

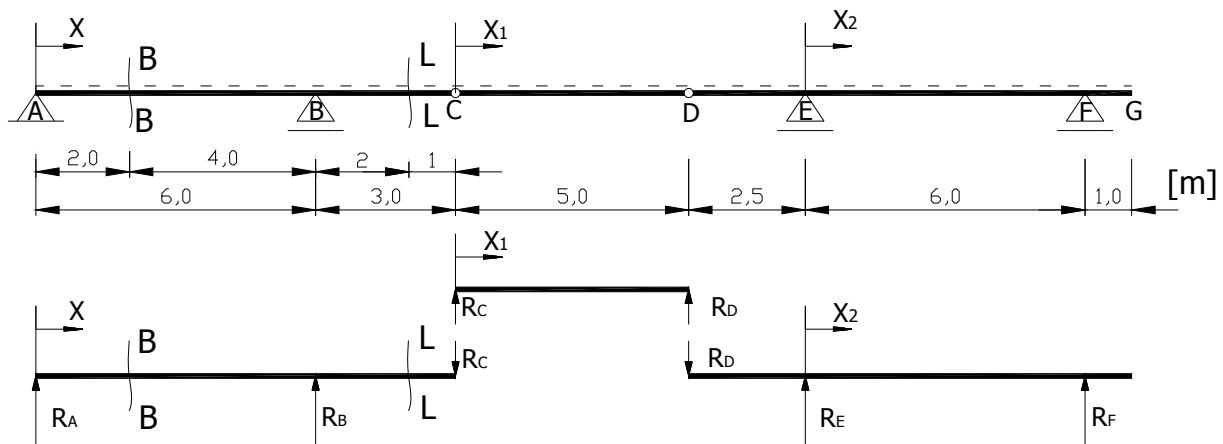
MECHANIKA BUDOWLI

Ćwiczenie nr 1

**LINIE WPŁYWOWE SIŁ W UKŁADACH STATYCZNIE
WYZNACZALNYCH**

Poznań 2004

BELKA nr 1. Wyznaczyć linie wpływu reakcji oraz sił przekrojowych:



L_{WR}A

$0 \leq x \leq 9$

$\sum M_B = 0$

$L_{WR}A \cdot 6 - P \cdot (6-x) = 0$

$L_{WR}A = 1 - \frac{x}{6}$

$L_{WR}A(0) = 1$

$L_{WR}A(6) = 0$

$L_{WR}A(9) = -\frac{1}{2}$

$0 \leq x_1 \leq 5$

$L_{WR}A = L_{WR}A(C) \cdot L_{WR}C$

$L_{WR}A = -\frac{1}{2} \cdot (1 - \frac{x_1}{5})$

$-2,5 \leq x_2 \leq 7$

$L_{WR}A = 0$

L_{WR}B

$0 \leq x \leq 9$

$\sum M_A = 0$

$-L_{WR}B \cdot 6 + P \cdot x = 0$

$L_{WR}B = \frac{x}{6}$

$L_{WR}B(0) = 0$

$L_{WR}B(6) = 1$

$L_{WR}B(9) = \frac{3}{2}$

$0 \leq x_1 \leq 5$

$L_{WR}B = L_{WR}B(C) \cdot L_{WR}C$

$L_{WR}B = \frac{3}{2} \cdot (1 - \frac{x_1}{5})$

$-2,5 \leq x_2 \leq 7$

$L_{WR}B = 0$

L_{WR}C

$0 \leq x \leq 9$

$L_{WR}C = 0$

$0 \leq x_1 \leq 5$

$\sum M_D = 0$

$L_{WR}C \cdot 5 - P \cdot (5-x_1) = 0$

$L_{WR}C = 1 - \frac{x_1}{5}$

$L_{WR}C(0) = 1$

$L_{WR}C(5) = 0$

$-2,5 \leq x_2 \leq 7$

$L_{WR}C = 0$

L_{WR}D

$0 \leq x \leq 9$

$L_{WR}D = 0$

$0 \leq x_1 \leq 5$

$\sum M_C = 0$

$-L_{WR}D \cdot 5 + P \cdot x_1 = 0$

$L_{WR}D = \frac{x_1}{5}$

$L_{WR}D(0) = 0$

$L_{WR}D(5) = 1$

$-2,5 \leq x_2 \leq 7$

$L_{WR}D = 0$

L_{WR}E

$-2,5 \leq x_2 \leq 7$

$\sum M_F = 0$

$L_{WR}E \cdot 6 - P \cdot (6-x_2) = 0$

$L_{WR}E = 1 - \frac{x_2}{6}$

$L_{WR}E(-2,5) = \frac{17}{12}$

$L_{WR}E(0) = 1$

$L_{WR}E(6) = 0$

$L_{WR}E(7) = -\frac{1}{6}$

$0 \leq x_1 \leq 5$

$L_{WR}E = L_{WR}E(D) \cdot L_{WR}D$

$L_{WR}E = \frac{17}{12} \cdot \frac{x_1}{5}$

$0 \leq x \leq 9$

$L_{WR}E = 0$

L_{WR}F

$-2,5 \leq x_2 \leq 7$

$\sum M_E = 0$

$-L_{WR}F \cdot 6 + P \cdot x_2 = 0$

$L_{WR}F = \frac{x_2}{6}$

$L_{WR}F(-2,5) = -\frac{5}{12}$

$L_{WR}F(0) = 0$

$L_{WR}F(6) = 1$

$L_{WR}F(7) = \frac{7}{6}$

$0 \leq x_1 \leq 5$

$L_{WR}F = L_{WR}F(D) \cdot L_{WR}D$

$L_{WR}F = -\frac{5}{12} \cdot \frac{x_1}{5}$

$0 \leq x \leq 9$

$L_{WR}F = 0$

L_{WMB}

$$0 \leq x \leq 2$$

$$\sum \mathbf{M}_B^P = \mathbf{0}$$

$$L_{WMB} = L_W R_B \cdot 4 = 0$$

$$L_{WMB} = \frac{x}{6} \cdot 4 = 0$$

$$L_{WMB}(0) = 0$$

$$L_{WMB}(2) = \frac{4}{3}$$

$$2 \leq x \leq 9$$

$$\sum \mathbf{M}_B^L = \mathbf{0}$$

$$L_{WMB} = L_W R_A \cdot 2 = 0$$

$$L_{WMB} = \left(1 - \frac{x}{6}\right) \cdot 2 = 0$$

$$L_{WMB}(2) = \frac{4}{3}$$

$$L_{WMB}(6) = 0$$

$$L_{WMB}(9) = -1$$

$$0 \leq x_1 \leq 5$$

$$L_{WMB} = L_{WMB}(C) \cdot L_W R_C$$

$$L_{WMB} = -1 \cdot \left(1 - \frac{x_1}{5}\right)$$

$$-2,5 \leq x_2 \leq 7$$

$$L_{WMB} = 0$$

L_{WML}

$$0 \leq x \leq 8$$

$$\sum \mathbf{M}_L^P = \mathbf{0}$$

$$L_{WML} = 0$$

$$8 \leq x \leq 9$$

$$\sum \mathbf{M}_L^L = \mathbf{0}$$

$$L_{WML} = R_A \cdot 8 + R_B \cdot 2$$

$$L_{WML} = \left(1 - \frac{x}{6}\right) \cdot 8 + \frac{x}{6} \cdot 2$$

$$L_{WML} = 8 - x$$

$$L_{WML}(8) = 0$$

$$L_{WML}(9) = -1$$

$$0 \leq x_1 \leq 5$$

$$L_{WML} = L_{WML}(C) \cdot L_W R_C$$

$$L_{WML} = -1 \cdot \left(1 - \frac{x_1}{5}\right)$$

$$-2,5 \leq x_2 \leq 7$$

$$L_{WML} = 0$$

L_{WTB}

$$0 \leq x < 2$$

$$\sum \mathbf{M}_B^P = \mathbf{0}$$

$$L_{WTB} = L_W R_B$$

$$L_{WTB} = \frac{x}{6}$$

$$L_{WTB}(0) = 0$$

$$L_{WTB}(2) = -\frac{1}{3}$$

$$2 < x \leq 9$$

$$\sum \mathbf{Y}_B^L = \mathbf{0}$$

$$L_{WTB} = L_W R_A$$

$$L_{WTB} = \left(1 - \frac{x}{6}\right)$$

$$L_{WTB}(2) = \frac{2}{3}$$

$$L_{WTB}(6) = 0$$

$$L_{WTB}(9) = -\frac{1}{2}$$

$$0 \leq x_1 \leq 5$$

$$L_{WTB} = L_{WTB}(C) \cdot L_W R_C$$

$$L_{WTB} = -\frac{1}{2} \cdot \left(1 - \frac{x_1}{5}\right)$$

$$-2,5 \leq x_2 \leq 7$$

$$L_{WTB} = 0$$

L_{WTL}

$$0 \leq x \leq 8$$

$$\sum \mathbf{Y}^P = \mathbf{0}$$

$$L_{WTL} = 0$$

$$8 \leq x \leq 9$$

$$\sum \mathbf{Y}^L = \mathbf{0}$$

$$L_{WTL} = R_A + R_B$$

$$L_{WTL} = \left(1 - \frac{x}{6}\right) + \frac{x}{6}$$

$$L_{WTL} = 1$$

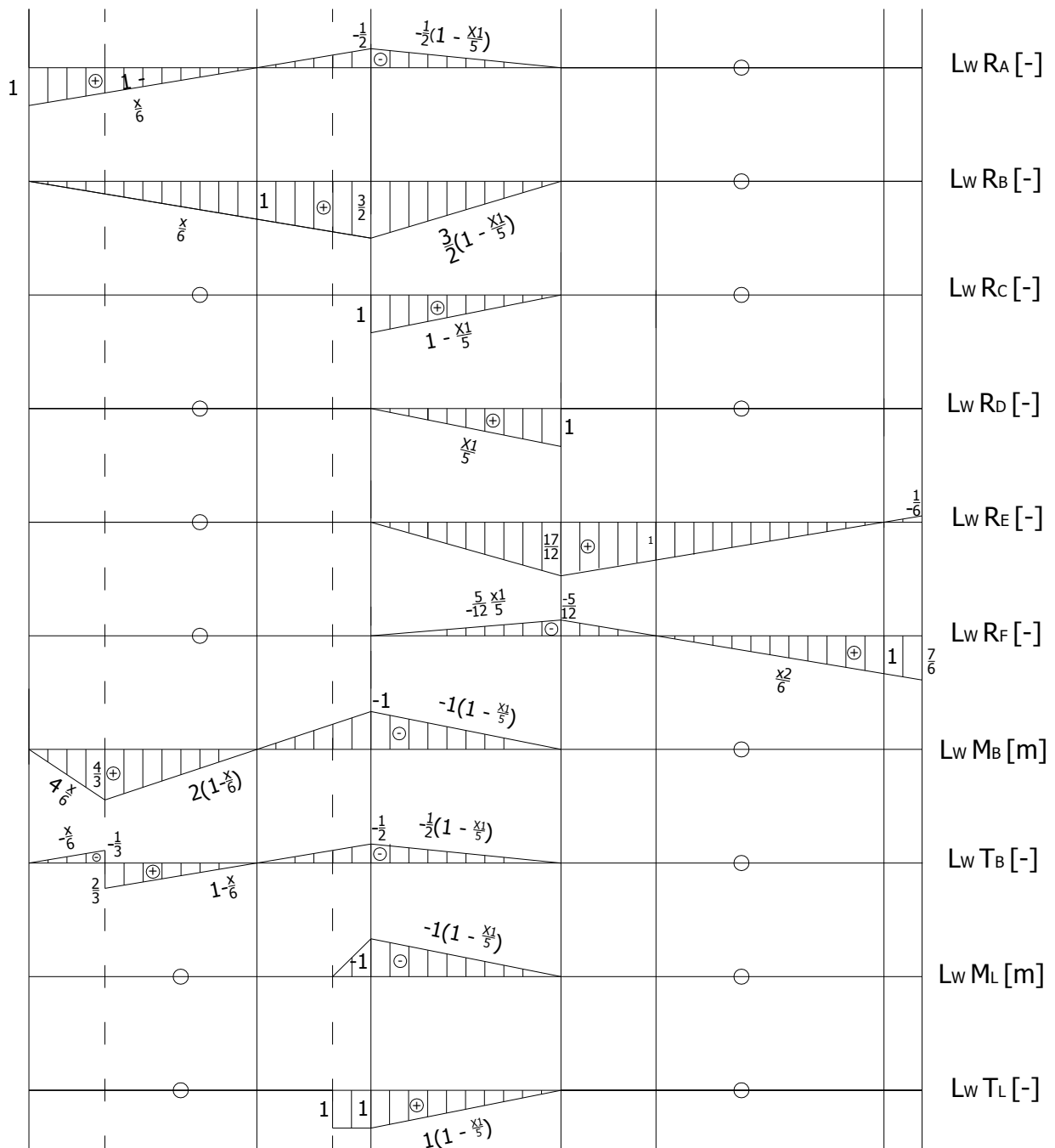
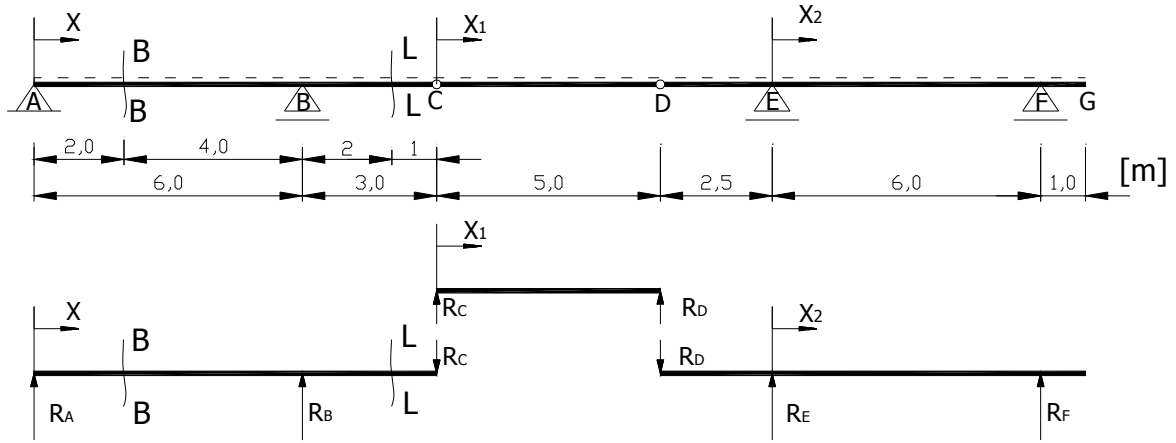
$$0 \leq x_1 \leq 5$$

$$L_{WTL} = L_{WTL}(C) \cdot L_W R_C$$

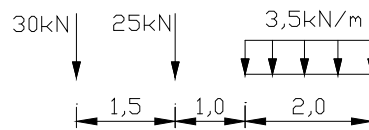
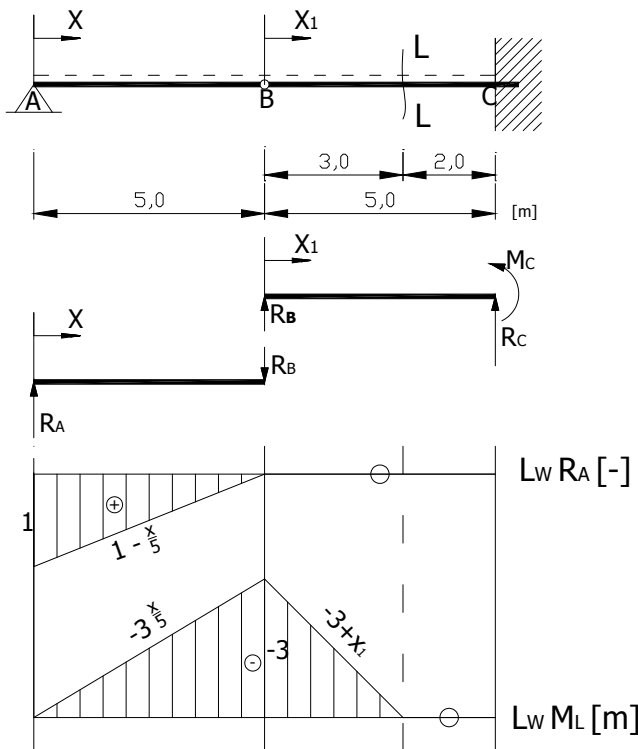
$$L_{WTL} = 1 \cdot \left(1 - \frac{x_1}{5}\right)$$

$$-2,5 \leq x_2 \leq 7$$

$$L_{WTL} = 0$$



BELKA nr 2. Wyznaczyć ekstremalne wartości R_A , M_L od zadanego obciążenia ruchomego:



▪ **Linie wpływu R_A, M_L :**

$L_W R_A$

$0 \leq x \leq 5$

$\sum M_B = 0$

$L_W R_A \cdot 5 - P(5-x) = 0$

$L_W R_A = (1 - \frac{x}{5})$

$L_W R_A(0) = 1$

$L_W R_A(5) = 0$

$0 \leq x_1 \leq 5$

$L_W R_A = 0$

$L_W M_L$

$0 \leq x_1 \leq 3$

$\sum M_L = 0$

$L_W M_L = -L_W R_B \cdot 3 - P(3-x_1)$

$L_W M_L = -3 + x_1$

$L_W M_L(0) = 3$

$L_W M_L(3) = 0$

$3 \leq x_1 \leq 5$

$\sum M_L = 0$

$L_W M_L = -L_W R_B \cdot 3$

$L_W M_L = 0$

$0 \leq x \leq 5$

$L_W M_L = 0$

▪ **Ekstrema R_A :**

Jazda w prawo:

$R_A = 3,5 \cdot 1 \cdot \frac{1}{2} \cdot (1 + \frac{4}{5}) = 3,15 \text{ kN}$

$R_A = 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (1 + \frac{3}{5}) = 5,6 \text{ kN}$

$R_A = 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (\frac{4}{5} + \frac{2}{5}) + 25 \cdot 1 = 29,2 \text{ kN}$

$R_A = 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (\frac{3}{5} + \frac{1}{5}) + 25 \cdot \frac{4}{5} = 22,8 \text{ kN}$

Jazda w lewo:

$R_A = 30 \cdot \frac{1}{5} = 6 \text{ kN}$

$R_A = 30 \cdot \frac{2}{5} + 25 \cdot \frac{1}{10} = 14,5 \text{ kN}$

$R_A = 30 \cdot \frac{3}{5} + 25 \cdot \frac{3}{10} + 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{10} = 25,587 \text{ kN}$

$R_A = 30 \cdot \frac{4}{5} + 25 \cdot \frac{5}{10} + 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{10} = 37,287 \text{ kN}$

$R_A = 30 \cdot 1 + 25 \cdot \frac{7}{10} + 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (\frac{5}{10} + \frac{1}{10}) = 49,6 \text{ kN}$

$R_A = 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \frac{2}{5} + 25 \cdot \frac{3}{5} + 30 \cdot \frac{9}{10} = 43,4 \text{ kN}$

$R_A = 3,5 \cdot 1 \cdot \frac{1}{2} \cdot \frac{1}{5} + 25 \cdot \frac{2}{5} + 30 \cdot \frac{5}{10} = 25,35 \text{ kN}$

$R_A = 25 \cdot \frac{1}{5} + 30 \cdot \frac{3}{10} = 14 \text{ kN}$

$R_A = 30 \cdot \frac{1}{10} = 3 \text{ kN}$

$R_A = 25 \cdot \frac{9}{10} + 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (\frac{7}{10} + \frac{3}{10}) = 26 \text{ kN}$

$R_A = 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (\frac{9}{10} + \frac{5}{10}) = 4,55 \text{ kN}$

$R_A = 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot (1 + \frac{7}{10}) = 4,462 \text{ kN}$

$R_A = 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot (1 + \frac{9}{10}) = 1,662 \text{ kN}$

**$R_{Amax} = 49,6 \text{ kN}$
 $R_{Amin} = 0 \text{ kN}$**

▪ Ekstrema M_L :

Jazda w prawo:

$$M_L = -3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{5} = -0,525 \text{ kNm}$$

$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot \frac{6}{5} = -4,2 \text{ kNm}$$

$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{9}{5} + \frac{3}{5}\right) = -8,4 \text{ kNm}$$

$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{12}{5} + \frac{6}{5}\right) - 25 \cdot \frac{3}{5} = -27,6 \text{ kNm}$$

$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(3 + \frac{9}{5}\right) - 25 \cdot \frac{6}{5} - 30 \cdot \frac{3}{10} = -55,8 \text{ kNm}$$

$$M_L = -3,5 \cdot 1 \cdot \frac{1}{2} \cdot \left(3 + \frac{12}{5}\right) - 3,5 \cdot 1 \cdot \frac{1}{2} \cdot (3 + 2) - 25 \cdot \frac{9}{5} - 30 \cdot \frac{9}{10} = -89,15 \text{ kNm}$$

$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot (3 + 1) - 25 \cdot \frac{12}{5} - 30 \cdot \frac{15}{10} = -119 \text{ kNm}$$

$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot 2 - 25 \cdot 3 - 30 \cdot \frac{21}{10} = -145 \text{ kNm}$$

$$M_L = -3,5 \cdot 1 \cdot \frac{1}{2} \cdot 1 - 25 \cdot 2 - 30 \cdot \frac{28}{10} = -135,75 \text{ kNm}$$

$$M_L = -25 \cdot 1 - 30 \cdot \frac{5}{2} = -100 \text{ kNm}$$

$$M_L = -30 \cdot \frac{3}{2} = -45 \text{ kNm}$$

Jazda w lewo:

$$M_L = -30 \cdot 1 = -30 \text{ kNm}$$

$$M_L = -30 \cdot 2 - 25 \cdot \frac{1}{2} = -72,5 \text{ kNm}$$

$$M_L = -30 \cdot 3 - 25 \cdot \frac{3}{2} - 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = -127,937 \text{ kNm}$$

$$M_L = -30 \cdot \frac{12}{5} - 25 \cdot \frac{5}{2} - 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} = 138,437 \text{ kNm}$$

$$M_L = -30 \cdot \frac{9}{5} - 25 \cdot \frac{28}{10} - 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \left(\frac{5}{2} + \frac{1}{2}\right) = -161,875 \text{ kNm}$$

$$M_L = -30 \cdot \frac{6}{5} - 25 \cdot \frac{21}{10} - 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \left(3 + \frac{28}{10}\right) - 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \left(3 + \frac{3}{2}\right) = -105,387 \text{ kNm}$$

$$M_L = -30 \cdot \frac{3}{5} - 25 \cdot \frac{15}{10} - 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \left(3 + \frac{21}{10}\right) - 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \left(3 + \frac{5}{2}\right) = -73,7 \text{ kNm}$$

$$M_L = -25 \cdot \frac{9}{10} - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{28}{10} + \frac{15}{10}\right) = -37,55 \text{ kNm}$$

$$M_L = -25 \cdot \frac{3}{10} - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{21}{10} + \frac{9}{10}\right) = -18 \text{ kNm}$$

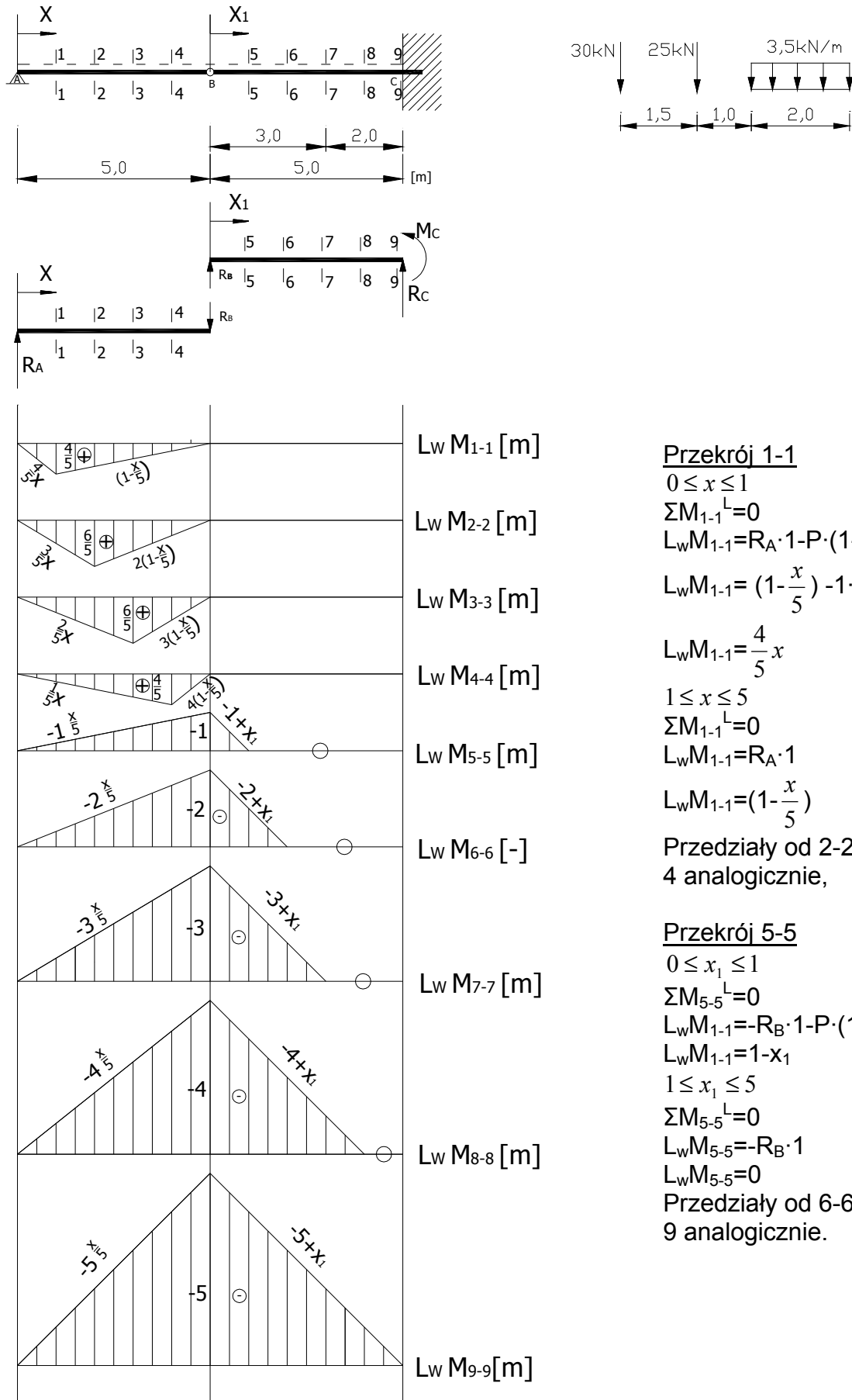
$$M_L = -3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{15}{10} + \frac{3}{10}\right) = -6,3 \text{ kNm}$$

$$M_L = -3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{9}{10} = -2,363 \text{ kNm}$$

$$M_L = -3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{10} = -0,263 \text{ kNm}$$

$M_{L\max} = 0$ $M_{L\min} = -161,875 \text{ kNm}$

▪ Obwiednia momentów (od zadanego obciążenia ruchomego)



Wartości ekstremalne M w poszczególnych przekrojach:

1-1

$$M_{\max} = 30 \cdot \frac{4}{5} + 25 \cdot \frac{5}{10} + 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{10}$$

$$M_{\max} = 37,2875 \text{ kNm}$$

$$M_{\min} = 0$$

2-2

$$M_{\max} = 30 \cdot \frac{6}{5} + 25 \cdot \frac{3}{5} + 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{5}$$

$$M_{\max} = 51,175 \text{ kNm}$$

$$M_{\min} = 0$$

3-3

$$M_{\max} = 30 \cdot \frac{6}{5} + 25 \cdot \frac{3}{5} + 3,5 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{5}$$

$$M_{\max} = 51,175 \text{ kNm}$$

$$M_{\min} = 0$$

4-4

$$M_{\max} = 30 \cdot \frac{4}{5} + 25 \cdot \frac{5}{10} + 3,5 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{10}$$

$$M_{\max} = 37,2875 \text{ kNm}$$

$$M_{\min} = 0$$

5-5

$$M_{\max} = 0$$

$$M_{\min} = -30 \cdot 1 - 25 \cdot \frac{7}{10} - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{1}{10} + \frac{5}{10}\right)$$

$$M_{\min} = -49,6 \text{ kNm}$$

6-6

$$M_{\max} = 0$$

$$M_{\min} = -30 \cdot 2 - 25 \cdot \frac{14}{10} - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(1 + \frac{1}{5}\right)$$

$$M_{\min} = -99,2 \text{ kNm}$$

7-7

$$M_{\max} = 0$$

$$M_{\min} = -30 \cdot 3 - 25 \cdot \frac{21}{10} - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(\frac{15}{10} + \frac{3}{10}\right)$$

$$M_{\min} = -148,8 \text{ kNm}$$

8-8

$$M_{\max} = 0$$

$$M_{\min} = -30 \cdot 4 - 25 \cdot \frac{28}{10} - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot \left(2 + \frac{2}{5}\right)$$

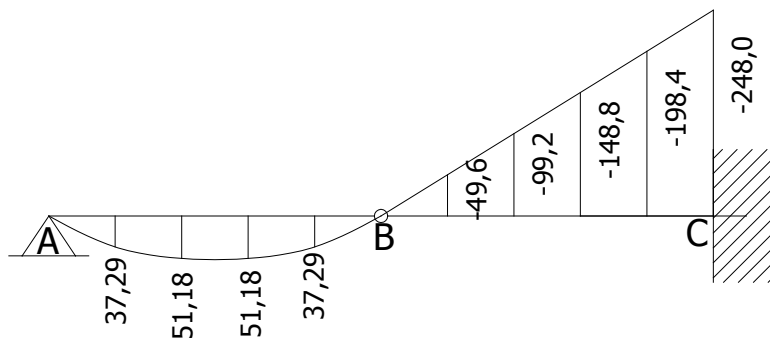
$$M_{\min} = -198,4 \text{ kNm}$$

9-9

$$M_{\max} = 0$$

$$M_{\min} = -30 \cdot 5 - 25 \cdot 3,5 - 3,5 \cdot 2 \cdot \frac{1}{2} \cdot (2,5 + 0,5)$$

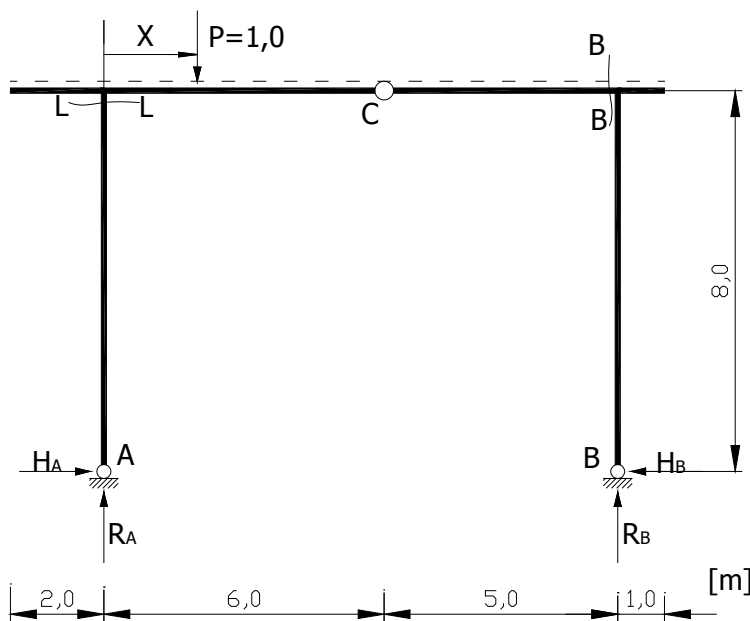
$$M_{\min} = -248 \text{ kNm}$$



Obwiednia momentów [kNm]

Rama.

Wyznaczyć linie wpływowe reakcji oraz sił przekrojowych:



$\underline{L_W R_A}$

$$\begin{aligned} & -2 \leq x \leq 12 \\ & \sum M_B = 0 \\ & L_W R_A \cdot 11 - P(11-x) = 0 \\ & L_W R_A = 1 - \frac{x}{11} \\ & L_W R_A(-2) = \frac{13}{11} \\ & L_W R_A(0) = 1 \\ & L_W R_A(11) = 0 \\ & L_W R_A(12) = -\frac{1}{11} \end{aligned}$$

$\underline{L_W R_B}$

$$\begin{aligned} & -2 \leq x \leq 12 \\ & \sum M_A = 0 \\ & -L_W R_B \cdot 11 + P \cdot x = 0 \\ & L_W R_B = \frac{x}{11} \\ & L_W R_B(-2) = -\frac{2}{11} \\ & L_W R_B(0) = 0 \\ & L_W R_B(11) = 1 \\ & L_W R_B(12) = \frac{12}{11} \end{aligned}$$

$\underline{L_W H_A} = \underline{L_W H_B} = \underline{L_W H}$

$$\begin{aligned} & \underline{L_W H} \\ & 6 \leq x \leq 12 \\ & \sum M_C^P = 0 \\ & -L_W H \cdot 8 + L_W R_A \cdot 6 = 0 \\ & L_W H = \left(1 - \frac{x}{11}\right) \cdot \frac{3}{4} \\ & L_W H(6) = \frac{15}{44} = 0,341 \\ & L_W H(11) = 0 \\ & L_W H(12) = -\frac{3}{44} \end{aligned}$$

$\underline{L_W H}$

$$\begin{aligned} & -2 \leq x \leq 6 \\ & \sum M_C^L = 0 \\ & L_W H \cdot 8 - L_W R_B \cdot 5 = 0 \\ & L_W H = \frac{x}{11} \cdot \frac{5}{8} \\ & L_W H(-2) = -\frac{10}{88} \\ & L_W H(0) = 0 \\ & L_W H(6) = \frac{30}{88} = 0,341 \end{aligned}$$

$\underline{L_W M_L}$

$$\begin{aligned} & -2 \leq x \leq 12 \\ & \sum M_L = 0 \\ & L_W M_L = -L_W H(x) \cdot 8 \\ & -2 \leq x \leq 6 \\ & L_W M_L = -\frac{5x}{11} \\ & L_W M_L(-2) = \frac{10}{11} = 0,909 \\ & L_W M_L(0) = 0 \\ & L_W M_L(6) = -\frac{30}{11} = -2,727 \end{aligned}$$

$\underline{L_W M_L}$

$$\begin{aligned} & 6 \leq x \leq 12 \\ & L_W M_L = -\left(1 - \frac{x}{11}\right) \cdot 6 \\ & L_W M_L(6) = -\frac{30}{11} \\ & L_W M_L(11) = 0 \\ & L_W M_L(12) = \frac{6}{11} \end{aligned}$$

$\underline{L_W N_L}$

$$\begin{aligned} & -2 \leq x \leq 12 \\ & \sum Y_L = 0 \\ & L_W N_L = -L_W R_A \\ & L_W N_L = -\left(1 - \frac{x}{11}\right) \\ & L_W N_L(-2) = -\frac{13}{11} \\ & L_W N_L(0) = -1 \\ & L_W N_L(6) = -\frac{5}{11} \\ & L_W N_L(11) = 0 \\ & L_W N_L(12) = \frac{1}{11} \end{aligned}$$

$L_W T_L$

$$-2 \leq x \leq 12$$

$$\sum Y_L = 0$$

$$L_W T_L = -L_W H(x)$$

$$-2 \leq x \leq 6$$

$$L_W T_L = -\frac{x}{11} \cdot \frac{5}{8}$$

$$L_W T_L(-2) = \frac{10}{88} = 0,114$$

$$L_W T_L(0) = 0$$

$$L_W T_L(6) = -\frac{30}{88} = -0,341$$

$$6 \leq x \leq 12$$

$$L_W T_L = -\left(1 - \frac{x}{11}\right) \cdot \frac{3}{4}$$

$$L_W T_L(6) = -\frac{15}{44} = -0,341$$

$$L_W T_L(11) = 0$$

$$L_W T_L(12) = \frac{3}{44} = 0,068$$

 $L_W M_B$

$$x \in <-2, 11>$$

$$\sum M_B^L = 0$$

$$L_W M_B = -L_W H(x) \cdot 8 - P \cdot (11 - x) + R_A \cdot 11$$

$$-2 \leq x \leq 6$$

$$L_W M_B = -\frac{5x}{11}$$

$$L_W M_B(-2) = \frac{10}{11} = 0,909$$

$$L_W M_B(0) = 0$$

$$L_W M_B(6) = -\frac{30}{11} = -2,727$$

 $L_W N_B$

$$x \in <-2, 11>$$

$$\sum Y_B^L = 0$$

$$L_W N_B = -L_W H(x)$$

$$-2 \leq x \leq 6$$

$$L_W N_B = -\frac{x}{11} \cdot \frac{5}{8}$$

$$L_W N_B(-2) = \frac{10}{88} = 0,114$$

$$L_W N_B(0) = 0$$

$$L_W N_B(6) = -\frac{30}{88} = -0,341$$

 $L_W T_B$

$$x \in <-2, 11>$$

$$\sum Y_B^L = 0$$

$$L_W T_L = L_W R_A - P$$

$$L_W T_B = -\frac{x}{11}$$

$$L_W T_B(-2) = \frac{2}{11}$$

$$L_W T_B(0) = 0$$

$$L_W T_B(6) = -\frac{6}{11}$$

$$L_W T_B(11) = -1$$

$$6 \leq x \leq 12$$

$$L_W M_B = -\left(1 - \frac{x}{11}\right) \cdot 6$$

$$L_W M_B(6) = -\frac{30}{11} = -2,727$$

$$L_W M_B(11) = 0$$

$$6 \leq x \leq 11$$

$$L_W N_B = -\frac{3}{4} \cdot \left(1 - \frac{x}{11}\right)$$

$$L_W N_B(6) = -\frac{15}{44} = -0,341$$

$$L_W N_B(11) = 0$$

$$x \in (11, 12 >$$

$$\sum Y_B^L = 0$$

$$L_W T_L = L_W R_A$$

$$L_W T_B = 1 - \frac{x}{11}$$

$$L_W T_B(11) = 0$$

$$L_W T_B(12) = -\frac{1}{11}$$

$$x \in <11, 12 >$$

$$\sum M_B^P = 0$$

$$L_W M_B = -L_W H(x) \cdot 8$$

$$L_W M_B = -6 \left(1 - \frac{x}{11}\right)$$

$$L_W M_B(11) = 0$$

$$L_W M_B(12) = -\frac{6}{11}$$

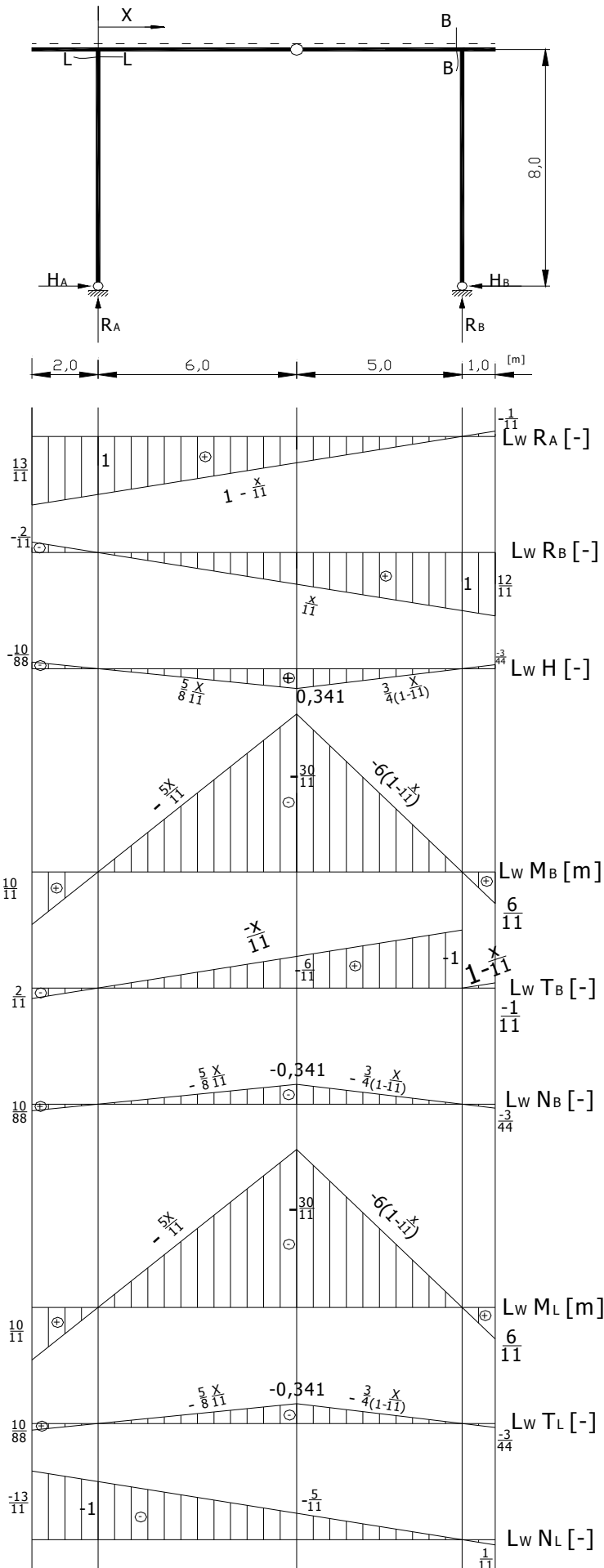
$$x \in <11, 12 >$$

$$\sum Y_B^P = 0$$

$$L_W N_B = -L_W H(x)$$

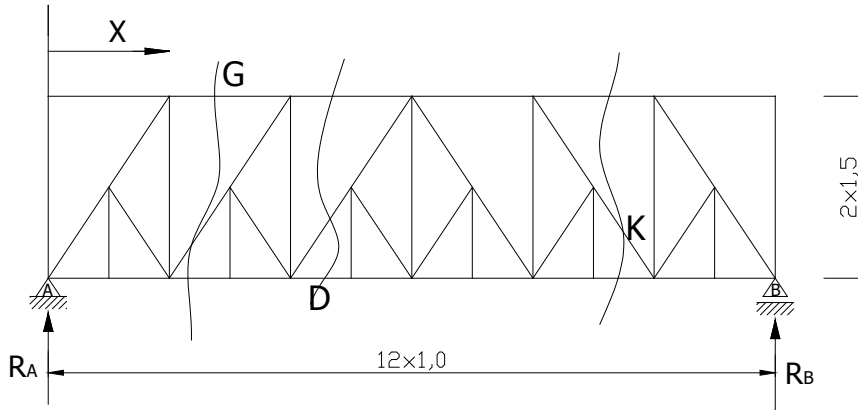
$$L_W N_B = -\frac{3}{4} \cdot \left(1 - \frac{x}{11}\right)$$

$$L_W N_B(12) = -\frac{3}{44}$$



Kratownica:

Wyznaczyć linie wpływowe reakcji oraz sił w zaznaczonych prętach:



L_{WR_A}

$0 \leq x \leq 12$

$\sum M_B = 0$

$L_{WR_A} \cdot 12 - P(12-x) = 0$

$L_{WR_A} = 1 - \frac{x}{12}$

$L_{WR_A}(0) = 1$

$L_{WR_A}(12) = 0$

L_{WR_B}

$0 \leq x \leq 12$

$\sum M_B = 0$

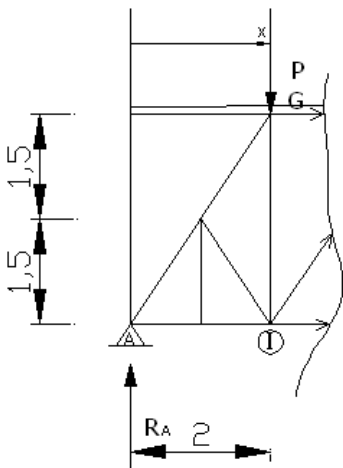
$-L_{WR_B} \cdot 12 + P \cdot x = 0$

$L_{WR_B} = \frac{x}{12}$

$L_{WR_B}(0) = 0$

$L_{WR_B}(12) = 1$

L_{WG}



$0 \leq X \leq 2$

$\sum M_I^L = 0$

$L_{WR_A} \cdot 2 - P(2-x) + L_{WG} \cdot 3 = 0$

$L_{WG} = -\frac{5}{18}x$

$L_{WG}(0) = 0$

$L_{WG}(2) = -\frac{5}{9}$

$4 \leq X \leq 12$

$\sum M_I^L = 0$

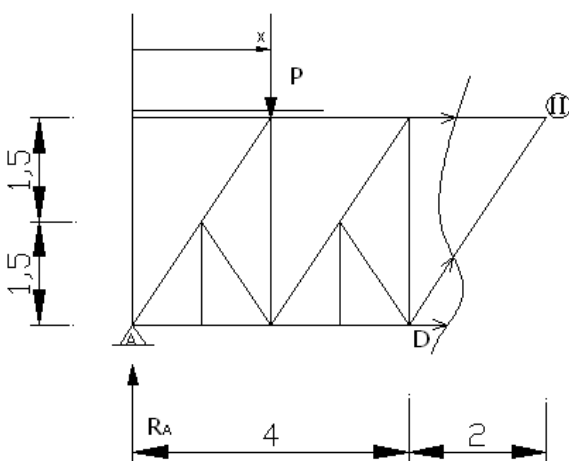
$L_{WR_A} \cdot 2 + L_{WG} \cdot 3 = 0$

$L_{WG} = -\frac{2}{3} \left(1 - \frac{X}{12}\right)$

$L_{WG}(4) = -\frac{4}{9}$

$L_{WG}(12) = 0$

L_{WD}



$0 \leq X \leq 4$

$\sum M_{II}^L = 0$

$L_{WR_A} \cdot 6 - P(6-x) - L_{WD} \cdot 3 = 0$

$L_{WD} = \frac{1}{6}x$

$L_{WD}(0) = 0$

$L_{WD}(4) = \frac{2}{3}$

$L_{WD}(12) = 0$

$5 \leq X \leq 12$

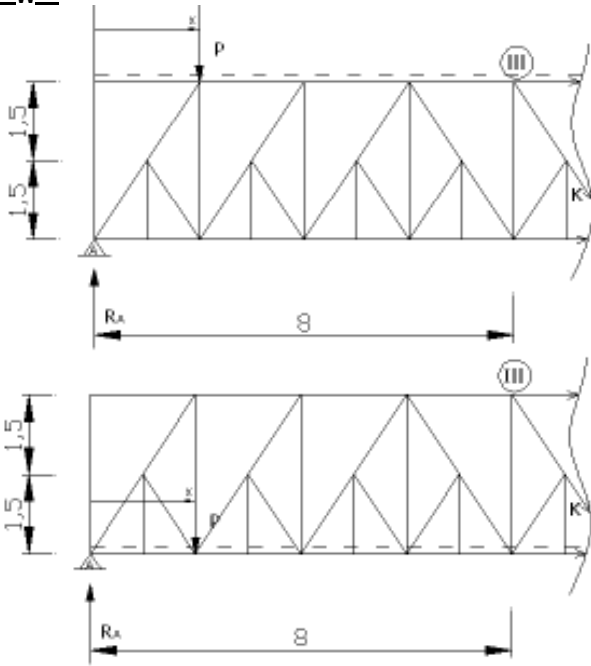
$\sum M_{II}^L = 0$

$L_{WR_A} \cdot 6 + L_{WD} \cdot 3 = 0$

$L_{WD} = 2 \left(1 - \frac{x}{12}\right)$

$L_{WD}(5) = \frac{7}{6}$

L_wK



$$0 \leq X \leq 9$$

$$\cos\alpha = \frac{3}{\sqrt{3^2 + 2^2}}$$

$$\sum Y^L = 0$$

$$L_W R_A - P - L_W K \cdot \cos\alpha = 0$$

$$L_W K = -\frac{x}{12} \cdot \frac{\sqrt{3^2 + 2^2}}{3}$$

$$L_W K(0) = 0$$

$$L_W K(8) = -0,8$$

$$L_W K(9) = -0,9$$

$$10 \leq X \leq 12$$

$$\sum M_{II}^L = 0$$

$$L_W R_A - L_W K \cdot \cos\alpha = 0$$

$$L_W K = \left(1 - \frac{x}{12}\right) \cdot \frac{\sqrt{3^2 + 2^2}}{3}$$

$$L_W K(10) = 0,2$$

$$L_W K(12) = 0$$

