Politechnika Poznańska

Instytut Konstrukcji Budowlanych

Zakład Mechaniki Budowli

Projekt wykonali: Stachowiak Apolonia

Stochmal Filip

Konsultacje: mgr inż. Anita Kaczor

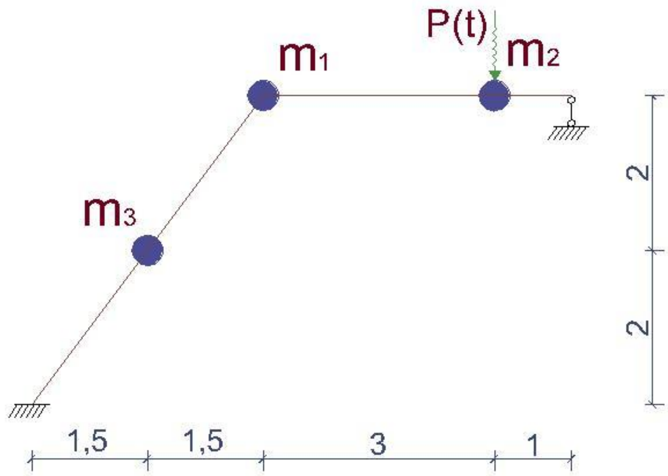
maj 2014

DYNAMIKA – UJĘCIE KLASYCZNE

1. Obliczyć częstości i postacie drgań własnych.

2. Obliczyć amplitudy drgań wymuszonych, siły bezwładności, obwiednię dynamicznych momentów zginających i sprawdzić maksymalne naprężenia normalne uwzględniając także obciążenie statyczne ciężarami mas.

Dla zadanego układu i następujących danych:



Dane:

$m_1 = 320$ kg

$m_2 = 500$ kg

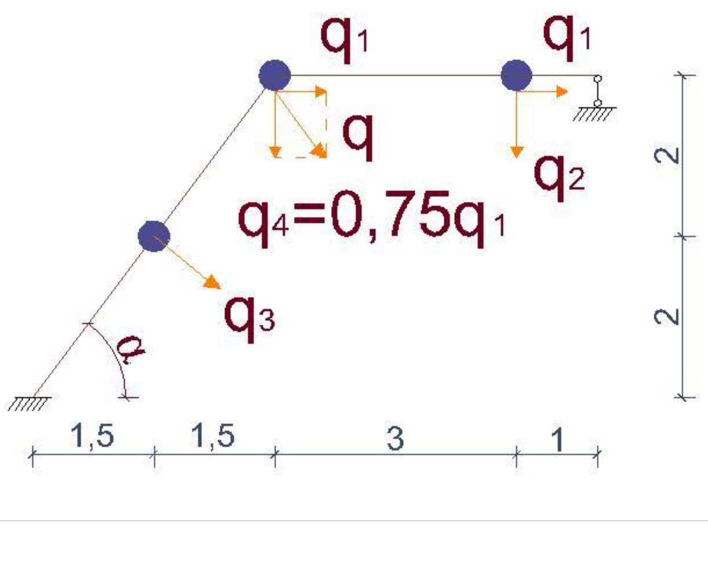
$m_3 = 600$ kg

$P(t) = P_0 \cos(pt)$

$P_0 = 25,2$ kN

$p = 10,8$ Hz

Przekrój IPN 180 $I = 1450\ cm^4$



SSD = 3

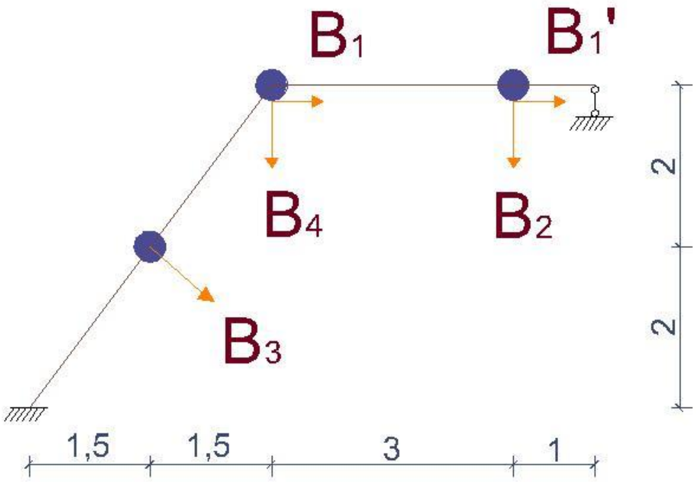
Zależności trygonometryczne:

$$sin\alpha = \frac{4}{5} = 0{,}8$$

$$cos\alpha = \frac{3}{5} = 0{,}6$$

$$tg\alpha = \frac{3}{4} = 0{,}75 \qquad ctg\alpha = \frac{4}{3} \approx 1{,}333$$

$$ctg\alpha = \frac{q_{1y}}{q_1} \quad \Rightarrow \quad q_{1y} = q_4 = q_1 \cdot ctg\alpha = \frac{3}{4} q_1$$



Siły bezwładności:

$$B_1 = -m_1 \ddot{q}_1$$

$$B_{1'} = -m_2 \ddot{q}_1$$

$$B_4 = -m_1 \ddot{q}_4$$

$$B_2 = -m_2 \ddot{q}_2$$

$$B_3 = -m_3 \ddot{q}_3$$

$$EI = 205 \cdot 10^9 \cdot 1450 \cdot 10^{-8} = 2\,972\,500\ Nm^2$$

Różniczkowe równania ruchu:

$$\begin{cases} q_1 = q_1{}^{B_1} + q_1{}^{B_1{}'} + q_1{}^{B_4} + q_1{}^{B_2} + q_1{}^{B_3} \\ q_2 = q_1{}^{B_1} + q_1{}^{B_1{}'} + q_1{}^{B_4} + q_1{}^{B_2} + q_1{}^{B_3} \\ q_3 = q_1{}^{B_1} + q_1{}^{B_1{}'} + q_1{}^{B_4} + q_1{}^{B_2} + q_1{}^{B_3} \end{cases}$$

$$\begin{cases} q_1 = \delta_{11} B_1 + \delta_{11} B_1{}' + \delta_{14} B_4 + \delta_{12} B_2 + \delta_{13} B_3 \\ q_2 = \delta_{21} B_1 + \delta_{21} B_1{}' + \delta_{24} B_4 + \delta_{22} B_2 + \delta_{23} B_3 \\ q_3 = \delta_{31} B_1 + \delta_{31} B_1{}' + \delta_{34} B_4 + \delta_{32} B_2 + \delta_{33} B_3 \end{cases}$$

$$\begin{cases} \ddot{q}_1 = A_1 \omega^2 cos(\omega t) \\ \ddot{q}_2 = A_2 \omega^2 cos(\omega t) \\ \ddot{q}_3 = A_3 \omega^2 cos(\omega t) \end{cases} \qquad \begin{cases} q_1 = A_1 cos(\omega t) \\ q_2 = A_2 cos(\omega t) \\ q_3 = A_3 cos(\omega t) \end{cases}$$

gdzie: ω – szukane częstości drgań własnych

A_i – amplituda drgań

Przyjęcie masy porównawczej m:

$$\begin{cases} m_1 = m = 320\ kg \\ m_2 = \dfrac{500}{320} m \approx 1{,}5625m \\ m_3 = \dfrac{600}{320} m \approx 1{,}875m \end{cases}$$

$$\begin{cases} A_1 cos(\omega t) = \delta_{11}(m_1 A_1 \omega^2 \cos(\omega t)_1 + m_2 A_1 \omega^2 \cos(\omega t)) + \delta_{14}(m_1 \cdot 1{,}3333 A_1 \omega^2 \cos(\omega t)) + \\ \qquad + \delta_{12}(m_2 A_2 \omega^2 \cos(\omega t)) + \delta_{13}(m_3 A_3 \omega^2 cos(\omega t)) \\ A_2 cos(\omega t) = \delta_{21}(m_1 A_1 \omega^2 \cos(\omega t)_1 + m_2 A_1 \omega^2 \cos(\omega t)) + \delta_{24}(m_1 \cdot 1{,}3333 A_1 \omega^2 \cos(\omega t)) + \\ \qquad + \delta_{22}(m_2 A_2 \omega^2 \cos(\omega t)) + \delta_{23}(m_3 A_3 \omega^2 cos(\omega t)) \\ A_3 cos(\omega t) = \delta_{31}(m_1 A_1 \omega^2 \cos(\omega t)_1 + m_2 A_1 \omega^2 \cos(\omega t)) + \delta_{34}(m_1 \cdot 1{,}3333 A_1 \omega^2 \cos(\omega t)) + \\ \qquad + \delta_{32}(m_2 A_2 \omega^2 \cos(\omega t)) + \delta_{33}(m_3 A_3 \omega^2 cos(\omega t)) \end{cases}$$

Podstawienie: $\frac{m\omega^2}{EI} = \lambda$ $\qquad \delta_{ik} = \frac{1}{EI}\delta_{ik}{}'$

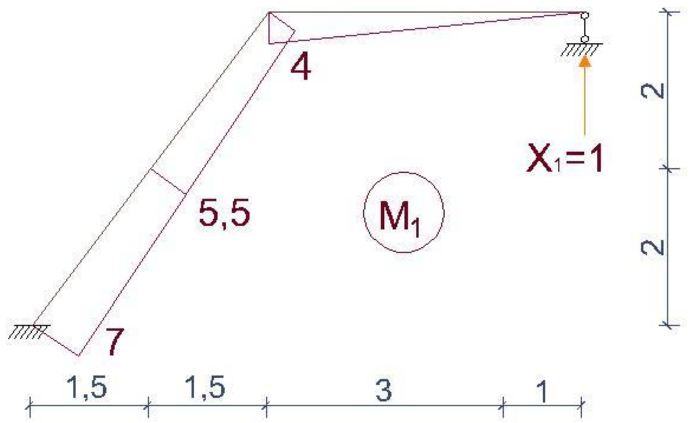
$$\begin{cases} A_1(1 - \delta_{11}{}'\lambda - 1{,}5625\delta_{11}{}'\lambda - 0{,}75\delta_{14}{}'\lambda) - A_2 1{,}5625\delta_{12}{}'\lambda - A_3 1{,}875\delta_{13}{}'\lambda = 0 \\ -A_1(\delta_{21}{}'\lambda + 1{,}5625\delta_{21}{}'\lambda + 0{,}75\delta_{24}{}'\lambda) + A_2(1 - 1{,}5625\delta_{22}{}'\lambda) - A_3 1{,}875\delta_{23}{}'\lambda = 0 \\ -A_1(\delta_{31}{}'\lambda + 1{,}5625\delta_{31}{}'\lambda + 0{,}75\delta_{34}{}'\lambda) - A_2 1{,}5625\delta_{32}{}'\lambda + A_3(1 - 1{,}875\delta_{13}{}'\lambda) = 0 \end{cases}$$

$$\begin{cases} A_1(1 - 2{,}5625\delta_{11}{}'\lambda - 0{,}75\delta_{14}{}'\lambda) - A_2 1{,}5625\delta_{12}{}'\lambda - A_3 1{,}875\delta_{13}{}'\lambda = 0 \\ -A_1(2{,}5625\delta_{21}{}'\lambda + 0{,}75\delta_{24}{}'\lambda) + A_2(1 - 1{,}5625\delta_{22}{}'\lambda) - A_3 1{,}875\delta_{23}{}'\lambda = 0 \\ -A_1(2{,}5625\delta_{31}{}'\lambda + 0{,}75\delta_{34}{}'\lambda) - A_2 1{,}5625\delta_{32}{}'\lambda + A_3(1 - 1{,}875\delta_{13}{}'\lambda) = 0 \end{cases}$$

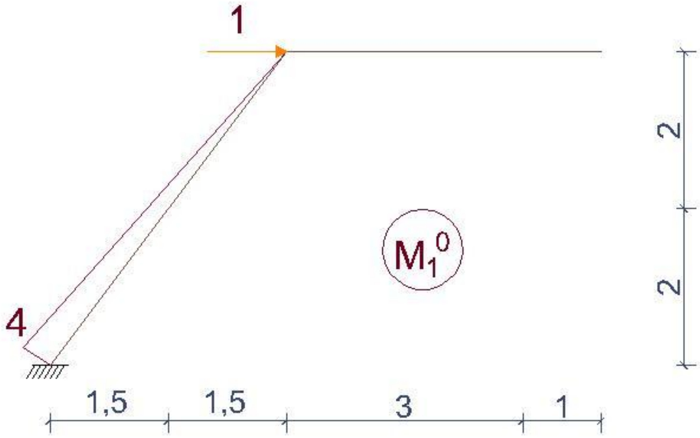
Obliczenie współczynników podatności z wykorzystaniem twierdzenia redukcyjnego:

$$\delta_{ik} = \sum \int \frac{M_i{}^n \cdot M_k{}^n}{EI} dx \xrightarrow{tw.red.} \sum \int \frac{M_i{}^0 \cdot M_k{}^n}{EI} dx$$

Siła po kierunku q_1:

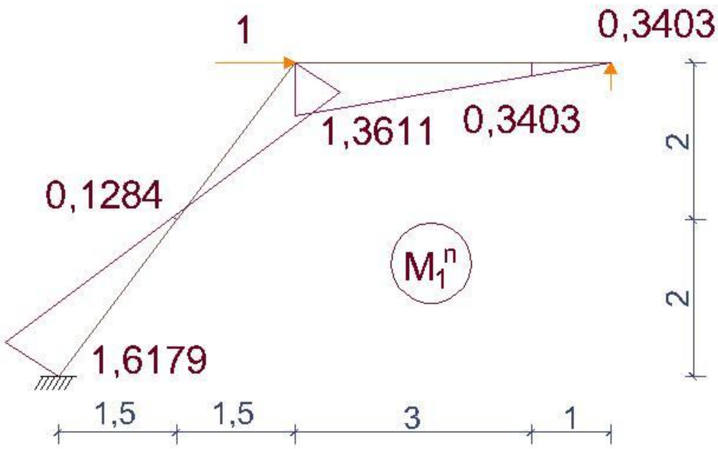


$$\delta_{11} = \frac{1}{EI}\left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + \frac{1}{2} \cdot 4 \cdot 5\left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 7\right) + \frac{1}{2} \cdot 5 \cdot 7\left(\frac{2}{3} \cdot 7 + \frac{1}{3} \cdot 4\right)\right) \approx \frac{176{,}3333}{EI}$$

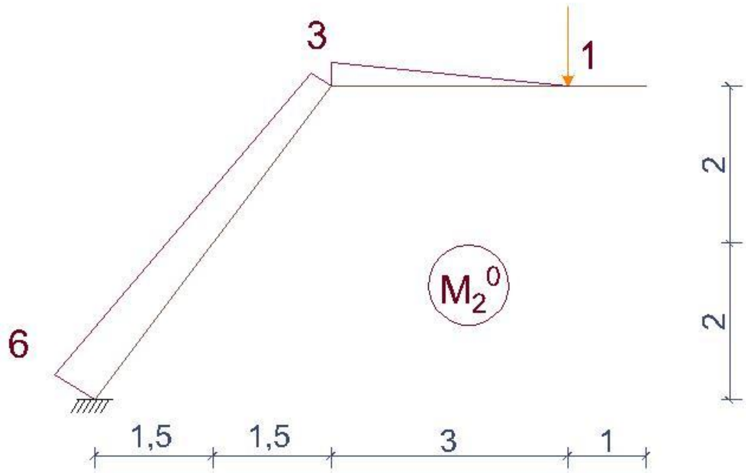


$\delta_{1P} = -\frac{1}{EI}\left(\frac{1}{2}\cdot 5\cdot 4\cdot\left(\frac{2}{3}\cdot 7+\frac{1}{3}\cdot 4\right)\right) = -\frac{60}{EI}$

$\delta_{11}X_1{}^1 + \delta_{1P} = 0 \qquad\qquad X_1{}^1 = 0{,}3403$

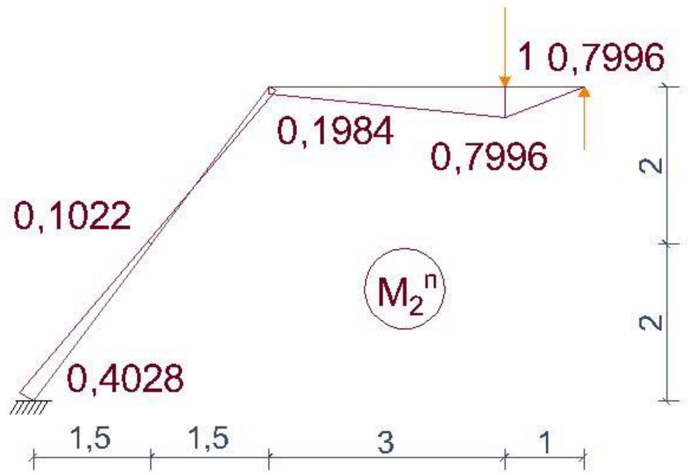


Siła po kierunku q_2:

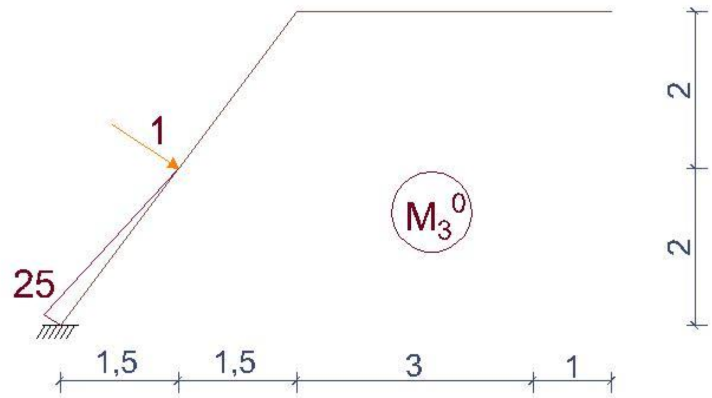


$$\delta_{1P} = -\frac{1}{EI}\left(\frac{1}{2}\cdot 5\cdot 4\cdot\left(\frac{2}{3}\cdot 7+\frac{1}{3}\cdot 4\right)\right) = -\frac{60}{EI}$$

$\delta_{11}{X_1}^2 + \delta_{1P} = 0 \qquad\qquad {X_1}^2 = 0{,}7996$

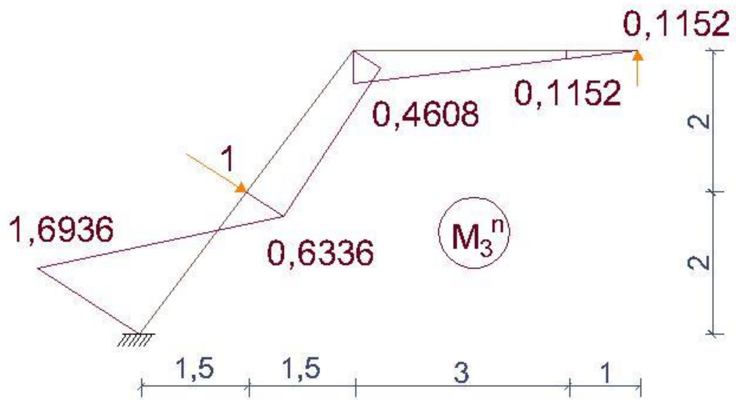


Siła po kierunku q_3:



$$\delta_{1P} = -\frac{1}{EI}\left(\frac{1}{2}\cdot 2{,}5\cdot 2{,}5\cdot\left(\frac{2}{3}\cdot 7 + \frac{1}{3}\cdot 5{,}5\right)\right) = -\frac{20{,}3125}{EI}$$

$\delta_{11}{X_1}^3 + \delta_{1P} = 0 \qquad\qquad {X_1}^3 = 0{,}1152$



$$\delta_{11}{}' = \sum \int \frac{M_1{}^0 \cdot M_1{}^n}{EI} dx = \frac{1}{2} \cdot 5 \cdot 4 \cdot \left(\frac{2}{3} \cdot 1{,}6179 - \frac{1}{3} \cdot 1{,}3611\right) = 6{,}2503$$

$$\delta_{12}{}' = \sum \int \frac{M_1{}^0 \cdot M_2{}^n}{EI} dx = \frac{1}{2} \cdot 5 \cdot 4 \cdot \left(\frac{2}{3} \cdot 0{,}407 - \frac{1}{3} \cdot 0{,}196\right) = 2{,}0227$$

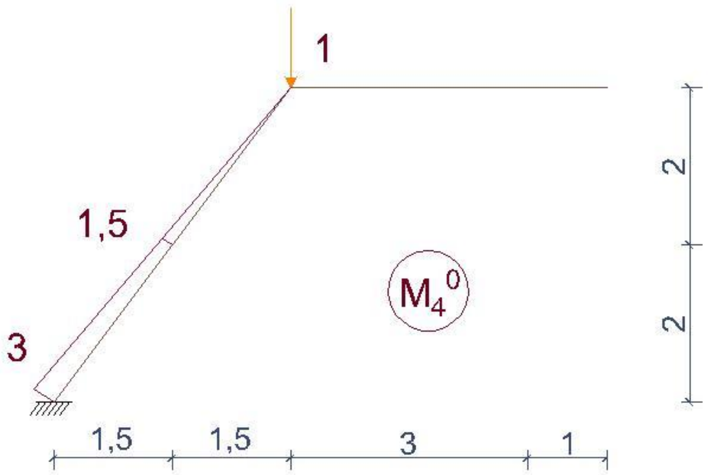
$$\delta_{13}{}' = \sum \int \frac{M_1{}^n \cdot M_3{}^0}{EI} dx = \frac{1}{2} \cdot 2{,}5 \cdot 2{,}5 \cdot \left(\frac{2}{3} \cdot 1{,}6179 + \frac{1}{3} \cdot 0{,}1284\right) = 3{,}5049$$

$$\delta_{23}{}' = \sum \int \frac{M_2{}^n \cdot M_3{}^0}{EI} dx \frac{1}{2} \cdot 2{,}5 \cdot 2{,}5 \cdot \left(\frac{2}{3} \cdot 0{,}4028 + \frac{1}{3} \cdot 0{,}1022\right) = 0{,}9453$$

$$\delta_{22}{}' = \sum \int \frac{M_2{}^0 \cdot M_2{}^n}{EI} dx = -\frac{1}{2} \cdot 3 \cdot 3 \cdot \left(\frac{2}{3} \cdot 0{,}1984 + \frac{1}{3} \cdot 0{,}7996\right) +$$

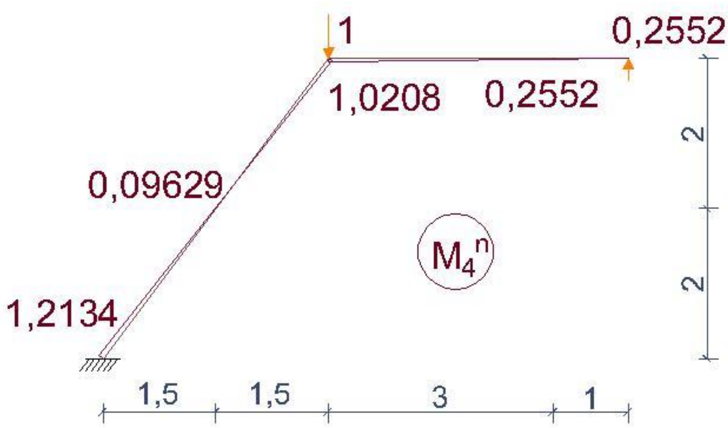
$$-\frac{1}{2} \cdot 5 \cdot 0{,}1984 \cdot \left(\frac{2}{3} \cdot 3 + \frac{1}{3} \cdot 6\right) + \frac{1}{2} \cdot 5 \cdot 0{,}4028 \cdot \left(\frac{2}{3} \cdot 6 + \frac{1}{3} \cdot 3\right) = 1{,}2553$$

$$\delta_{33}{}' = \sum \int \frac{M_3{}^0 \cdot M_3{}^n}{EI} dx = \frac{1}{2} \cdot 2{,}5 \cdot 2{,}5 \cdot \left(\frac{2}{3} \cdot 1{,}6936 - \frac{1}{3} \cdot 0{,}6336\right) = 2{,}8684$$



$$\delta_{1P} = -\frac{1}{EI}\left(\frac{1}{2} \cdot 3 \cdot 5 \cdot \left(\frac{2}{3} \cdot 7 + \frac{1}{3} \cdot 4\right)\right) = -\frac{45}{EI}$$

$\delta_{11} X_1{}^4 + \delta_{1P} = 0$ $\qquad$ $X_1{}^4 = 0{,}2552$



$$\delta_{14}' = \sum\int \frac{M_1{}^n \cdot M_4{}^0}{EI}dx = \frac{1}{2}\cdot 5\cdot 3\cdot\left(\frac{2}{3}\cdot 1{,}6179 - \frac{1}{3}\cdot 1{,}3611\right) = 4{,}6868$$

$$\delta_{24}' = \sum\int \frac{M_1{}^n \cdot M_4{}^0}{EI}dx = \frac{1}{2}\cdot 5\cdot 3\cdot\left(\frac{2}{3}\cdot 0{,}407 - \frac{1}{3}\cdot 0{,}196\right) = 1{,}5475$$

$$\delta_{34}' = \sum\int \frac{M_3{}^0 \cdot M_4{}^n}{EI}dx = \frac{1}{2}\cdot 2{,}5\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 0{,}6336 + \frac{1}{3}\cdot 0{,}4608\right) +$$

$$+\frac{1}{2}\cdot 2{,}5\cdot 1{,}5\cdot\left(\frac{1}{3}\cdot 1{,}6936 - \frac{2}{3}\cdot 0{,}6336\right) + \frac{1}{2}\cdot 2{,}5\cdot 3\cdot\left(\frac{2}{3}\cdot 1{,}6936 - \frac{1}{3}\cdot 0{,}6336\right) = 2{,}6255$$

Podstawiając obliczone δ_{ik} do układu równań ruchu otrzymujemy:

$$\begin{cases} A_1(1 - 19{,}5315\cdot\lambda) - A_2\cdot 3{,}1605\cdot\lambda - A_3\cdot 6{,}5717\cdot\lambda = 0 \\ -A_1\cdot 6{,}3438\cdot\lambda + A_2(1 - 1{,}96142\cdot\lambda) - A_3\cdot 1{,}7724\cdot\lambda = 0 \\ -A_1\cdot 10{,}9504\cdot\lambda - A_2\cdot 1{,}47703\cdot\lambda + A_3(1 - 5{,}3783\cdot\lambda) = 0 \end{cases}$$

Jest to układ równań jednorodnych, który posiada niezerowe rozwiązanie, gdy jego wyznacznik główny równy jest zero. Otrzymujemy równanie charakterystyczne:

$$(1 - 19{,}5315\cdot\lambda)(1 - 1{,}96142\cdot\lambda)(1 - 5{,}3783\cdot\lambda) + (-3{,}1605\cdot\lambda\cdot 1{,}7724\cdot\lambda\cdot 10{,}9504) +$$

$$+(-6{,}5717\cdot\lambda\cdot 6{,}3438\cdot\lambda\cdot 1{,}47703\cdot\lambda) - \big((1 - 1{,}96142\cdot\lambda)\cdot 6{,}5717\cdot\lambda\cdot 10{,}9504\cdot\lambda\big) +$$

$$-((1 - 19{,}5315\cdot\lambda)\cdot 1{,}7724\cdot\lambda\cdot 1{,}47703\cdot\lambda) - (1 - 5{,}3783\cdot\lambda)\cdot 3{,}1605\cdot\lambda\cdot 6{,}3438\cdot\lambda = 0$$

$$-27{,}2859\cdot\lambda^3 + 59{,}2742\cdot\lambda^2 - 26{,}8712\cdot\lambda + 1 = 0$$

Układ posiada rozwiązanie dla:

$$\lambda_3 = 1{,}553663$$

$$\lambda_1 = 0{,}040821$$

$$\lambda_2 = 0{,}577854$$

$$\omega_3 = \sqrt{92891{,}0625\cdot 1{,}553663} = 379{,}8966\ \frac{rad}{s}$$

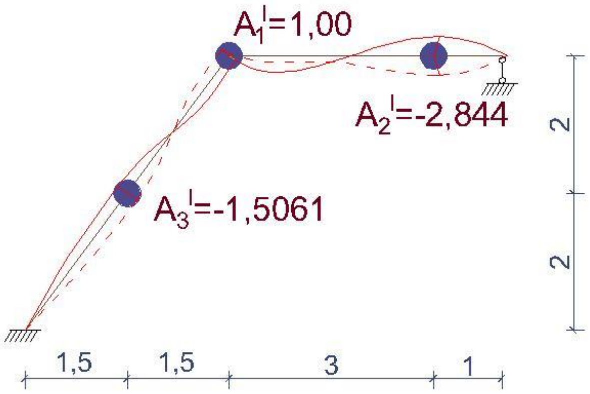
$$\omega_i = \sqrt{\frac{\lambda_i\cdot EI}{m}} \qquad \omega_1 = \sqrt{92891{,}0625\cdot 0{,}040821} = 61{,}5785\ \frac{rad}{s}$$

$$\omega_2 = \sqrt{92891{,}0625\cdot 0{,}577854} = 231{,}68399\ \frac{rad}{s}$$

Postacie drgań własnych wyznaczymy korzystając z wcześniejszego układu równań:

$$\begin{cases} A_1(1 - 19{,}5315\cdot\lambda) - A_2\cdot 3{,}1605\cdot\lambda - A_3\cdot 6{,}5717\cdot\lambda = 0 \\ -A_1\cdot 6{,}3438\cdot\lambda + A_2(1 - 1{,}96142\cdot\lambda) - A_3\cdot 1{,}7724\cdot\lambda = 0 \\ -A_1\cdot 10{,}9504\cdot\lambda - A_2\cdot 1{,}47703\cdot\lambda + A_3(1 - 5{,}3783\cdot\lambda) = 0 \end{cases}$$

III postać drgań własnych: ($\lambda_1 = 1{,}553663$)

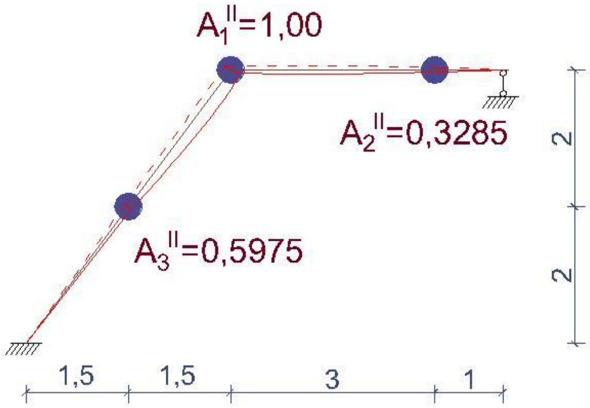


$A_1{}^I = 1{,}0$

$$\begin{cases} -29{,}3461 - A_2 \cdot 4{,}9105 - A_3 \cdot 10{,}2105 = 0 \\ 9{,}8564 + A_2(-2{,}007) - A_3 2{,}7538 = 0 \end{cases}$$

$$\begin{cases} A_1{}^I = 1{,}0 \\ A_2{}^I = -2{,}8444 \\ A_3{}^I = -1{,}5061 \end{cases}$$

I postać drgań własnych: ($\lambda_2 = 0{,}040821$)

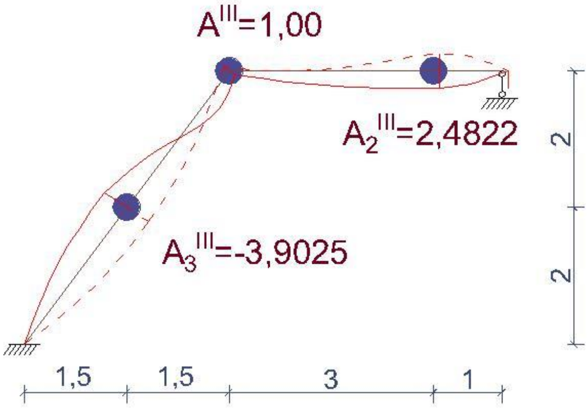


$A_1{}^{II} = 1{,}0$

$$\begin{cases} 0{,}2027 - A_2 \cdot 0{,}1290 - A_3 \cdot 0{,}2683 = 0 \\ -0{,}25896 + A_2 0{,}9199 - A_3 \cdot 0{,}0724 = 0 \end{cases}$$

$$\begin{cases} A_1{}^{II} = 1{,}0 \\ A_2{}^{II} = 0{,}3285 \\ A_3{}^{II} = 0{,}5975 \end{cases}$$

II postać drgań własnych: ($\lambda_3 = 0{,}577854$)

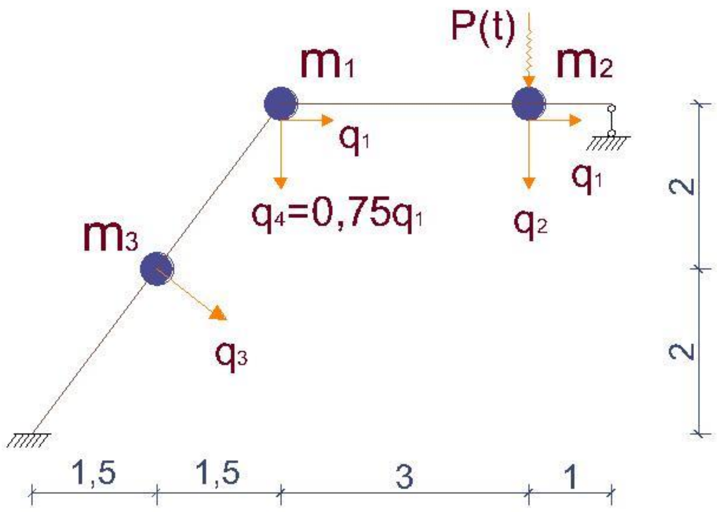


$A_1{}^I = 1{,}0$

$$\begin{cases} -10{,}2864 - A_2 \cdot 1{,}8263 - A_3 \cdot 3{,}7975 = 0 \\ -3{,}6658 + A_2(-0{,}1334) - A_3 \cdot 1{,}0242 = 0 \end{cases}$$

$$\begin{cases} A_1{}^{III} = 1{,}0 \\ A_2{}^{III} = 2{,}4822 \\ A_3{}^{III} = -3{,}9025 \end{cases}$$

DRGANIA WYMUSZONE:



Dane:

P(t) = P_0 cos(pt)

$P_0 = 25{,}2\ kN = 25\ 200\ N$

$p = 10{,}8\ Hz = 10{,}8 \cdot 2\pi =$

$= 67{,}8584\ \frac{rad}{s}$

m_1 = 320 kg

m_2 = 500 kg

m_3 = 600 kg

Obliczenie amplitud drgań wymuszonych.

$$\begin{cases} q_1 = \delta_{11}B_1 + \delta_{11}{B_1}' + \delta_{14}B_4 + \delta_{12}B_2 + \delta_{13}B_3 + \delta_{12}P(t) \\ q_2 = \delta_{21}B_1 + \delta_{21}{B_1}' + \delta_{24}B_4 + \delta_{22}B_2 + \delta_{23}B_3 + \delta_{22}P(t) \\ q_3 = \delta_{31}B_1 + \delta_{31}{B_1}' + \delta_{34}B_4 + \delta_{32}B_2 + \delta_{33}B_3 + \delta_{32}P(t) \end{cases}$$

$$\begin{cases} q_1 = A_1 cos(pt) \\ q_2 = A_2 cos(pt) \\ q_3 = A_3 cos(pt) \end{cases} \qquad \begin{cases} \ddot{q_1} = -A_1 p^2 cos(pt) \\ \ddot{q_2} = -A_2 p^2 cos(pt) \\ \ddot{q_3} = -A_3 p^2 cos(pt) \end{cases}$$

Rozwiązujemy układ równań stosując podstawienie: $\lambda' = \frac{mp^2}{EI} = 0{,}4957$ i obliczamy amplitudy drgań wymuszonych.

$$\begin{cases} A_1(1 - 19{,}5315 \cdot \lambda') - A_2 \cdot 3{,}1605 \cdot \lambda' - A_3 \cdot 6{,}5717 \cdot \lambda' = \delta_{12}P_o \\ -A_1 \cdot 6{,}3438 \cdot \lambda' + A_2(1 - 1{,}96142 \cdot \lambda') - A_3 \cdot 1{,}7724 \cdot \lambda' = \delta_{22}P_o \\ -A_1 \cdot 10{,}9504 \cdot \lambda' - A_2 \cdot 1{,}47703 \cdot \lambda' + A_3(1 - 5{,}3783 \cdot \lambda') = \delta_{32}P_o \end{cases}$$

$$\begin{cases} -8{,}6818 \cdot A_1 - 1{,}5667 \cdot A_2 - 3{,}2576 \cdot A_3 = 0{,}01715 \\ -3{,}1446 \cdot A_1 - 0{,}02773 \cdot A_2 - 0{,}8786 \cdot A_3 = 0{,}0106 \\ -5{,}4281 \cdot A_1 - 0{,}7322 \cdot A_2 - 1{,}66602 \cdot A_3 = 0{,}008014 \end{cases}$$

$$\begin{cases} A_1 = -0{,}005172\ m \\ A_2 = -0{,}084972\ m \\ A_3 = 0{,}049387\ m \end{cases}$$

Obliczenie amplitud sił bezwładności:

$$B_1 = -m_1\ddot{q}_1 = m_1 A_1 p^2 cos(pt)$$

$$B_{1'} = -m_2\ddot{q}_1 = m_2 A_1 p^2 cos(pt)$$

$$B_4 = -m_1\ddot{q}_4 = 1{,}3333 m_1 A_1 p^2 cos(pt)$$

$$B_2 = -m_2\ddot{q}_2 = m_2 A_2 p^2 cos(pt)$$

$$B_3 = -m_3\ddot{q}_3 = m_3 A_3 p^2 cos(pt)$$

Siły dynamiczne wyznaczone dla $cos(pt) = 1$:

$B_1 = -7{,}6211\ kN$

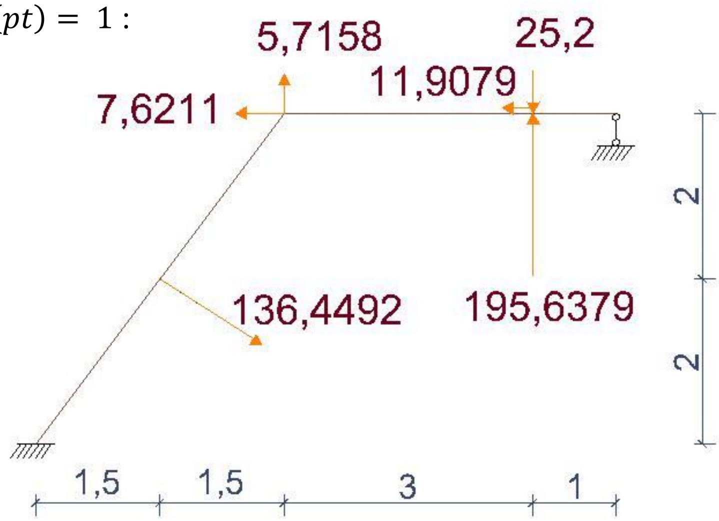
$B_{1'} = -11{,}9079\ kN$

$B_4 = -5{,}7158\ kN$

$B_2 = -195{,}6379\ kN$

$B_3 = 136{,}4492\ kN$

$P = P_0$ =25,2 kN



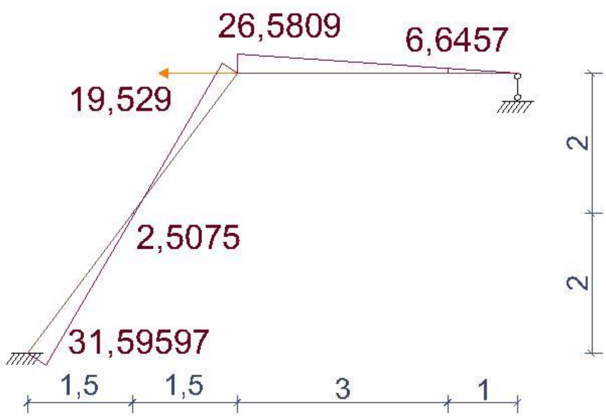
$\omega_1 = 61{,}5785\,\frac{rad}{s}$

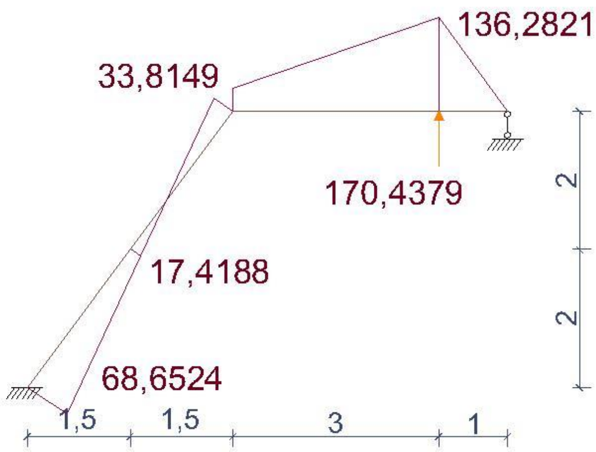
$p = 67{,}8584\ \frac{rad}{s}$

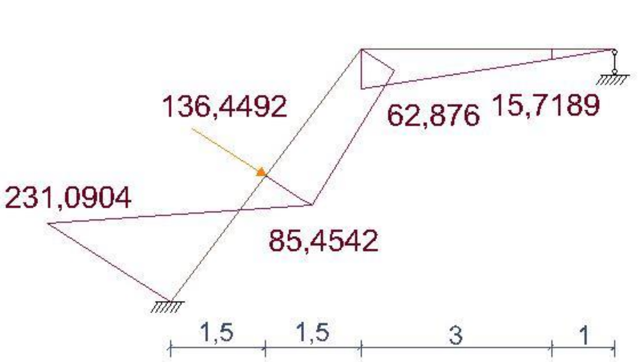
$$\eta = \frac{p}{\omega} = \frac{67{,}8584}{61{,}5785} \approx 1{,}102$$

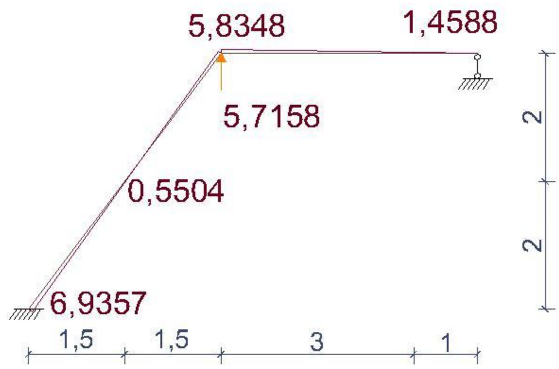
Drgania wymuszone znajdują się w strefie rezonansowej (0,75 < η < 1,25).

Wykresy cząstkowe od poszczególnych sił (wykorzystano wykresy momentów zginających $M_1^n, M_2^n, M_3^n, M_4^n$).

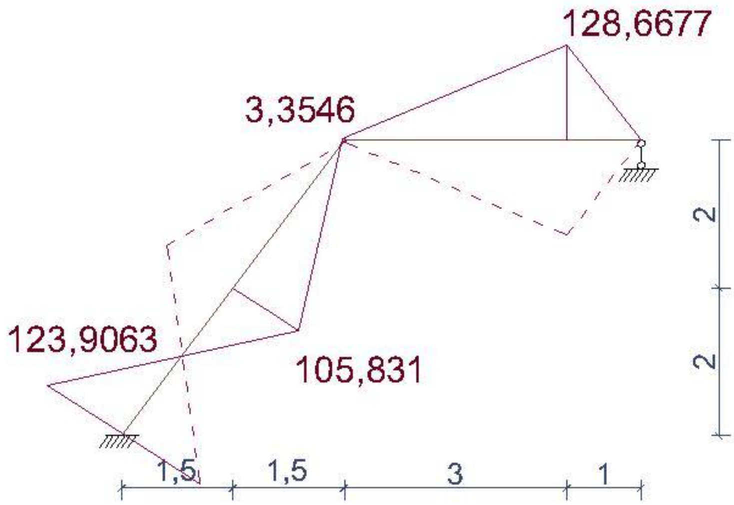








Obwiednia momentów dynamicznych:

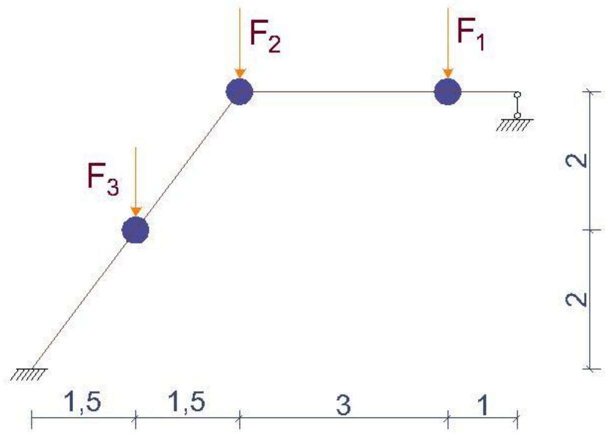


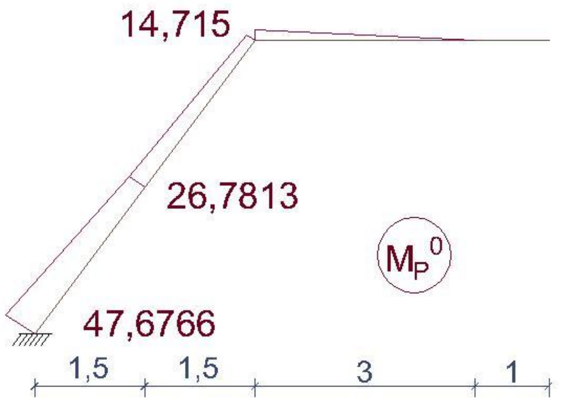
Obciążenie statyczne ciężarami mas:

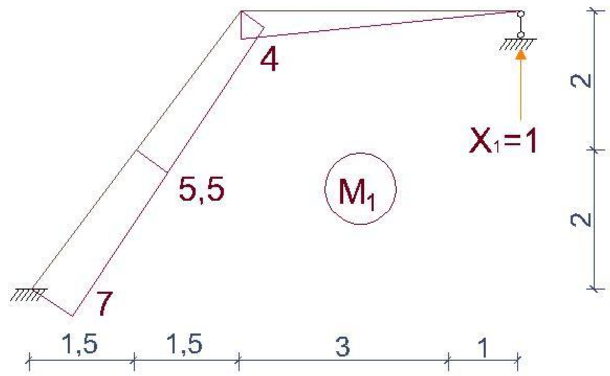
$F_1 = 320 \cdot 9{,}81 = 3{,}1392\ kN$

$F_2 = 500 \cdot 9{,}81 = 4{,}905\ kN$

$F_3 = 600 \cdot 9{,}81 = 5{,}886\ kN$





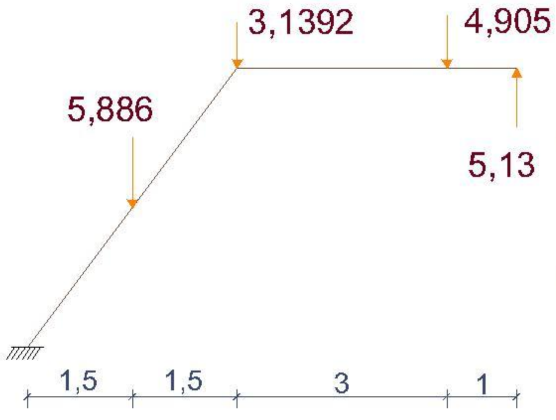


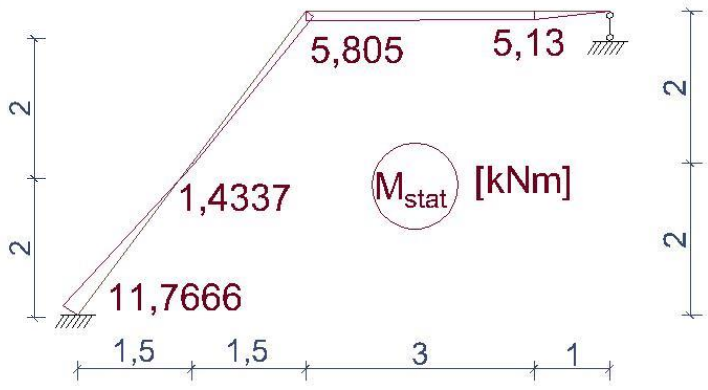
$$\delta_{1P} = \frac{1}{EI} \cdot (-\frac{1}{2} \cdot 3 \cdot 14{,}715 \cdot \left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 1\right) - \frac{1}{2} \cdot 2{,}5 \cdot 26{,}7813 \cdot \left(\frac{2}{3} \cdot 5{,}5 + \frac{1}{3} \cdot 4\right) +$$

$$-\frac{1}{2} \cdot 2{,}5 \cdot 14{,}715 \cdot \left(\frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 5{,}5\right) - \frac{1}{2} \cdot 2{,}5 \cdot 26{,}7813 \cdot \left(\frac{2}{3} \cdot 5{,}5 + \frac{1}{3} \cdot 7\right) +$$

$$-\frac{1}{2} \cdot 2{,}5 \cdot 47{,}6766 \cdot \left(\frac{2}{3} \cdot 7 + \frac{1}{3} \cdot 5{,}5\right)) = -\frac{1}{EI} \cdot (904{,}6047)$$

$$X_1{}^P = \frac{904{,}6047}{176{,}3333} = 5{,}13$$





Sprawdzenie naprężeń normalnych w przekroju:

$$M_{max} = 1{,}2 \cdot 5{,}13 + 5 \cdot 128{,}6677 = 649{,}4945 \text{ kNm}$$

$$\frac{M_{max}}{W} = \frac{649{,}4945\ kNm}{161 \cdot 10^{-6}\ m^3} \approx 4\,037\,127{,}33\ \frac{kN}{m^2} \approx 4\,037{,}13\ MPa > 215\ MPa$$

Dla zadanych sił przekrój ramy nie jest odpowiednio zaprojektowany, ponieważ naprężenia przekraczają naprężenia dopuszczalne.