac{48,16}{139,2}=-0,3460$$

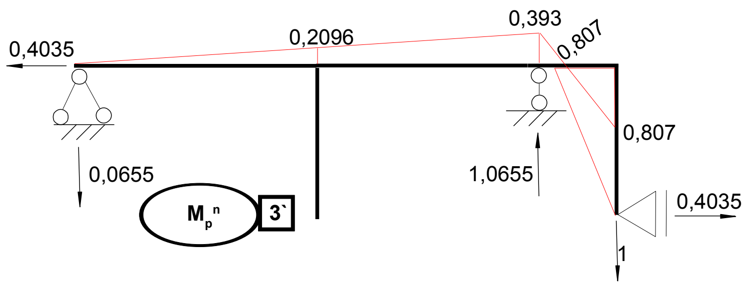


Siła jednostkowa po kierunku q_3:



$$\delta_{1P}=\sum\int\frac{M_1\cdot M_P}{EI}dx=\frac{1}{EI}\cdot(-\frac{1}{2}\cdot 6\cdot 6\cdot\frac{2}{3}\cdot 6-1,2\cdot 6\cdot(\frac{1}{2}\cdot 6+\frac{1}{2}\cdot 7,2)-\frac{1}{2}\cdot 7,2\cdot 2\cdot\frac{2}{3}\cdot 6)=-\frac{148,32}{EI}$$

$$X_1=-\frac{\delta_{1P}}{\delta_{11}}=\frac{148,32}{139,2}=1,0655$$



Przyjmuję masę porównawczą:

$M=5\ kg$

$m_1=65\,M$

$m_2=80\,M$

$m_3=112\,M$

$$A_1(1-\delta_{11}(112\,M+65\,M)\omega^2)-A_2\delta_{12}65\mathrm{M}\cdot\omega^2-A_3\delta_{13}(80\mathrm{M}+112\mathrm{M})\omega^2=0$$
$$-A_1\delta_{21}(112\mathrm{M}+65\mathrm{M})\omega^2+A_2(1-\delta_{22}65\mathrm{M}\,\omega^2)-A_3\delta_{23}(80\mathrm{M}+112\mathrm{M})\omega^2=0$$
$$-A_1\delta_{31}(112\mathrm{M}+65\mathrm{M})\omega^2-A_2\delta_{32}65\mathrm{M}\,\omega^2+A_3(1-\delta_{33}(80\mathrm{M}+112\mathrm{M})\omega^2)=0$$

$$A_1(1-\delta_{11}177\mathrm{M}\,\omega^2)-A_2\delta_{12}65\mathrm{M}\,\omega^2-A_3\delta_{13}192\mathrm{M}\,\omega^2=0$$
$$-A_1\delta_{21}177\mathrm{M}\,\omega^2+A_2(1-\delta_{22}65\mathrm{M}\,\omega^2)-A_3\delta_{23}192\,M\,\omega^2=0$$
$$-A_1\delta_{31}177\mathrm{M}\,\omega^2-A_2\delta_{32}65\,M\,\omega^2+A_3(1-\delta_{33}192\,M\,\omega^2)=0$$

Obliczenie współczynników macierzy podatności:

$$\delta_{11}'=\sum\int M_1^2dx=\frac{1}{2}\cdot 1{,}1776\cdot 3{,}2\cdot\frac{2}{3}\cdot 1{,}1776+\frac{1}{2}\cdot 1{,}1776\cdot 2{,}8\cdot(\frac{2}{3}\cdot 1{,}1776-\frac{1}{3}\cdot 0{,}592)$$
$$+\frac{1}{2}\cdot 2{,}8\cdot 0{,}592\cdot(\frac{2}{3}\cdot 0{,}592-\frac{1}{3}\cdot 1{,}1776)+1{,}2\cdot 0{,}592\cdot 0{,}592+\frac{1}{2}\cdot 0{,}592\cdot 2\cdot\frac{2}{3}\cdot 0{,}592=3{,}1041$$

$$\delta_{22}'=\sum\int M_2^2dx=\frac{1}{2}\cdot 3{,}2\cdot 1{,}1072\cdot\frac{2}{3}\cdot 1{,}1072+\frac{1}{2}\cdot 2\cdot 2\cdot\frac{2}{3}\cdot 2+\frac{1}{2}\cdot 2{,}8\cdot 1{,}1072\cdot(\frac{2}{3}\cdot 1{,}1072+\frac{1}{3}\cdot 0{,}076)$$
$$+\frac{1}{2}\cdot 2{,}8\cdot 0{,}076\cdot(\frac{2}{3}\cdot 0{,}076+\frac{1}{3}\cdot 1{,}1072)+0{,}076\cdot 1{,}2\cdot 0{,}076+\frac{1}{2}\cdot 0{,}076\cdot 2\cdot\frac{2}{3}\cdot 0{,}076=5{,}2131$$

$$\delta_{33}'=\sum\int M_3^2dx=\frac{1}{2}\cdot 0{,}393\cdot 6\cdot\frac{2}{3}\cdot 0{,}393+\frac{1}{2}\cdot 0{,}393\cdot 1{,}2\cdot(\frac{2}{3}\cdot 0{,}393-\frac{1}{3}\cdot 0{,}807)$$
$$+\frac{1}{2}\cdot 1{,}2\cdot 0{,}807\cdot(\frac{2}{3}\cdot 0{,}807-\frac{1}{3}\cdot 0{,}393)+\frac{1}{2}\cdot 0{,}807\cdot 2\cdot\frac{2}{3}\cdot 0{,}807=0{,}9385$$

$$\delta_{12}'=\sum\int M_1\cdot M_2dx=\frac{1}{2}\cdot 1{,}1776\cdot 3{,}2\cdot\frac{2}{3}\cdot 1{,}1072+\frac{1}{2}\cdot 1{,}1776\cdot 2{,}8\cdot(\frac{2}{3}\cdot 1{,}1072+\frac{1}{3}\cdot 0{,}076)$$
$$-\frac{1}{2}\cdot 0{,}592\cdot 2{,}8\cdot(\frac{2}{3}\cdot 0{,}076+\frac{1}{3}\cdot 1{,}1072)-0{,}592\cdot 1{,}2\cdot 0{,}076-\frac{1}{2}\cdot 0{,}076\cdot 2\cdot\frac{2}{3}\cdot 0{,}592=2{,}2176$$

$$\delta_{13}'=\sum\int M_1\cdot M_3dx=-\frac{1}{2}\cdot 1{,}1776\cdot 3{,}2\cdot\frac{2}{3}\cdot 0{,}2096-1{,}1776\cdot\frac{1}{2}\cdot 2{,}8\cdot(\frac{2}{3}\cdot 0{,}2096+\frac{1}{3}\cdot 0{,}393)$$
$$+\frac{1}{2}\cdot 2{,}8\cdot 0{,}592\cdot(\frac{2}{3}\cdot 0{,}393+\frac{1}{3}\cdot 0{,}2096)+1{,}2\cdot 0{,}592\cdot(\frac{1}{2}\cdot 0{,}393-\frac{1}{2}\cdot 0{,}807)-\frac{1}{2}\cdot 2\cdot 0{,}592\cdot\frac{2}{3}\cdot 0{,}807=-0{,}9001$$

$$\delta_{23}'=\sum\int M_2\cdot M_3dx=-\frac{1}{2}\cdot 1{,}1072\cdot 3{,}2\cdot\frac{2}{3}\cdot 0{,}2096-\frac{1}{2}\cdot 1{,}1072\cdot 2{,}8\cdot(\frac{2}{3}\cdot 0{,}2096+\frac{1}{3}\cdot 0{,}393)$$
$$-\frac{1}{2}\cdot 0{,}076\cdot 2{,}8\cdot(\frac{2}{3}\cdot 0{,}393+\frac{1}{3}\cdot 0{,}2096)+1{,}2\cdot 0{,}076\cdot(-\frac{1}{2}\cdot 0{,}393+\frac{1}{2}\cdot 0{,}807)+\frac{1}{2}\cdot 2\cdot 0{,}076\cdot\frac{2}{3}\cdot 0{,}807=-0{,}6427$$

$\delta_{11}'=3{,}1041$
$\delta_{22}'=5{,}2131$
$\delta_{33}'=0{,}9385$
$\delta_{12}'=2{,}2176$
$\delta_{13}'=-0{,}9001$
$\delta_{23}'=-0{,}6427$

Podstawiamy:

$$\lambda=\frac{M\cdot\omega^2}{EI}$$

$$A_1(1-3{,}1041\cdot 177\lambda)-A_2 2{,}2176\cdot 65\lambda+A_3 0{,}9001\cdot 192\lambda=0$$
$$-A_1 2{,}2176\cdot 177\lambda+A_2(1-5{,}2131\cdot 65\lambda)+A_3 0{,}6427\cdot 192\lambda=0$$
$$+A_1 0{,}9001\cdot 177\lambda+A_2 0{,}6427\cdot 65\lambda+A_3(1-180{,}192\cdot 192\lambda)=0$$

$$A_1(1-549{,}4257\lambda)-A_2\cdot 144{,}144\lambda+A_3\cdot 172{,}3984\lambda=0$$
$$-A_1\cdot 392{,}5152\lambda+A_2(1-338{,}8515\lambda)+A_3\cdot 123{,}3984\lambda=0$$
$$A_1\cdot 159{,}3177\lambda+A_2 41{,}7755\lambda+A_3\cdot(1-180{,}1920\lambda)=0$$

$$\begin{vmatrix} 1-549{,}4257\lambda & -144{,}144\lambda & 172{,}3984\lambda \\ -392{,}5152\lambda & 1-338{,}8515\lambda & 123{,}3984\lambda \\ 159{,}3177\lambda & 41{,}7755\lambda & 1-180{,}1920\lambda \end{vmatrix}=0$$

$$(1-549{,}4257\lambda)\cdot(1-338{,}8515\lambda)\cdot(1-180{,}1920\lambda)-144{,}144\lambda\cdot 123{,}3984\lambda\cdot 159{,}3177\lambda+$$
$$-392{,}5152\lambda\cdot 41{,}7755\lambda\cdot 172{,}3984\lambda-172{,}3984\lambda\cdot(1-338{,}8515\lambda)\cdot 159{,}3177\lambda+$$
$$-123{,}3984\lambda\cdot 41{,}7755\lambda\cdot(1-549{,}4257\lambda)-(1-180{,}1920\lambda)\cdot(-144{,}144\lambda)\cdot(-392{,}5152\lambda)=0$$

Otrzymane z równania lambdy to:

$$\lambda_1=0{,}0013171$$
$$\lambda_2=0{,}0051070$$
$$\lambda_3=0{,}0088193$$

Podstawiono lambdy do wzoru:

$$\omega=\sqrt{\frac{\lambda EI}{M}}$$

$$\omega_1=\sqrt{\frac{0{,}0013171\cdot 6273000}{5}}=40{,}65\frac{rad}{s}$$

$$\omega_2=\sqrt{\frac{0{,}0051070\cdot 6273000}{5}}=80{,}05\frac{rad}{s}$$

$$\omega_3=\sqrt{\frac{0{,}0088193\cdot 6273000}{5}}=105{,}19\frac{rad}{s}$$

I postać drgań własnych:

$Dla: A_1 = 1\ ;\ \lambda = \lambda_1$

$$A_1(1-3{,}1041\cdot 177\lambda) - A_2 2{,}2176\cdot 65\lambda + A_3 0{,}9001\cdot 192\lambda = 0$$
$$-A_1\cdot 2{,}2176\cdot 177\lambda + A_2(1-5{,}2131\cdot 65\lambda) + A_3 0{,}6427\cdot 192\lambda = 0$$

$$(1-549{,}4257\cdot 0{,}0013171) - A_2\cdot 144{,}144\cdot 0{,}0013171 + A_3\cdot 172{,}3984\cdot 0{,}0013171 = 0$$
$$-392{,}5152\cdot 0{,}0013171 + A_2(1-338{,}8515\cdot 0{,}0013171) + A_3\cdot 123{,}3984\cdot 0{,}0013171 = 0$$

$A_1 = 1$
$A_2 = 1{,}0363$
$A_3 = -0{,}3497$

II postać drgań własnych:

$Dla: A_1 = 1\ ;\ \lambda = \lambda_2$

$$A_1(1-3{,}1041\cdot 177\lambda) - A_2 2{,}2176\cdot 65\lambda + A_3 0{,}9001\cdot 192\lambda = 0$$
$$-A_1\cdot 2{,}2176\cdot 177\lambda + A_2(1-5{,}2131\cdot 65\lambda) + A_3 0{,}6427\cdot 192\lambda = 0$$

$$(1-549{,}4257\cdot 0{,}005107) - A_2\cdot 144{,}144\cdot 0{,}005107 + A_3\cdot 172{,}3984\cdot 0{,}005107 = 0$$
$$-392{,}5152\cdot 0{,}005107 + A_2(1-338{,}8515\cdot 0{,}005107) + A_3\cdot 123{,}3984\cdot 0{,}005107 = 0$$

$A_1 = 1$
$A_2 = -3{,}4902$
$A_3 = -0{,}8649$

III postać drgań własnych:

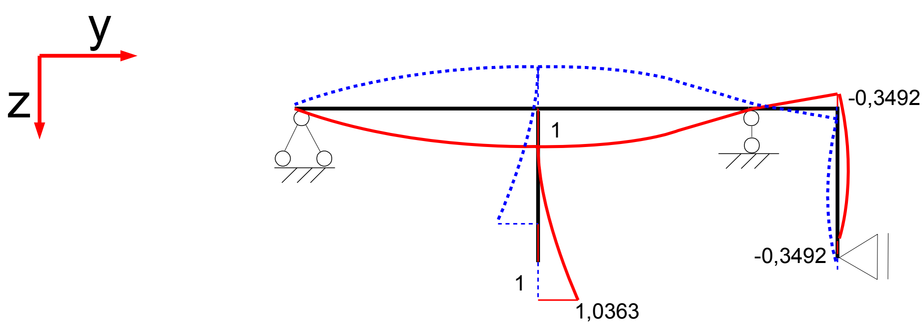
$Przyjęto: A_1 = 1\ ;\ \lambda = \lambda_3$

$$A_1(1-3{,}1041\cdot 177\lambda) - A_2 2{,}2176\cdot 65\lambda + A_3 0{,}9001\cdot 192\lambda = 0$$
$$-A_1\cdot 2{,}2176\cdot 177\lambda + A_2(1-5{,}2131\cdot 65\lambda) + A_3 0{,}6427\cdot 192\lambda = 0$$

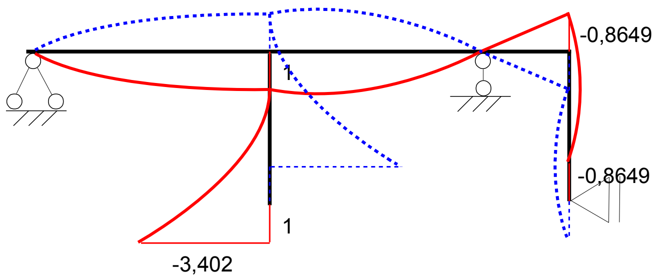
$$(1-549{,}4257\cdot 0{,}0088193) - A_2\cdot 144{,}144\cdot 0{,}0088193 + A_3\cdot 172{,}3984\cdot 0{,}0088193 = 0$$
$$-392{,}5152\cdot 0{,}0088193 + A_2(1-338{,}8515\cdot 0{,}0088193) + A_3\cdot 123{,}3984\cdot 0{,}0088193 = 0$$

$A_1 = 1$
$A_2 = -0{,}6624$
$A_3 = 1{,}9706$

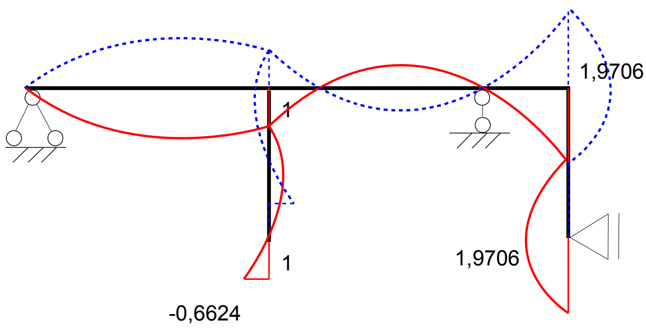
Postacie drgań własnych



I postać drgań własnych

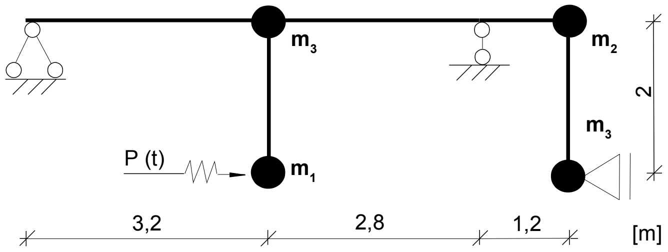


II postać drgań własnych



III postać drgań własnych

Obliczenie amplitud drgań wymuszonych:



$P(t) = Pcospt$
$m_1 = 325\ kg$
$m_2 = 400\ kg$
$m_3 = 560\ kg$
$P -$ *amplituda siły wymuszającej* $\rightarrow P = 26{,}5\ kN$
$p -$ *częstotliwość siły wymuszającej* $\rightarrow p = 20{,}6\ Hz$
$EI = 6273\ kNm^2 = 6273000\ Nm^2$
$p = 20{,}6 \cdot 2 \cdot \pi = 129{,}43\ \frac{rad}{s}$

Układ sił $\rightarrow\ B_i + P(t)$

$$q_1 = \delta_{11} \cdot B_1 + \delta_{12} \cdot B_2 + \delta_{13} \cdot B_3 + \delta_{12} \cdot P(t)$$
$$q_2 = \delta_{21} \cdot B_1 + \delta_{22} \cdot B_2 + \delta_{23} \cdot B_3 + \delta_{22} \cdot P(t)$$
$$q_3 = \delta_{31} \cdot B_1 + \delta_{32} \cdot B_2 + \delta_{33} \cdot B_3 + \delta_{32} \cdot P(t)$$

$$q_1 + \delta_{11} \cdot (m_3 + m_1) \cdot \ddot{q}_1 + \delta_{12} \cdot m_1 \cdot \ddot{q}_2 + \delta_{13} \cdot (m_2 + m_3) \cdot \ddot{q}_3 = \delta_{12} \cdot P(t)$$
$$q_2 + \delta_{21} \cdot (m_3 + m_1) \cdot \ddot{q}_1 + \delta_{22} \cdot m_1 \cdot \ddot{q}_2 + \delta_{23} \cdot (m_2 + m_3) \cdot \ddot{q}_3 = \delta_{22} \cdot P(t)$$
$$q_3 + \delta_{31} \cdot (m_3 + m_1) \cdot \ddot{q}_1 + \delta_{32} \cdot m_1 \cdot \ddot{q}_2 + \delta_{33} \cdot (m_2 + m_3) \cdot \ddot{q}_3 = \delta_{32} \cdot P(t)$$

$$q_1 = A_1 \cdot \cos pt \quad \rightarrow \quad \ddot{q}_1 = -A_1 p^2 \cos pt$$
$$q_2 = A_2 \cdot \cos pt \quad \rightarrow \quad \ddot{q}_2 = -A_2 p^2 \cos pt$$
$$q_3 = A_3 \cdot \cos pt \quad \rightarrow \quad \ddot{q}_3 = -A_3 p^2 \cos pt$$

$$A_1 \cos pt - \delta_{11}(m_3 + m_1) A_1 p^2 \cos pt - \delta_{12} m_1 A_2 p^2 \cos pt - \delta_{13}(m_2 + m_3) A_3 p^2 \cos pt = \delta_{12} \cdot Pcospt$$
$$A_2 \cos pt - \delta_{21}(m_3 + m_1) \cdot A_1 p^2 \cos pt - \delta_{22} m_1 A_2 p^2 \cos pt - \delta_{23}(m_2 + m_3) A_3 p^2 \cos pt = \delta_{22} \cdot Pcospt$$
$$A_3 \cos pt - \delta_{31}(m_3 + m_1) \cdot A_1 p^2 \cos pt - \delta_{32} m_1 A_2 p^2 \cos pt - \delta_{33}(m_2 + m_3) A_3 p^2 \cos pt = \delta_{32} \cdot Pcospt$$

$$A_1(1 - \delta_{11}(m_3 + m_1) p^2) - A_2 \delta_{12} m_1 p^2 - A_3 \delta_{13}(m_2 + m_3) p^2 = \delta_{12} \cdot P$$
$$-A_1 \delta_{21}(m_3 + m_1) p^2 + A_2(1 - \delta_{22} m_1 p^2) - A_3 \delta_{23}(m_2 + m_3) p^2 = \delta_{22} \cdot P$$
$$-A_1 \delta_{31}(m_3 + m_1) p^2 - A_2 \delta_{32} m_1 p^2 + A_3(1 - \delta_{33}(m_2 + m_3) p^2) = \delta_{32} \cdot P$$

Przyjmuję masę porównawczą:
$M = 5\ kg$
$m_1 = 65\ M$
$m_2 = 80\ M$
$m_3 = 112\ M$

$$A_1\left(1-549{,}4257\cdot\frac{M}{EI}\cdot p^2\right)-A_2\cdot 144{,}144\frac{M}{EI}\cdot p^2+A_3\cdot 172{,}3984\frac{M}{EI}\cdot p^2=\frac{\delta'_{12}}{EI}\cdot P$$
$$-A_1\cdot 392{,}5152\frac{M}{EI}\cdot p^2+A_2\left(1-338{,}8515\frac{M}{EI}\cdot p^2\right)+A_3\cdot 123{,}3984\frac{M}{EI}\cdot p^2=\frac{\delta'_{22}}{EI}\cdot P$$
$$A_1\cdot 159{,}3177\frac{M}{EI}\cdot p^2+A_2 41{,}7755\frac{M}{EI}\cdot p^2+A_3\cdot\left(1-180{,}1920\frac{M}{EI}\cdot p^2\right)=\frac{\delta'_{32}}{EI}\cdot P$$

$$A_1\left(1-549{,}4257\cdot\frac{5}{6273000}\cdot 129{,}43^2\right)-A_2\cdot 144{,}144\frac{5}{6273000}\cdot 129{,}43^2+A_3\cdot 172{,}3984\frac{5}{6273000}\cdot 129{,}43^2=\frac{2{,}2176}{6273000}\cdot 26500$$
$$-A_1\cdot 392{,}5152\frac{5}{6273000}\cdot 129{,}43^2+A_2\left(1-338{,}8515\frac{5}{6273000}\cdot 129{,}43^2\right)+A_3\cdot 123{,}3984\frac{5}{6273000}\cdot 129{,}43^2=\frac{5{,}2133}{6273000}\cdot 26500$$
$$A_1\cdot 159{,}3177\frac{5}{6273000}\cdot 129{,}43^2+A_2 41{,}7755\frac{5}{6273000}\cdot 129{,}43^2+A_3\cdot\left(1-180{,}1920\frac{5}{6273000}\cdot 129{,}43^2\right)=\frac{-0{,}6427}{6273000}\cdot 26500$$

$$A_1\cdot(-6{,}3362)-A_2\cdot 1{,}9247+A_3\cdot 2{,}30200=0{,}0093681$$
$$-A_1\cdot 5{,}2411+A_2(-3{,}5245)+A_3\cdot 1{,}6477=0{,}02202$$
$$A_1\cdot 2{,}1273+A_2\cdot 0{,}5578+A_3\cdot(-1{,}4060)=-0{,}002715$$

wyniki:

$A_1=0{,}0009118\,m$

$A_2=-0{,}0074390\,m$

$A_3=0{,}0003602\,m$

Obliczenie amplitud sił bezwładności:

$B_i=m_1\cdot p^2\cdot A_i\cos(pt)$

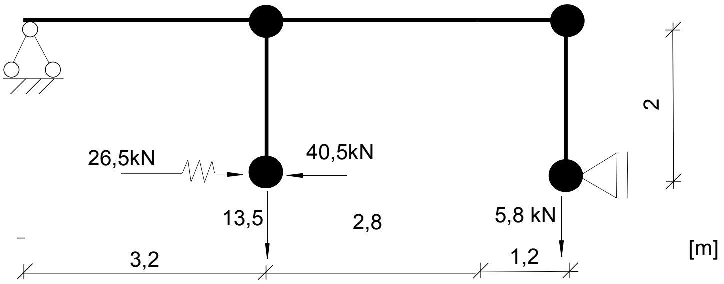
założono: $\cos(pt)=1$

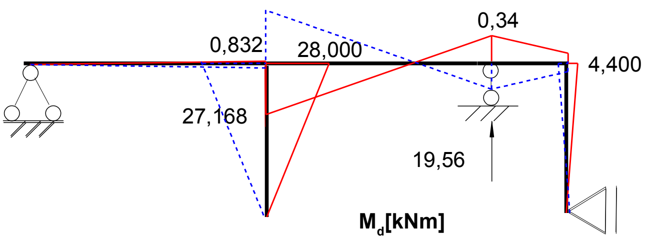
$B_1=885\cdot 129{,}4^2\cdot 0{,}0009118=13511{,}7\,N=13{,}5\,kN$

$B_2=300\cdot 129{,}4^2\cdot(-0{,}0074390)=-40482{,}4\,N=-40{,}5\,kN$

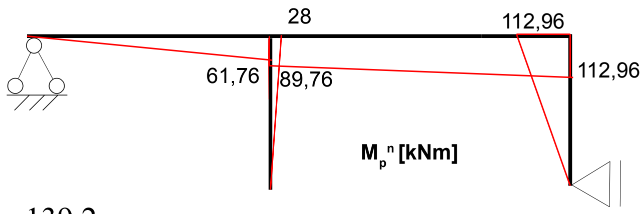
$B_3=960\cdot 129{,}4^2\cdot 0{,}0003602=5790{,}06\,N=5{,}8\,kN$

Obwiednia momentów dynamicznych:





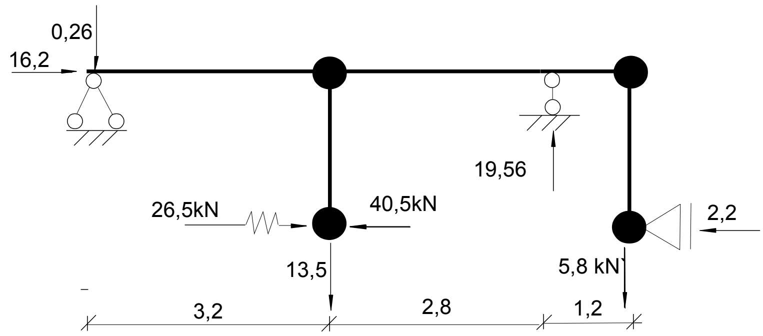
Stan "p"

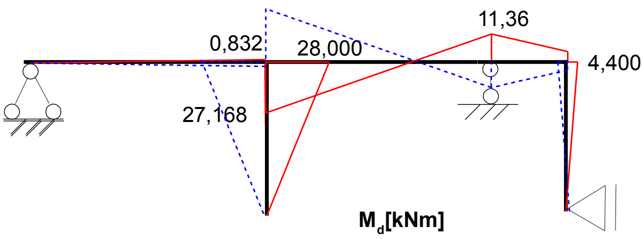


$$\delta_{11}=\frac{139{,}2}{EI}$$

$$\delta_{1p}=\frac{1}{EI}\cdot\left[-\frac{1}{2}\cdot 112{,}96\cdot 2\cdot\frac{2}{3}-3\cdot 1{,}2\cdot\frac{1}{2}\cdot(106+112{,}96)\right]+$$
$$+\left[-\frac{1}{2}\cdot 3{,}2\cdot 2{,}8\cdot\frac{2\cdot 89{,}76+106}{3}\right]+$$
$$+\left[-\frac{1}{2}\cdot 6\cdot 2{,}8\cdot\frac{2\cdot 106+89{,}76}{3}\right]+$$
$$+\left[\frac{1}{2}\cdot 3{,}2\cdot 3{,}2\cdot\frac{2}{3}\cdot 61{,}76\right]=\frac{-2722{,}2080}{EI}$$

$$x_1=-\frac{\delta_{1p}}{\delta_{11}}=\frac{2722{,}2080}{139{,}2000}=19{,}56\,kN$$





Wykres momentów od obciążenia statycznego:

$F_1 = m_1 \cdot g = 325 \cdot 9{,}81 = 3188\,N = 3{,}188\,kN$

$F_2 = m_2 \cdot g = 400 \cdot 9{,}81 = 3924\text{N} = 3{,}924\,kN$

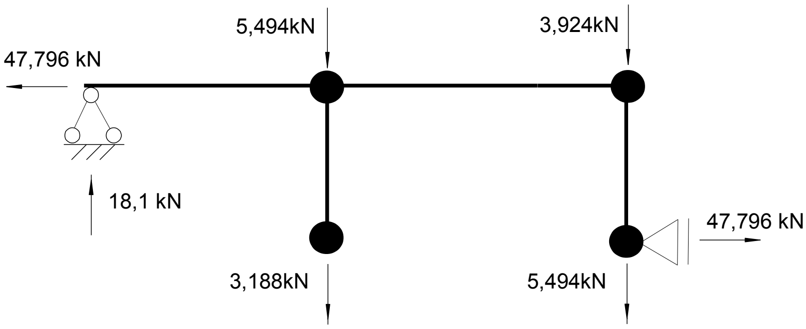
$F_3 = m_3 \cdot g = 560 \cdot 9{,}81 = 5494\text{N} = 5{,}494\,kN$

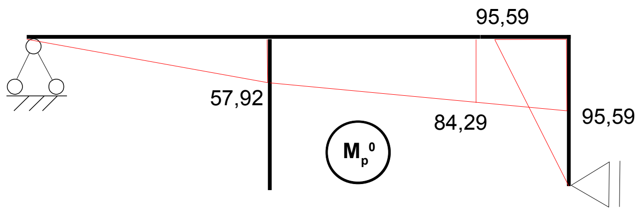
$\delta_{11} \cdot X_1 + \delta_{1P} = 0$

$X_1 = -\dfrac{\delta_{1P}}{\delta_{11}}$

$\delta_{11} = \dfrac{139{,}2}{EI}$

Stan ,, P''

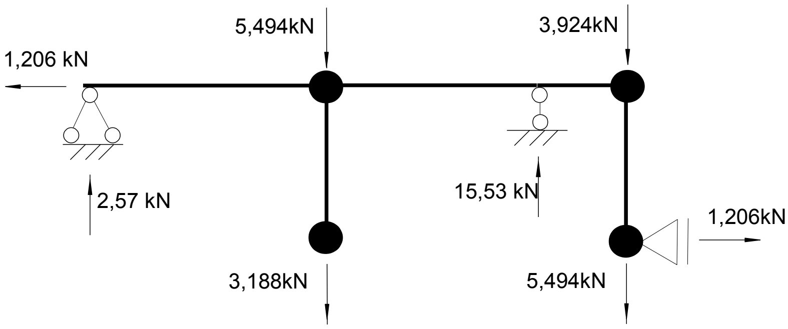


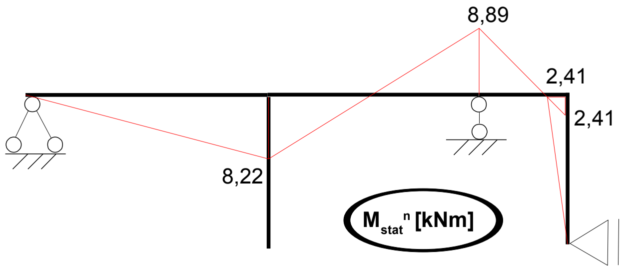


$$\delta_{1P} = \sum \int \frac{M_1 \cdot M_P}{EI} dx = -\frac{1}{2} \cdot 3{,}2 \cdot 3{,}2 \cdot \frac{2}{3} \cdot 57{,}92 - \frac{1}{2} \cdot 3{,}2 \cdot 2{,}8 \cdot \left(\frac{2}{3} \cdot 57{,}92 + \frac{1}{3} \cdot 84{,}29\right)$$

$$-\frac{1}{2} \cdot 6 \cdot 2{,}8 \cdot \left(\frac{2}{3} \cdot 84{,}29 + \frac{1}{3} \cdot 57{,}92\right) - \frac{1}{2} \cdot 84{,}29 \cdot 1{,}2 \cdot 6 - \frac{1}{2} \cdot 95{,}59 \cdot 1{,}2 \cdot 6 - \frac{1}{2} \cdot 6 \cdot 2 \cdot \frac{2}{3} \cdot 95{,}59 = -\frac{2161{,}6}{EI}$$

$$X_1 = \frac{2161{,}6}{139{,}2} = 15{,}53\,kN$$





Sprawdzenie naprężeń normalnych:

$$M_{MAX} = 1{,}2 \cdot M_{STMAX} + 5 \cdot M_{DYNMAX} = 1{,}2 \cdot 8{,}22 + 5 \cdot 27{,}17 = 145{,}72\,kNm$$

$$\frac{M_{MAX}}{W} = \frac{145720}{278} = 524\,MPa > 215\mathrm{MPa}$$

Naprężenia występujące w przekroju są zdecydowanie większe niż dopuszczalne, należy przyjąć inny przekrój.