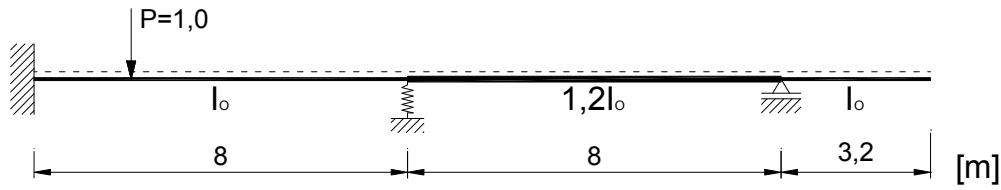
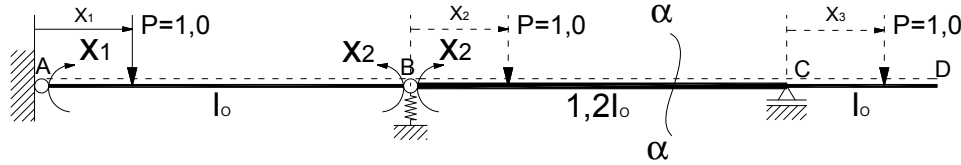


Schemat układu:



SSN=2

Układ podstawowy:

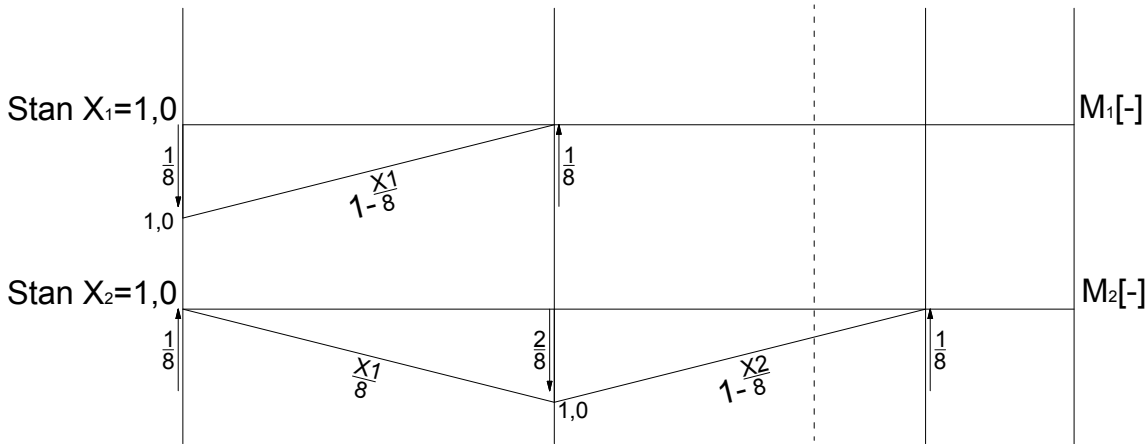
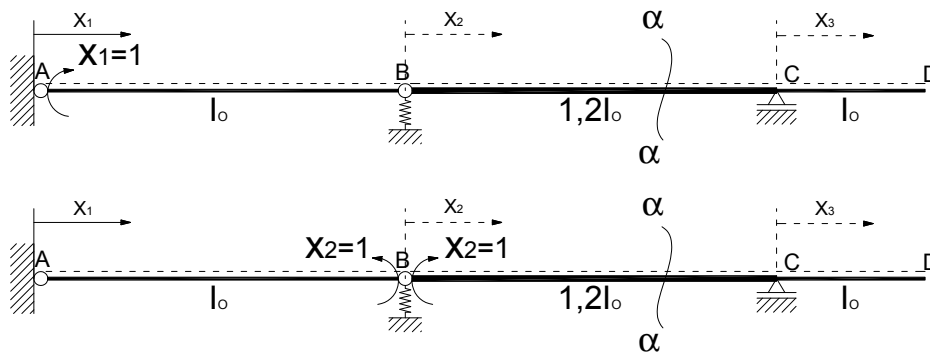


Układ równań kanonicznych:

$$\begin{cases} \delta_{11}X_1(x) + \delta_{12}X_2(x) + \delta_{1P}(x) = 0 \\ \delta_{21}X_1(x) + \delta_{22}X_2(x) + \delta_{2P}(x) = 0 \end{cases}$$

$$\delta_{ik} = \int_s \frac{M_i \cdot M_k}{EI} ds + \sum R \cdot R \cdot \frac{1}{k}, \quad \text{gdzie } k = \frac{1}{8}EI_0$$

$$\delta_{iP} = \int_s \frac{M_i \cdot M_P}{EI} ds$$



Korzystając z twierdzenia Wereszczagina-Mohra wyznaczam:

$$\delta_{11} = \frac{1}{EI_0} \left[\frac{1}{2} \cdot 8 \cdot 1 \cdot \frac{2}{3} \cdot 1 \right] + \frac{1}{8} \cdot \frac{1}{8} \cdot \frac{8}{EI_0} = \frac{2,791667}{EI_0}$$

$$\delta_{22} = \frac{1}{EI_0} \left[\frac{1}{2} \cdot 8 \cdot 1 \cdot \frac{2}{3} \cdot 1 \right] + \frac{1}{1,2EI_0} \left[\frac{1}{2} \cdot 8 \cdot 1 \cdot \frac{2}{3} \cdot 1 \right] + \frac{2}{8} \cdot \frac{2}{8} \cdot \frac{8}{EI_0} = \frac{5,388889}{EI_0}$$

$$\delta_{12} = \frac{1}{EI_0} \left[\frac{1}{2} \cdot 8 \cdot 1 \cdot \frac{1}{3} \cdot 1 \right] - \frac{1}{8} \cdot \frac{2}{8} \cdot \frac{8}{EI_0} = \frac{1,083333}{EI_0}$$

Korzystając z twierdzenia Maxwella wyznaczam:

$$\delta_{iP}(x) = \delta_{Pi}(x)$$

$$EI_0 \frac{d^2 y}{dx^2} = -M(x)$$

$$\delta_{P_1}(x) \in \langle A, B \rangle$$

$$M(x) = 1 - \frac{x_1}{8}$$

$$EI_0 \frac{d^2 y}{dx^2} = \frac{x_1}{8} - 1$$

$$EI_0 \frac{dy}{dx} = \frac{1}{2} \cdot \frac{x_1^2}{8} - 1 \cdot x_1 + C$$

$$EI_0 \cdot y = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{x_1^3}{8} - 1 \cdot \frac{1}{2} \cdot x_1^2 + Cx_1 + D$$

$$EI_0 \cdot y = \frac{1}{48} \cdot x_1^3 - \frac{1}{2} \cdot x_1^2 + Cx_1 + D$$

$$y = \delta_{P_1}(x) = \frac{1}{EI_0} \left[\frac{1}{48} \cdot x_1^3 - \frac{1}{2} \cdot x_1^2 + \frac{67}{24} x_1 \right]$$

$$\delta_{P_1}(x) \in \langle B, C \rangle$$

$$M(x) = 0$$

$$1,2EI_0 \frac{d^2 y}{dx^2} = 0$$

$$1,2EI_0 \frac{dy}{dx} = C$$

$$1,2EI_0 \cdot y = Cx_2 + D$$

$$y = \delta_{P_1}(x) = \frac{1}{1,2EI_0} \left[-\frac{6}{40} x_2 + \frac{6}{5} \right] = \frac{1}{EI_0} \left[-\frac{1}{8} x_2 + 1 \right]$$

$$\delta_{P_1}(x) \in \langle C, D \rangle$$

$$M(x) = 0$$

$$EI_0 \frac{d^2 y}{dx^2} =$$

$$EI_0 \frac{dy}{dx} = C$$

$$EI_0 \cdot y = Cx_3 + D$$

$$y = \delta_{P_1}(x) = \frac{1}{EI_0} \left[-\frac{1}{8} x_3 \right]$$

Warunki brzegowe:

$$x_1 = 0 \quad y = 0 \quad \rightarrow \quad D = 0$$

$$x_1 = 8 \quad y = \frac{1}{k} \cdot \frac{1}{8} = \frac{8}{EI_0} \cdot \frac{1}{8} = \frac{1}{EI_0} \quad \rightarrow \quad C = \frac{67}{24}$$

Warunki brzegowe:

$$x_2 = 0 \quad y = \frac{1}{EI_0} \quad \rightarrow \quad D = 1,2 = \frac{6}{5}$$

$$x_8 = 8 \quad y = 0 \quad \rightarrow \quad C = -\frac{1,2}{8} = -\frac{6}{40}$$

Warunki brzegowe:

$$x_3 = 0 \quad y = 0 \quad \rightarrow \quad D = 0$$

$$x_3 = 0 \quad \varphi_C^L = \varphi_C^P$$

$$1,2EI_0 \frac{dy}{dx} = C, \quad C = -\frac{6}{40}$$

$$EI_0 \frac{dy}{dx} = -\frac{1}{8}$$

$$\delta_{p_2}(\mathbf{x}) \in \langle \mathbf{A}, \mathbf{B} \rangle$$

$$M(x) = \frac{x_1}{8}$$

$$EI_0 \frac{d^2 y}{dx^2} = -\frac{x_1}{8}$$

$$EI_0 \frac{dy}{dx} = -\frac{1}{2} \cdot \frac{x_1^2}{8} + C$$

$$EI_0 \cdot y = -\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{x_1^3}{8} + Cx_1 + D$$

$$EI_0 \cdot y = -\frac{1}{48} \cdot x_1^3 + Cx_1 + D$$

$$y = \delta_{p_2}(x) = \frac{1}{EI_0} \left[-\frac{1}{48} \cdot x_1^3 + \frac{13}{12} x_1 \right]$$

Warunki brzegowe:

$$x_1 = 0 \quad y = 0 \rightarrow D = 0$$

$$x_1 = 8 \quad y = \frac{1}{k} \cdot \left(-\frac{2}{8} \right) = \frac{8}{EI_0} \cdot \left(-\frac{2}{8} \right) = \frac{-2}{EI_0} \rightarrow C = \frac{13}{12}$$

$$\delta_{p_2}(\mathbf{x}) \in \langle \mathbf{B}, \mathbf{C} \rangle$$

$$M(x) = 1 - \frac{x_2}{8}$$

$$1,2EI_0 \frac{d^2 y}{dx^2} = \frac{x_2}{8} - 1$$

$$1,2EI_0 \frac{dy}{dx} = \frac{1}{2} \cdot \frac{x_2^2}{8} - 1 \cdot x_2 + C$$

$$1,2EI_0 \cdot y = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{x_2^3}{8} - 1 \cdot \frac{1}{2} \cdot x_2^2 + Cx_2 + D$$

$$1,2EI_0 \cdot y = \frac{1}{48} \cdot x_2^3 - \frac{1}{2} \cdot x_2^2 + Cx_2 + D$$

$$y = \delta_{p_2}(x) = \frac{1}{1,2EI_0} \left[\frac{1}{48} \cdot x_2^3 - \frac{1}{2} \cdot x_2^2 + \frac{89}{30} x_2 - \frac{12}{5} \right] = \frac{1}{EI_0} \left[\frac{5}{288} \cdot x_2^3 - \frac{5}{12} \cdot x_2^2 + \frac{89}{36} \cdot x_2 - 2 \right]$$

Warunki brzegowe:

$$x_2 = 0 \quad y = -\frac{2}{EI_0} \rightarrow D = -\frac{12}{5}$$

$$x_2 = 8 \quad y = 0 \rightarrow C = \frac{89}{30}$$

$$\delta_{p_2}(\mathbf{x}) \in \langle \mathbf{C}, \mathbf{D} \rangle$$

$$M(x) = 0$$

$$EI_0 \frac{d^2 y}{dx^2} = 0$$

$$EI_0 \frac{dy}{dx} = C$$

$$EI_0 \cdot y = Cx_3 + D$$

Warunki brzegowe:

$$x_3 = 0 \quad y = 0 \rightarrow D = 0$$

$$x_3 = 0 \quad \varphi_C^L = \varphi_C^P$$

$$1,2EI_0 \frac{dy}{dx} = \frac{1}{2} \cdot \frac{x_2^2}{8} - 1 \cdot x_2 + C$$

$$x_2 = 8, \quad C = \frac{89}{30}$$

$$EI_0 \frac{dy}{dx} = -\frac{31}{36}$$

$$y = \delta_{p_1}(x) = \frac{1}{EI_0} \left[-\frac{31}{36} x_3 \right]$$

$$\begin{cases} \frac{2,791667}{EI_0} X_1(x) + \frac{1,083333}{EI_0} X_2(x) + \delta_{1P}(x) = 0 / \cdot EI_0 \\ \frac{1,083333}{EI_0} X_1(x) + \frac{5,388889}{EI_0} X_2(x) + \delta_{2P}(x) = 0 / \cdot EI_0 \end{cases}$$

$$\begin{cases} 2,791667 \cdot X_1(x) + 1,083333 \cdot X_2(x) = -\delta_{1P}(x) \cdot EI_0 \\ 1,083333 \cdot X_1(x) + 5,388889 \cdot X_2(x) = -\delta_{2P}(x) \cdot EI_0 \end{cases}$$

⟨A,B⟩

$$EI_0 \cdot \delta_{p1}(x) = \frac{1}{48} \cdot x_1^3 - \frac{1}{2} \cdot x_1^2 + \frac{67}{24} x_1$$

$$EI_0 \cdot \delta_{p2}(x) = -\frac{1}{48} \cdot x_1^3 + \frac{13}{12} x_1$$

⟨B,C⟩

$$EI_0 \cdot \delta_{p1}(x) = -\frac{1}{8} x_2 + 1$$

$$EI_0 \cdot \delta_{p2}(x) = \frac{5}{288} \cdot x_2^3 - \frac{5}{12} \cdot x_2^2 + \frac{89}{36} \cdot x_2 - 2$$

⟨C,D⟩

$$EI_0 \cdot \delta_{p1}(x) = -\frac{1}{8} x_3$$

$$EI_0 \cdot \delta_{p1}(x) = -\frac{31}{36} x_3$$

$$W = \begin{vmatrix} 2,791667 & 1,083333 \\ 1,083333 & 5,388889 \end{vmatrix} = \underline{13,87037}$$

$$W = \begin{vmatrix} -\delta_{p1}(x) & 1,083333 \\ -\delta_{p2}(x) & 5,388889 \end{vmatrix} = -5,388889 \cdot \delta_{p1}(x) \cdot EI_0 + 1,083333 \cdot \delta_{p2}(x) \cdot EI_0$$

$$W = \begin{vmatrix} 2,791667 & -\delta_{p1}(x) \\ 1,083333 & -\delta_{p2}(x) \end{vmatrix} = -2,791667 \cdot \delta_{p2}(x) \cdot EI_0 + 1,083333 \cdot \delta_{p1}(x) \cdot EI_0$$

$$X_1 = \frac{W_1}{W} = \frac{1}{13,87037} \cdot (-5,388889 \cdot \delta_{p1}(x) \cdot EI_0 + 1,083333 \cdot \delta_{p2}(x) \cdot EI_0)$$

$$X_2 = \frac{W_2}{W} = \frac{1}{13,87037} \cdot (-2,791667 \cdot \delta_{p2}(x) \cdot EI_0 + 1,083333 \cdot \delta_{p1}(x) \cdot EI_0)$$

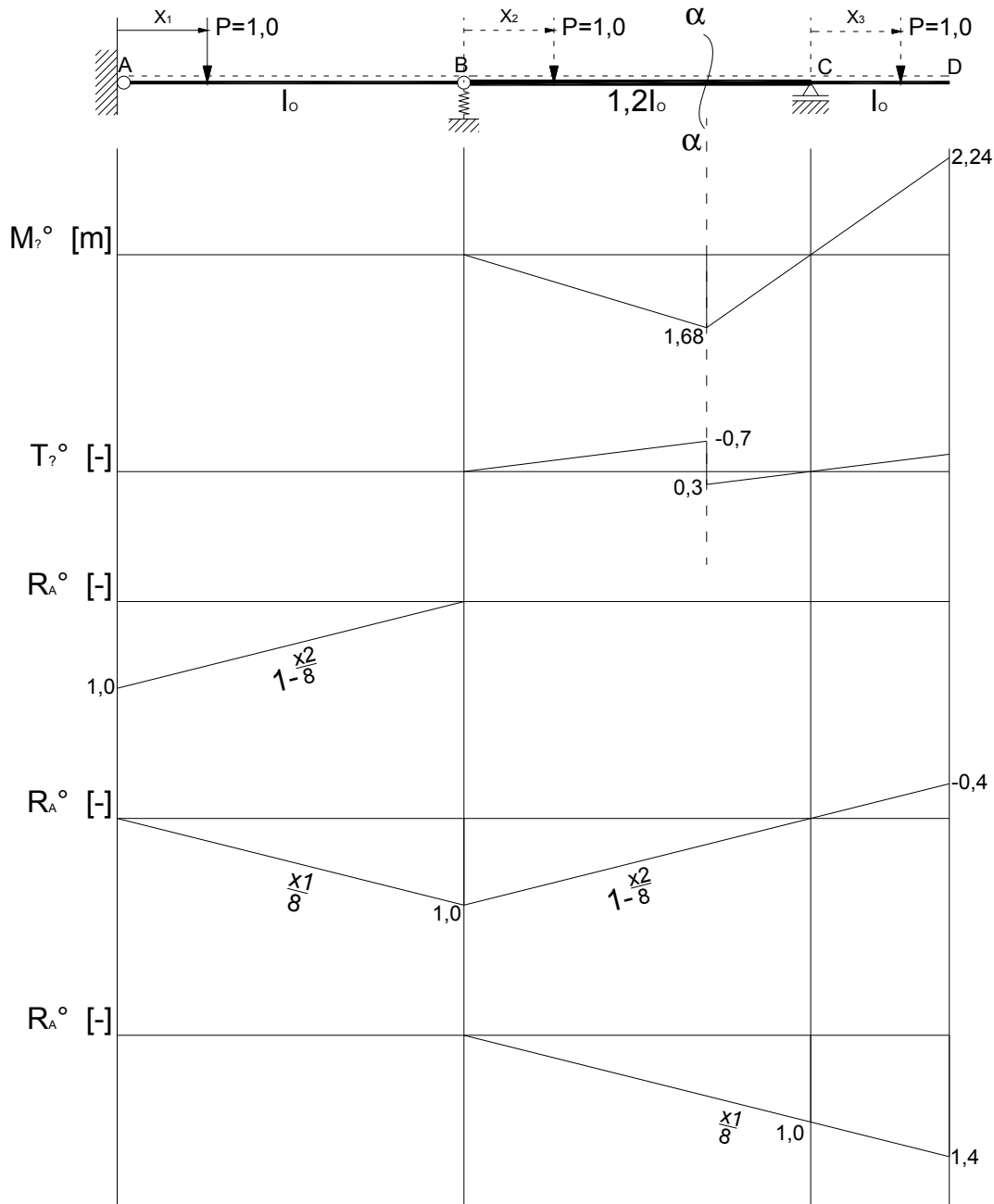
$$X_1 = -0,388518 \cdot \delta_{p1}(x) \cdot EI_0 + 0,078104 \cdot \delta_{p2}(x) \cdot EI_0$$

$$X_2 = -0,201268 \cdot \delta_{p2}(x) \cdot EI_0 + 0,078104 \cdot \delta_{p1}(x) \cdot EI_0$$

$$L_W M_\alpha = L_W M_\alpha^0 + L_W M_\alpha^{X_1=1} L_W X_1 + L_W M_\alpha^{X_2=1} L_W X_2 + \dots + L_W M_\alpha^{X_n=1} L_W X_n$$

$$L_W T_\alpha = L_W T_\alpha^0 + L_W T_\alpha^{X_1=1} L_W X_1 + L_W T_\alpha^{X_2=1} L_W X_2 + \dots + L_W T_\alpha^{X_n=1} L_W X_n$$

$$L_W R_1 = L_W R_1^0 + L_W R_1^{X_1=1} L_W X_1 + L_W R_1^{X_2=1} L_W X_2 + \dots + L_W R_1^{X_n=1} L_W X_n$$



$$L_W M_\alpha^{X_1=1} = -\frac{1}{8} \cdot 13,6 + \frac{1}{8} \cdot 5,6 + 1 = \underline{0,0}$$

$$L_W T_\alpha^{X_1=1} = \frac{1}{8} - \frac{1}{8} = \underline{0,0}$$

$$L_W M_\alpha^{X_2=1} = \frac{1}{8} \cdot 13,6 - \frac{2}{8} \cdot 5,6 = \underline{0,3}$$

$$L_W T_\alpha^{X_2=1} = \frac{1}{8} - \frac{2}{8} = \underline{-0,125}$$

$$L_W R_A^{X_1=1} = \underline{-0,125}$$

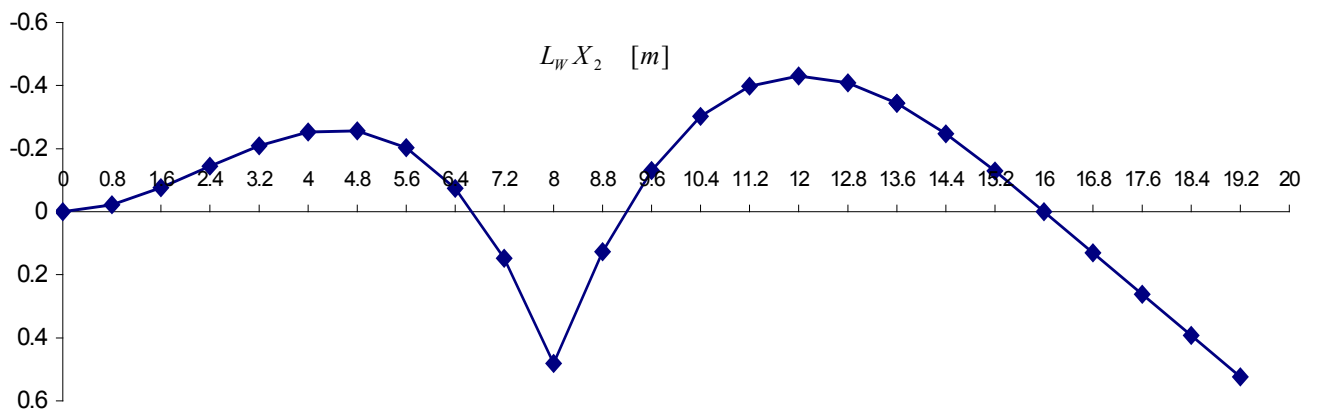
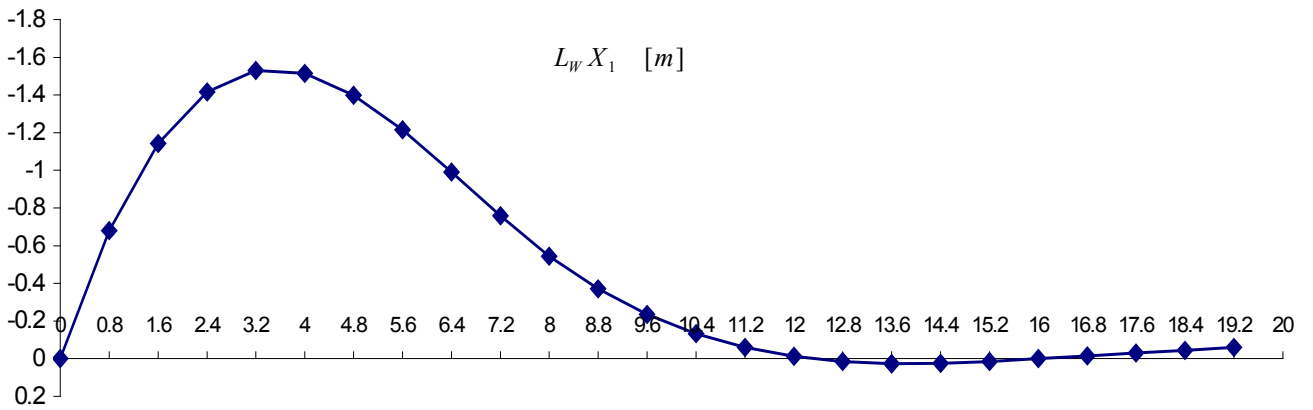
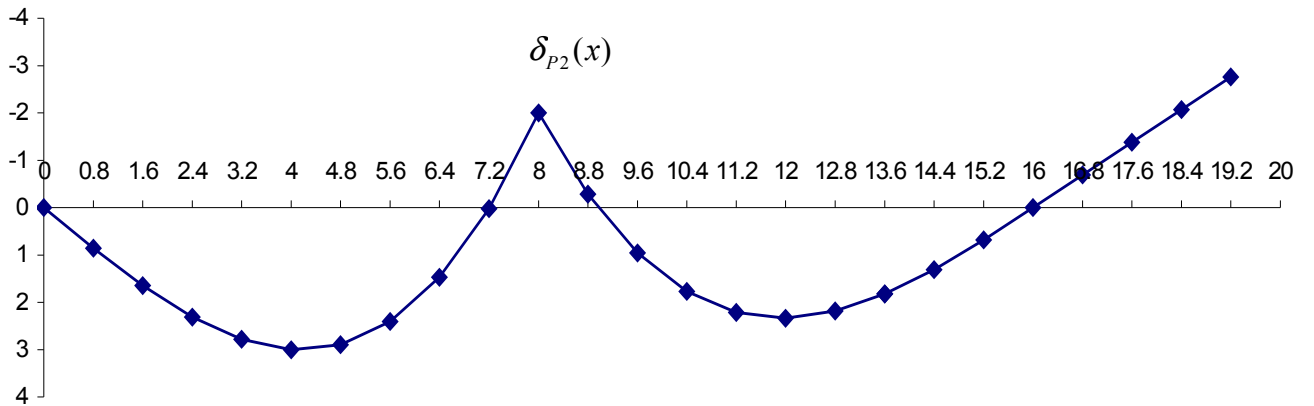
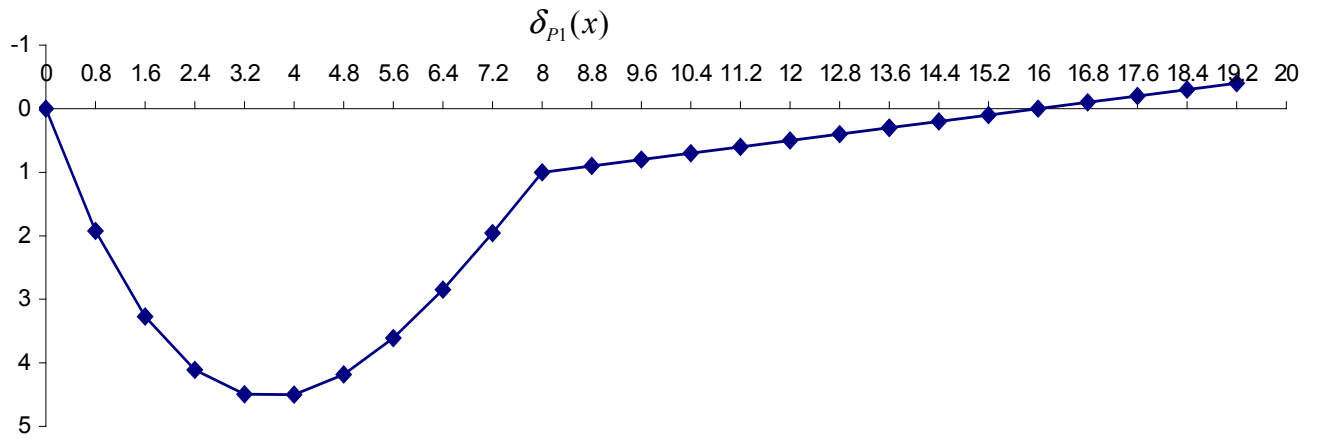
$$L_W R_B^{X_1=1} = \underline{0,125}$$

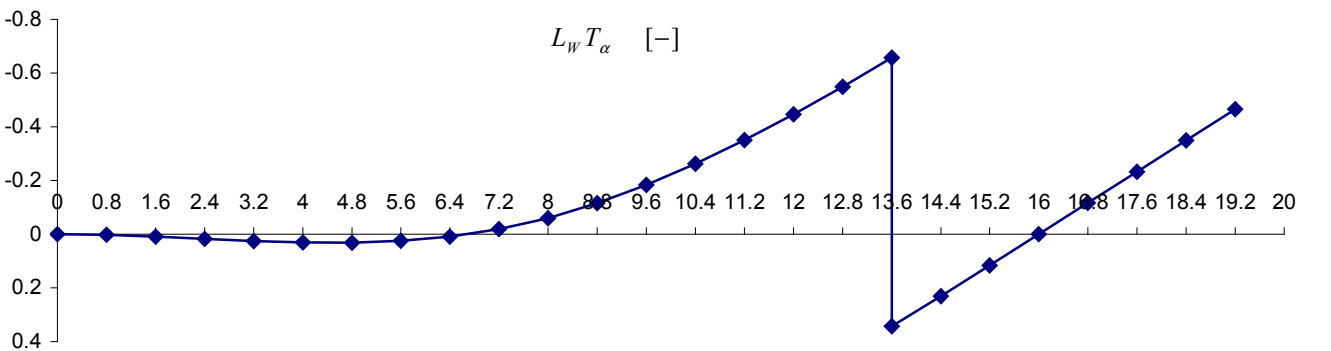
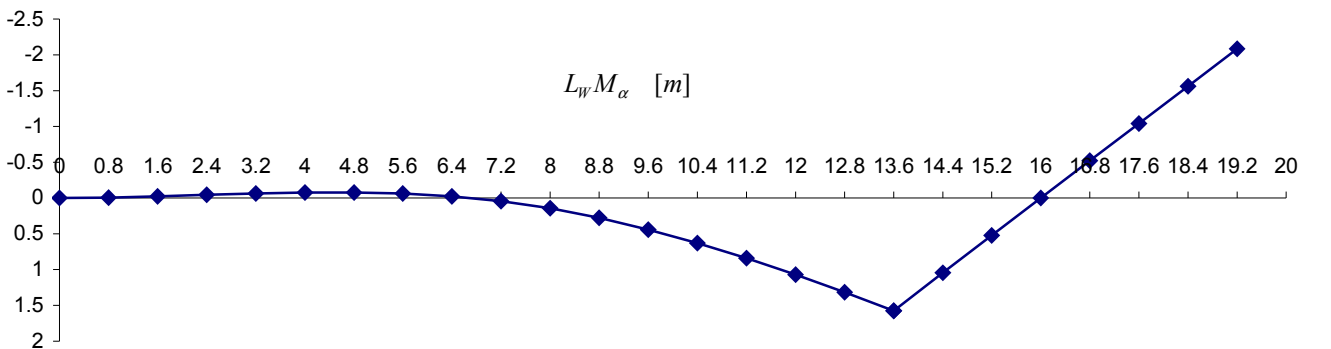
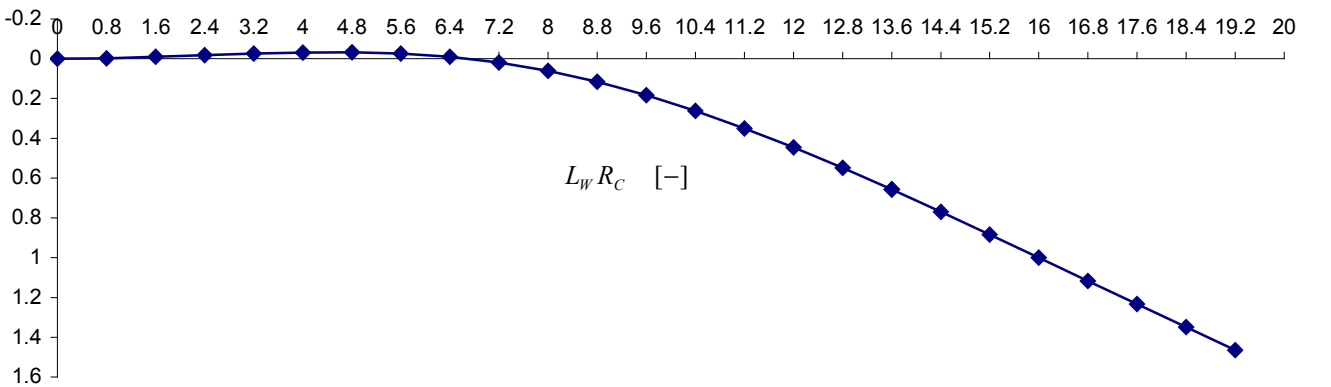
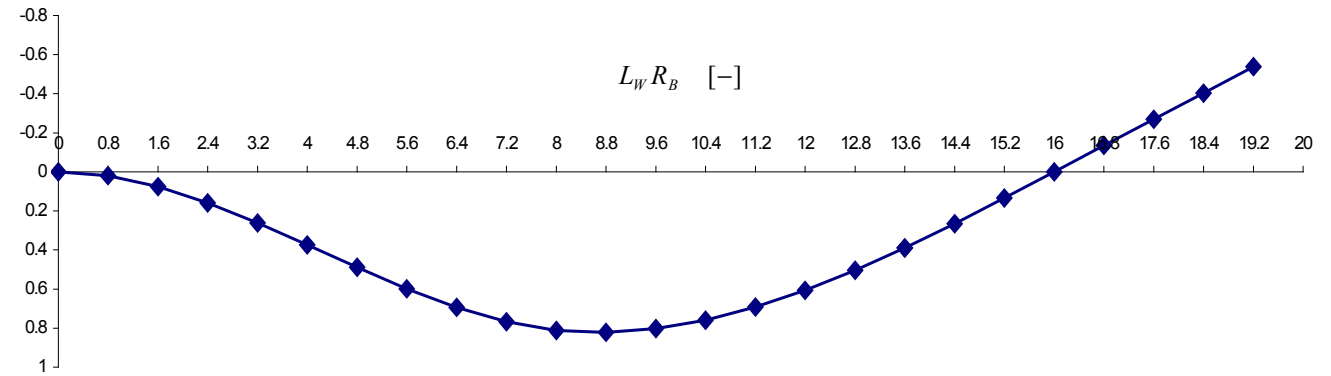
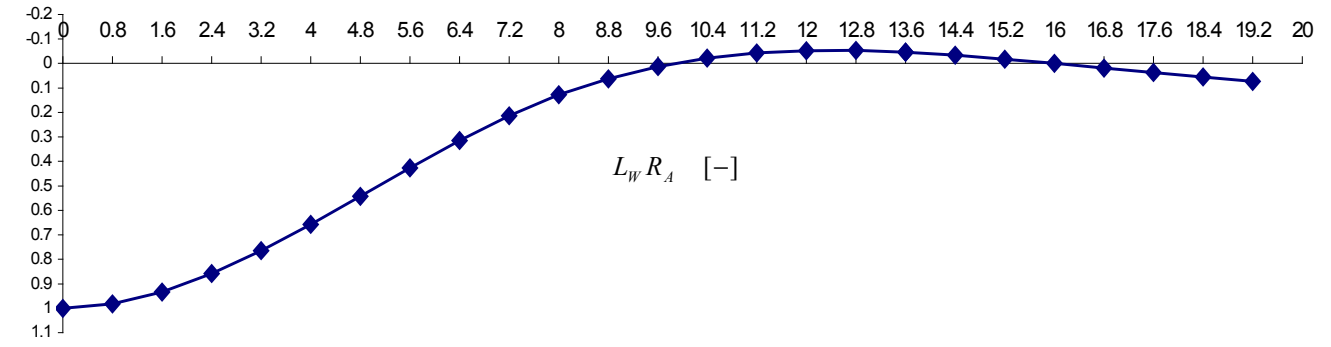
$$L_W R_C^{X_1=1} = \underline{0,0}$$

$$L_W R_A^{X_2=1} = \underline{0,125}$$

$$L_W R_B^{X_2=1} = \underline{-0,25}$$

$$L_W R_C^{X_2=1} = \frac{1}{8} - \frac{2}{8} = \underline{0,125}$$





x	$EI\delta_{p1}(X)$	$EI\delta_{p2}(X)$	$L_w X_1$	$L_w X_2$	$L_w M_\alpha^o$	$L_w T_\alpha^o$	$L_w R_A^o$	$L_w R_B^o$	$L_w R_C^o$	$L_w M_\alpha$	$L_w T_\alpha$	$L_w R_A$	$L_w R_B$	$L_w R_C$
0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
0.8	1.924	0.856	-0.680652	-0.02201	0	0	0.9	0.1	0	-0.0066	0.002752	0.98233	0.020422	-0.00275
1.6	3.272	1.648	-1.142515	-0.07613	0	0	0.8	0.2	0	-0.02284	0.009517	0.933298	0.076219	-0.00952
2.4	4.108	2.312	-1.415455	-0.14448	0	0	0.7	0.3	0	-0.04334	0.01806	0.858872	0.159188	-0.01806
3.2	4.496	2.784	-1.529335	-0.20917	0	0	0.6	0.4	0	-0.06275	0.026147	0.76502	0.261127	-0.02615
4	4.5	3	-1.514019	-0.25234	0	0	0.5	0.5	0	-0.0757	0.031542	0.65771	0.373832	-0.03154
4.8	4.184	2.896	-1.39937	-0.25609	0	0	0.4	0.6	0	-0.07683	0.032011	0.542911	0.4891	-0.03201
5.6	3.612	2.408	-1.215252	-0.20254	0	0	0.3	0.7	0	-0.06076	0.025318	0.426589	0.598729	-0.02532
6.4	2.848	1.472	-0.99153	-0.07383	0	0	0.2	0.8	0	-0.02215	0.009228	0.314713	0.694515	-0.00923
7.2	1.956	0.024	-0.758067	0.147941	0	0	0.1	0.9	0	0.044382	-0.01849	0.213251	0.768256	0.018493
8	1	-2	-0.544726	0.480641	0	0	0	1	0	0.144192	-0.06008	0.128171	0.811749	0.06008
8.8	0.9	-0.28	-0.371535	0.126649	0.24	-0.1	0	0.9	0.1	0.277995	-0.11583	0.062273	0.821896	0.115831
9.6	0.8	0.96	-0.235834	-0.13073	0.48	-0.2	0	0.8	0.2	0.44078	-0.18366	0.013138	0.803204	0.183658
10.4	0.7	1.773333	-0.133458	-0.30224	0.72	-0.3	0	0.7	0.3	0.629327	-0.26222	-0.0211	0.758879	0.26222
11.2	0.6	2.213333	-0.06024	-0.39861	0.96	-0.4	0	0.6	0.4	0.840417	-0.35017	-0.0423	0.692123	0.350174
12	0.5	2.333333	-0.012016	-0.43057	1.2	-0.5	0	0.5	0.5	1.070828	-0.44618	-0.05232	0.606142	0.446178
12.8	0.4	2.186667	0.0153805	-0.40887	1.44	-0.6	0	0.4	0.6	1.31734	-0.54889	-0.05303	0.504139	0.548892
13.6	0.3	1.826667	0.0261148	-0.34422	1.68	-0.7	0	0.3	0.7	1.576734	-0.65697	-0.04629	0.389319	0.656973
13.6	0.3	1.826667	0.0261148	-0.34422	1.68	0.3	0	0.3	0.7	1.576734	0.343027	-0.04629	0.389319	0.656973
14.4	0.2	1.306667	0.0243525	-0.24737	1.12	0.2	0	0.2	0.8	1.045789	0.230921	-0.03397	0.264887	0.769079
15.2	0.1	0.68	0.014259	-0.12905	0.56	0.1	0	0.1	0.9	0.521284	0.116132	-0.01791	0.134045	0.883868
16	0	0	0	0	0	0	0	0	1	0	0	0	0	1
16.8	-0.1	-0.68889	-0.014953	0.130841	-0.56	-0.1	0	-0.1	1.1	-0.52075	-0.11636	0.018224	-0.13458	1.116355
17.6	-0.2	-1.37778	-0.029907	0.261682	-1.12	-0.2	0	-0.2	1.2	-1.0415	-0.23271	0.036449	-0.26916	1.23271
18.4	-0.3	-2.06667	-0.04486	0.392523	-1.68	-0.3	0	-0.3	1.3	-1.56224	-0.34907	0.054673	-0.40374	1.349065
19.2	-0.4	-2.75556	-0.059813	0.523364	-2.24	-0.4	0	-0.4	1.4	-2.08299	-0.46542	0.072897	-0.53832	1.465421