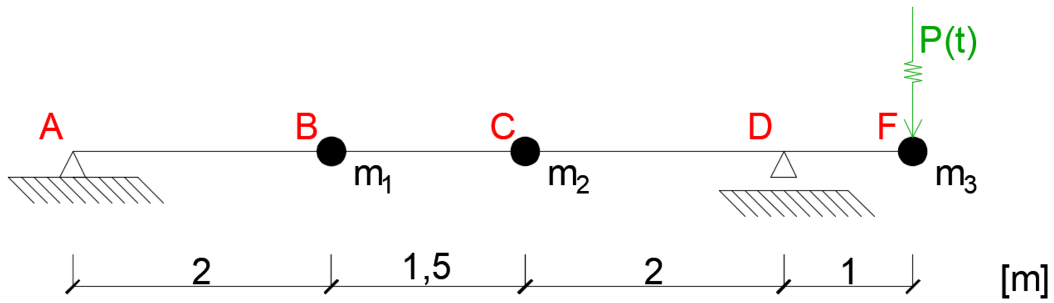
Wydział Inżynierii Lądowej i Transportu

MECHANIKA BUDOWLI – PROJEKTY
DYNAMIKA – BELKA

Prowadząca: mgr inż. Anita Kaczor

Wykonał:
Filip Maciejewski
Gr. B5
Rok akademicki 2019/2020

Schemat układu



Dane:

m_1= 140 kg

m_2 = 240 kg

m_3= 220 kg

P(t) = Psin(pt)

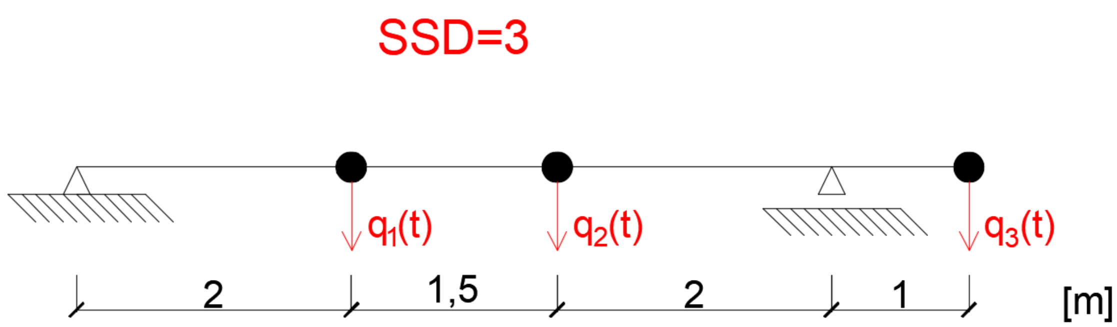
P= 12 kN

p = 83 Hz

Przekrój dwuteowy 180

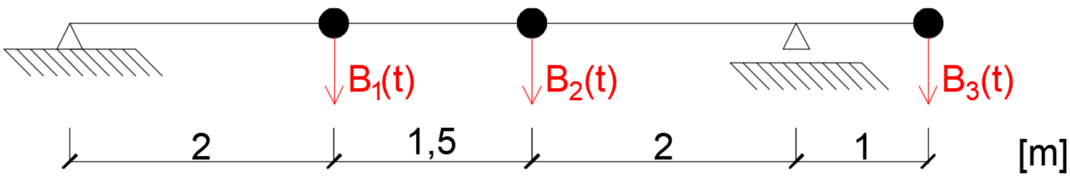
- I = 1450 cm^4
- E = 210 GPa
- w_x = 161 cm^3

Stopień swobody dynamicznej układu



DRGANIA SWOBODNE (WŁASNE)

Siły działające na układ



Równania ruchu

$$\begin{cases} q_1(t) = \delta_{11} \cdot B_1(t) + \delta_{12} \cdot B_2(t) + \delta_{13} \cdot B_3(t) \\ q_2(t) = \delta_{21} \cdot B_1(t) + \delta_{22} \cdot B_2(t) + \delta_{23} \cdot B_3(t) \\ q_3(t) = \delta_{31} \cdot B_1(t) + \delta_{32} \cdot B_2(t) + \delta_{33} \cdot B_3(t) \end{cases}$$

$$q_i(t) = a_i \cdot \sin(\omega t)$$

$$B_i(t) = -m_i \cdot \ddot{q}_i(t)$$

$$\ddot{q}_i(t) = -a_i \cdot \omega^2 \cdot \sin(\omega t)$$

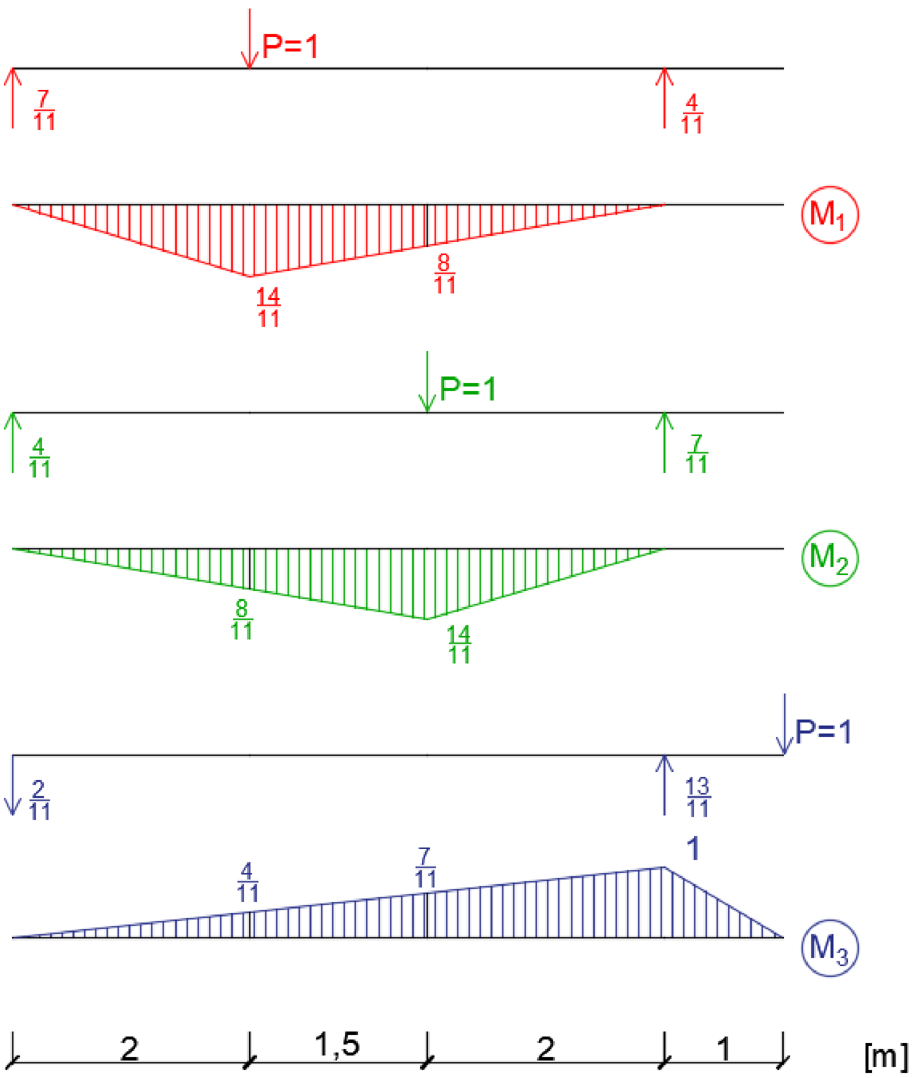
$$B_1(t) = m_1 \cdot a_1 \cdot \omega^2 \cdot \sin(\omega t)$$
$$B_2(t) = m_2 \cdot a_2 \cdot \omega^2 \cdot \sin(\omega t)$$
$$B_3(t) = m_3 \cdot a_3 \cdot \omega^2 \cdot \sin(\omega t)$$

Przyjęto masę porównawczą: m = 10 kg

$$m_1 = 14m$$
$$m_2 = 24m$$
$$m_3 = 22m$$

$$\begin{cases} a_1 = \delta_{11} \cdot 14m \cdot a_1 \cdot \omega^2 + \delta_{12} \cdot 24m \cdot a_2 \cdot \omega^2 + \delta_{13} \cdot 22m \cdot a_3 \cdot \omega^2 \\ a_2 = \delta_{21} \cdot 14m \cdot a_1 \cdot \omega^2 + \delta_{22} \cdot 24m \cdot a_2 \cdot \omega^2 + \delta_{23} \cdot 22m \cdot a_3 \cdot \omega^2 \\ a_3 = \delta_{31} \cdot 14m \cdot a_1 \cdot \omega^2 + \delta_{32} \cdot 24m \cdot a_2 \cdot \omega^2 + \delta_{33} \cdot 22m \cdot a_3 \cdot \omega^2 \end{cases}$$

Obliczenie współczynników δ_{ik} (równanie prac wirtualnych)



$$\delta_{11}=\frac{1}{EI}\left(\frac{1}{2}\cdot 2\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{14}{11}+\frac{1}{2}\cdot 3{,}5\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{14}{11}\right)=\frac{98}{33EI}=\frac{2{,}970}{EI}$$

$$\delta_{12}=\frac{1}{EI}\left(\frac{1}{2}\cdot 2\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{8}{11}+\frac{1}{2}\cdot 1{,}5\cdot\frac{14}{11}\cdot\left(\frac{2}{3}\cdot\frac{8}{11}+\frac{1}{3}\cdot\frac{14}{11}\right)+\frac{1}{2}\cdot 2\cdot\frac{8}{11}\cdot\frac{2}{3}\cdot\frac{14}{11}+\frac{1}{2}\cdot 1{,}5\cdot\frac{8}{11}\cdot\left(\frac{2}{3}\cdot\frac{14}{11}+\frac{1}{3}\cdot\frac{8}{11}\right)\right)=\frac{89}{33EI}=\frac{2{,}697}{EI}$$

$$\delta_{13}=-\frac{1}{EI}\left(\frac{1}{2}\cdot 2\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{4}{11}+\frac{1}{2}\cdot 1{,}5\cdot\frac{14}{11}\cdot\left(\frac{2}{3}\cdot\frac{4}{11}+\frac{1}{3}\cdot\frac{7}{11}\right)+\frac{1}{2}\cdot 1{,}5\cdot\frac{8}{11}\cdot\left(\frac{2}{3}\cdot\frac{7}{11}+\frac{1}{3}\cdot\frac{4}{11}\right)+\frac{1}{2}\cdot 2\cdot\frac{8}{11}\cdot\left(\frac{2}{3}\cdot\frac{7}{11}+\frac{1}{3}\cdot 1\right)\right)=-\frac{35}{22EI}=-\frac{1{,}591}{EI}$$

$$\delta_{22}=\frac{1}{EI}\left(\frac{1}{2}\cdot 3{,}5\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{14}{11}+\frac{1}{2}\cdot 2\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{14}{11}\right)=\frac{98}{33EI}=\frac{2{,}970}{EI}$$

$$\delta_{23}=-\frac{1}{EI}\left(\frac{1}{2}\cdot 3{,}5\cdot\frac{14}{11}\cdot\frac{2}{3}\cdot\frac{7}{11}+\frac{1}{2}\cdot 2\cdot\frac{14}{11}\cdot\left(\frac{2}{3}\cdot\frac{7}{11}\cdot\frac{1}{3}\cdot 1\right)\right)=-\frac{21}{11EI}=-\frac{1{,}909}{EI}$$

$$\delta_{33} = \frac{1}{EI}\left(\frac{1}{2} \cdot 5{,}5 \cdot 1 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 1 \cdot 1 \cdot \frac{2}{3} * 1\right) = \frac{13}{6EI} = \frac{2{,}167}{EI}$$

$$\delta_{21} = \delta_{12}$$

$$\delta_{32} = \delta_{23}$$

Do układu równań wprowadzamy stałą:

$$\lambda = \frac{m\omega^2}{EI}$$

$$\begin{cases} a_1 = 14 \cdot 2{,}970 \cdot a_1 \cdot \lambda + 24 \cdot 2{,}967 \cdot a_2 \cdot \lambda - 22 \cdot 1{,}591 \cdot a_3 \cdot \lambda \\ a_2 = 14 \cdot 2{,}697 \cdot a_1 \cdot \lambda + 24 \cdot 2{,}970 \cdot a_2 \cdot \lambda - 22 \cdot 1{,}909 \cdot a_3 \cdot \lambda \\ a_3 = -14 \cdot 1{,}591 \cdot a_1 \cdot \lambda - 24 \cdot 1{,}909 \cdot a_2 \cdot \lambda + 22 \cdot 2{,}167 \cdot a_3 \cdot \lambda \end{cases}$$

$$\begin{cases} a_1 = 41{,}58 \cdot a_1 \cdot \lambda + 64{,}73 \cdot a_2 \cdot \lambda - 35 \cdot a_3 \cdot \lambda \\ a_2 = 37{,}76 \cdot a_1 \cdot \lambda + 71{,}28 \cdot a_2 \cdot \lambda - 42 \cdot a_3 \cdot \lambda \\ a_3 = -22{,}27 \cdot a_1 \cdot \lambda - 45{,}82 \cdot a_2 \cdot \lambda + 47{,}67 \cdot a_3 \cdot \lambda \end{cases}$$

$$\begin{cases} (1 - 41{,}58 \cdot \lambda) \cdot a_1 - 64{,}73 \cdot \lambda \cdot a_2 + 35 \cdot \lambda \cdot a_3 = 0 \\ -37{,}76 \cdot \lambda \cdot a_1 + (1 - 71{,}28 \cdot \lambda) \cdot a_2 + 42 \cdot \lambda \cdot a_3 = 0 \\ 22{,}27 \cdot \lambda \cdot a_1 + 45{,}82 \cdot \lambda \cdot a_2 + (1 - 47{,}67 \cdot \lambda) \cdot a_3 = 0 \end{cases}$$

Otrzymano układ równań jednorodnych, który ma rozwiązanie jeżeli:

$$\begin{vmatrix} (1 - 41{,}58 \cdot \lambda) & -64{,}73\lambda & 35 \cdot \lambda \\ -37{,}76 \cdot \lambda & (1 - 71{,}28 \cdot \lambda) & 42 \cdot \lambda \\ 22{,}27 \cdot \lambda & 45{,}82\lambda & (1 - 47{,}67 \cdot \lambda) \end{vmatrix} = 0$$

Korzystamy z programu UPW – obliczone wartości charakterystyczne:

$$\lambda_1 = 0{,}007252$$
$$\lambda_2 = 0{,}05368$$
$$\lambda_3 = 0{,}2493$$

Obliczenie częstości drgań własnych

$$\lambda = \frac{m\omega^2}{EI} \rightarrow \omega = \sqrt{\frac{\lambda EI}{m}}$$

$$m = 10\ kg\ (masa\ porównawcza)$$

$$EI = 1450 \cdot 10^{-8} \cdot 210 \cdot 10^{9} = 3045000\ Nm^2$$

$$\omega_1 = 46{,}99\ \frac{rad}{s}$$
$$\omega_2 = 127{,}9\ \frac{rad}{s}$$

$$\omega_3 = 275{,}5\ \frac{rad}{s}$$

Sprawdzenie czy układ znajduje się w strefie rezonansowej

$$0{,}75 < \frac{p}{\omega} < 1{,}25$$

$$p = 83Hz = 83 \cdot 2\pi = 521{,}5\ \frac{rad}{s}$$

$$\frac{p}{\omega_1} = 11{,}09$$
$$\frac{p}{\omega_2} = 4{,}079$$
$$\frac{p}{\omega_3} = 1{,}893$$

Wniosek: Układ znajduje się poza strefą rezonansową.

Postacie drgań własnych

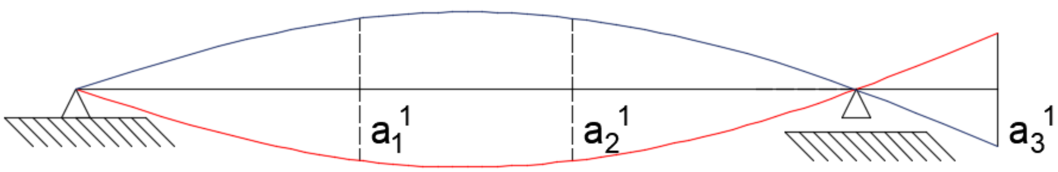
I postać

$$\lambda = 0{,}007252$$

$$a_1^1 = 1$$

$$\begin{cases} 0{,}6985 - 0{,}4694 \cdot a_2^1 + 0{,}02538 \cdot a_3^1 = 0 \\ -0{,}2738 + 0{,}4831 \cdot a_2^1 + 0{,}3045 \cdot a_3^2 = 0 \end{cases}$$

$$\begin{cases} a_2^1 = 1{,}062 \\ a_3^1 = -0{,}7865 \end{cases}$$



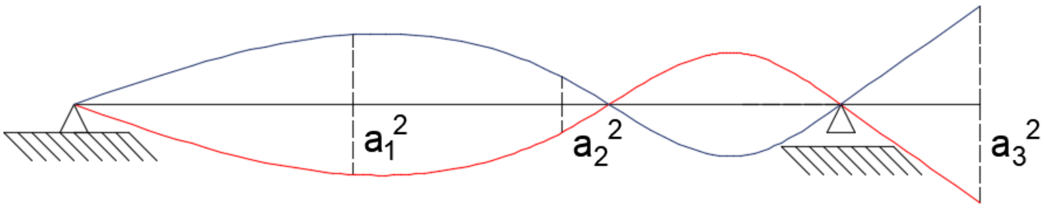
II postać

$$\lambda = 0{,}05368$$

$$a_1^2 = 1$$

$$\begin{cases} -2{,}928 - 3{,}475 \cdot a_2^2 + 1{,}879 \cdot a_3^2 = 0 \\ -2{,}027 — 2{,}827 \cdot a_2^2 + 2{,}255 \cdot a_3^2 = 0 \end{cases}$$

$$\begin{cases} a_2^2 = 0{,}4085 \\ a_3^2 = 1{,}411 \end{cases}$$



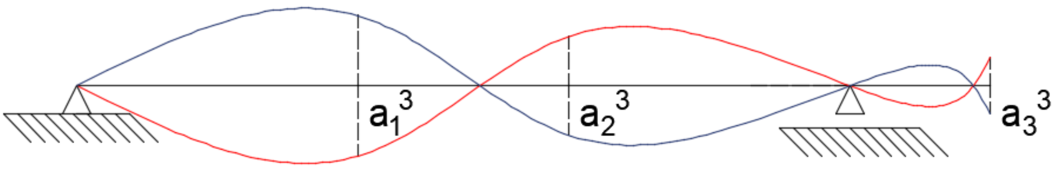
III postać

$$\lambda = 0{,}2493$$

$$a_1^3 = 1$$

$$\begin{cases} -9{,}366 - 16{,}14 \cdot a_2^3 + 8{,}726 \cdot a_3^3 = 0 \\ -9{,}413 — 16{,}77 \cdot a_2^3 + 10{,}47 \cdot a_3^3 = 0 \end{cases}$$

$$\begin{cases} a_2^2 = -0{,}7040 \\ a_3^2 = -0{,}2286 \end{cases}$$



Warunek ortogonalności drgań

$$\Sigma m_i a_i^j a_i^k = 0$$

$a_1^1 = 1$	$a_1^2 = 1$	$a_1^3 = 1$	$m_1 = 140kg$
$a_2^1 = 1{,}063$	$a_2^2 = 0{,}4085$	$a_2^2 = -0{,}7040$	$m_2 = 240kg$
$a_3^1 = -0{,}7865$	$a_3^2 = 1{,}411$	$a_3^2 = -0{,}2286$	$m_3 = 220kg$

I i II postać

$$140 + 240 \cdot 1{,}063 \cdot 0{,}4085 + 220 \cdot (-0{,}7865) \cdot 1{,}411 = 6{,}708 \cdot 10^{-3} \approx 0$$

I i III postać

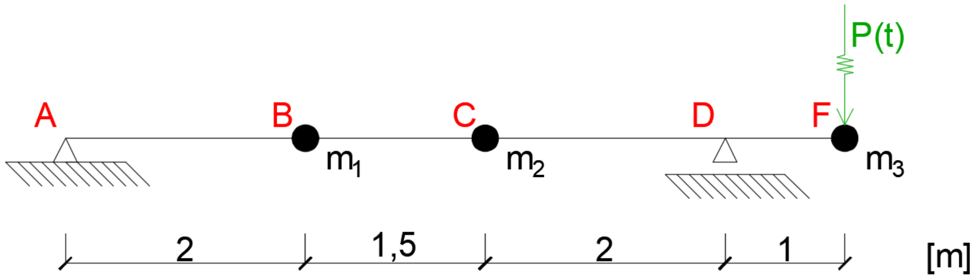
$$140 + 240 \cdot 1{,}063 \cdot (-0{,}7040) + 220 \cdot (-0{,}7865) \cdot (-0{,}2286) = -4{,}698 \cdot 10^{-4} \approx 0$$

II i III postać

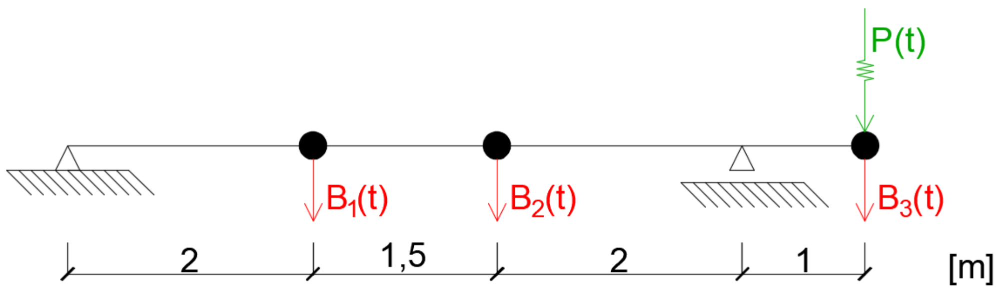
$$140 + 240 \cdot (0{,}4085) \cdot (-0{,}7040) + 220 \cdot 1{,}411 \cdot (-0{,}2286) = -0{,}0143 \approx 0$$

DRGANIA WYMUSZONE

$$P = 12000\ N$$
$$EI = 3045000\ Nm^2$$
$$p = 521{,}5\ \frac{rad}{s}$$



Układ sił działających na belkę



Równania ruchu

$$\begin{cases} q_1(t) = \delta_{11} \cdot B_1(t) + \delta_{12} \cdot B_2(t) + \delta_{13} \cdot [B_3(t) + P(t)] \\ q_2(t) = \delta_{21} \cdot B_1(t) + \delta_{22} \cdot B_2(t) + \delta_{23} \cdot [B_3(t) + P(t)] \\ q_3(t) = \delta_{31} \cdot B_1(t) + \delta_{32} \cdot B_2(t) + \delta_{33} \cdot [B_3(t) + P(t)] \end{cases}$$

$$q_i(t) = A_i \cdot \sin(pt)$$

$$B_i(t) = -m_i \cdot \ddot{q}_i(t)$$

$$\ddot{q}_{i1}(t) = -A_i \cdot p^2 \cdot \sin(pt)$$

$$B_1(t) = m_1 \cdot A_1 \cdot p^{2\cdot} \cdot \sin(pt)$$
$$B_2(t) = m_2 \cdot A_2 \cdot p^2 \cdot \sin(pt)$$
$$B_3(t) = m_3 \cdot A_3 \cdot p^2 \cdot \sin(pt)$$

$$P(t) = P \cdot \sin(pt)$$

Przyjmujemy masę porównawczą *m*=10 kg

$$m_1 = 14m$$
$$m_2 = 24m$$
$$m_3 = 22m$$

Po podstawieniu otrzymujemy:

$$\begin{cases} A_1 = \delta_{11} \cdot 14m \cdot A_1 \cdot p^2 + \delta_{12} \cdot 24m \cdot A_2 \cdot p^2 + \delta_{13} \cdot (22m \cdot A_3 \cdot p^2 + P) \\ A_2 = \delta_{21} \cdot 14m \cdot A_1 \cdot p^2 + \delta_{22} \cdot 24m \cdot A_2 \cdot p^2 + \delta_{23} \cdot (22m \cdot A_3 \cdot p^2 + P) \\ A_3 = \delta_{31} \cdot 14m \cdot A_1 \cdot p^2 + \delta_{32} \cdot 24m \cdot A_2 \cdot p^2 + \delta_{33} \cdot (22m \cdot A_3 \cdot p^2 + P) \end{cases}$$

Korzystamy ze wcześniej obliczonych współczynników:

$$\begin{cases} A_1 = 14m \cdot \frac{2,970}{EI} \cdot A_1 \cdot p^2 + 24m \cdot \frac{2,967}{EI} \cdot A_2 \cdot p^2 - \frac{1,591}{EI} \cdot (22m \cdot A_3 \cdot p^2 + P) \\ A_2 = 14m \cdot \frac{2,697}{EI} \cdot A_1 \cdot p^2 + 24m \cdot \frac{2,970}{EI} \cdot A_2 \cdot p^2 - \frac{1,909}{EI} \cdot (22m \cdot A_3 \cdot p^2 + P) \\ A_3 = -14m \cdot \frac{1,591}{EI} \cdot A_1 \cdot p^2 - 24m \cdot \frac{1,909}{EI} \cdot A_2 \cdot p^2 + \frac{2,167}{EI} \cdot (22m \cdot A_3 \cdot p^2 + P) \end{cases}$$

$$\begin{cases} A_1 = 37,14 \cdot A_1 + 57,81 \cdot A_2 - 31,26 \cdot A_3 - 0,006270 \\ A_2 = 33,72 \cdot A_1 + 63,66 \cdot A_2 - 37,51 \cdot A_3 - 0,007523 \\ A_3 = -19,89 \cdot A_1 - 40,92 \cdot A_2 + 42,58 \cdot A_3 + 0,00854 \end{cases}$$

$$\begin{cases} 36,14 \cdot A_1 + 57,81 \cdot A_2 - 31,26 \cdot A_3 = 0,006270 \\ 33,72 \cdot A_1 + 62,66 \cdot A_2 - 37,51 \cdot A_3 = 0,007523 \\ -19,89 \cdot A_1 - 40,92 \cdot A_2 + 41,58 \cdot A_3 = -0,00854 \end{cases}$$

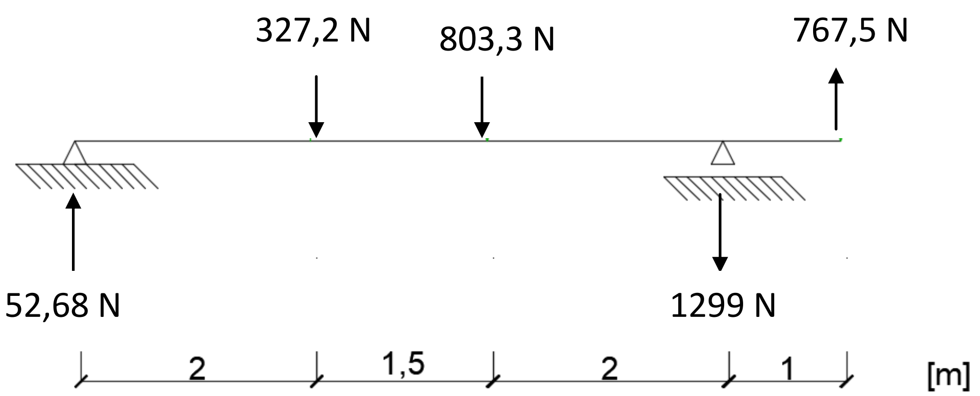
$$A_1 = 8,595 \cdot 10^{-6}\, m$$
$$A_2 = -0,00001231\, m$$
$$A_3 = -0,0002134\, m$$

Siły dynamiczne

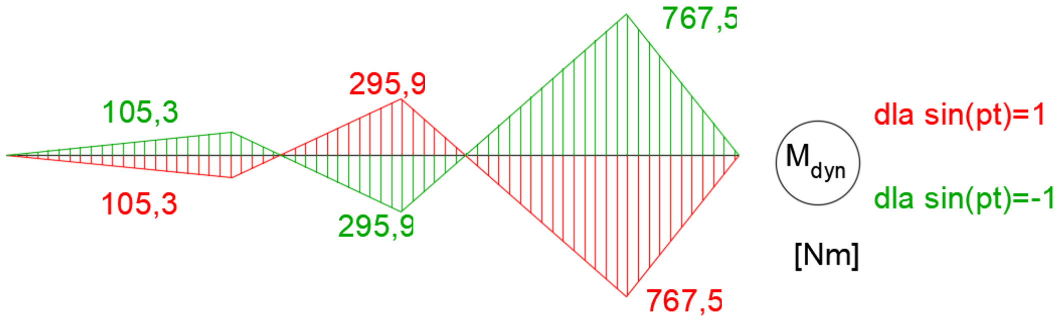
$$B_1(t) = m_1 \cdot A_1 \cdot p^2 \cdot \sin(pt)$$
$$B_2(t) = m_2 \cdot A_2 \cdot p^2 \cdot \sin(pt)$$
$$B_3(t) + P(t) = m_3 \cdot A_3 \cdot p^2 \cdot \sin(pt) + P \cdot sin(pt)$$

Dla $sin(pt) = 1$ otrzymujemy:

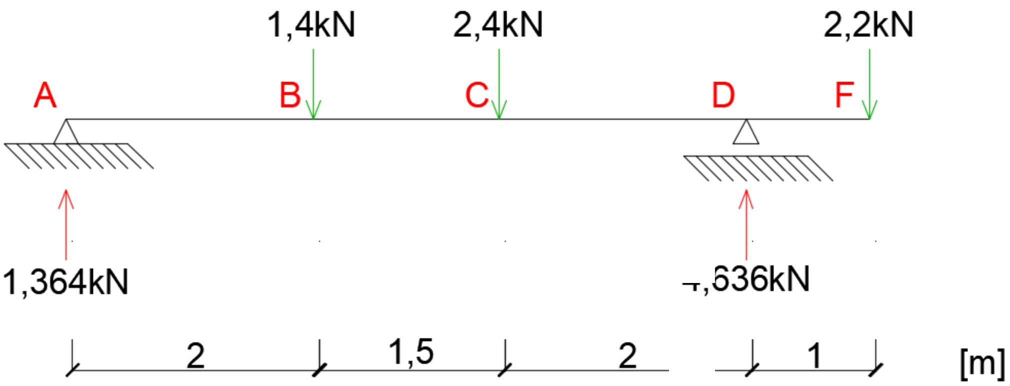
$$B_1(t) = 140 \cdot 8,595 \cdot 10^{-6} \cdot 521,5^2 \cdot 1 = 327,2\, N$$
$$B_2(t) = 240 \cdot (-0,00001231) \cdot 521,5^2 \cdot 1 = -803,3\, N$$
$$B_3(t) + P(t) = 220 \cdot (-0,0002134\,) \cdot 521,5^2 \cdot 1 + 12000 = -767,5\, N$$

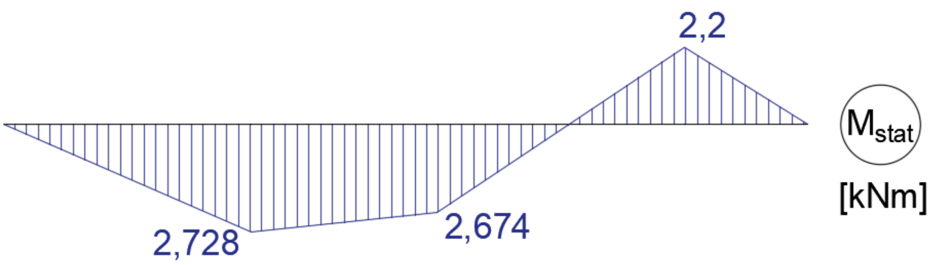


Obwiednia momentów dynamicznych



Obciążenie statyczne siłami ciężkości





Szukamy maksymalnego momentu uwzględniając współczynniki obliczeniowe

$$M_D = 1{,}2 \cdot 2{,}2 + 5 \cdot 0{,}7675 = 6{,}478\ kNm$$
$$M_C = 1{,}2 \cdot 2{,}674 + 5 \cdot 0{,}296 = 4{,}689\ kNm$$
$$M_B = 1{,}2 \cdot 2{,}728 + 5 \cdot 0{,}1054 = 3{,}801\ kNm$$

Maksymalne naprężenie

$$\sigma = \frac{6{,}478\ kNm}{161\ cm^3} = \frac{647{,}8 kNcm}{161\ cm^3} = 4{,}023\ \frac{kN}{cm^2} = 40{,}23\ MPa$$

$$\sigma_{dop} = 215\ MPa$$

$$\sigma < \sigma_{dop}$$