POLITECHNIKA POZNAŃSKA
WYDZIAŁ BUDOWNICTWA I INŻYNIERII ŚRODOWISKA
INSTYTUT KONSTRUKCJI *BUDOWLANYCH*
ZAKŁAD MECHANIKI BUDOWLI

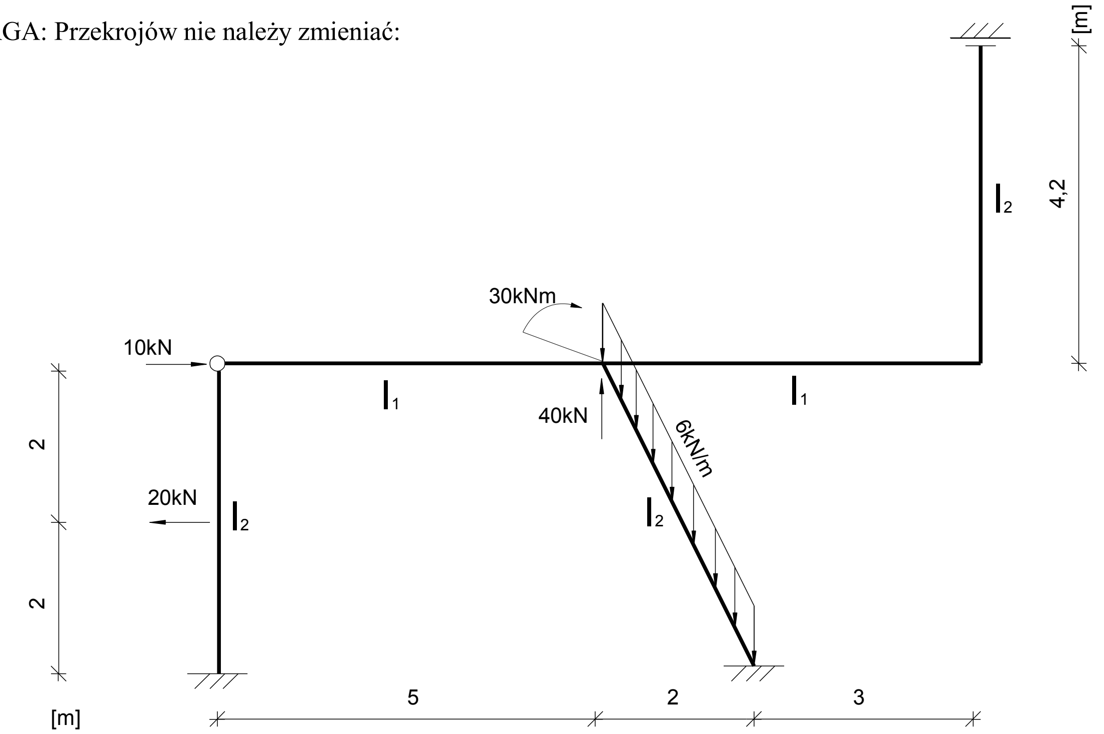
Projekt nr 3

Obliczenie sił przekrojowych przy użyciu metody przemieszczeń

Konrad Kaczmarek
Albert Kawałek
Grupa B3
Semestr IV
Rok akademicki 2013/2014

Dla podanego układu należy:
1. Przyjąć przekroje I_1 i I_2 z profili dwuteowych (IN, IPE, HEB, HEA).
2. Korzystając z metody przemieszczeń obliczyć siły przekrojowe (M, T, N) od zadanego obciążenia oraz wykonać kontrolę kinematyczną i kontrolę statyczną.
3. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2 porównać je ze wartościami dopuszczalnymi naprężeń (215 Mpa) sformułować wnioski.

UWAGA: Przekrojów nie należy zmieniać:



Przyjęto:

$I_1 = 4369\text{cm}^2 \rightarrow \text{IPE240}$

$I_2 = 3830\text{cm}^2 \rightarrow \text{HEB180}$

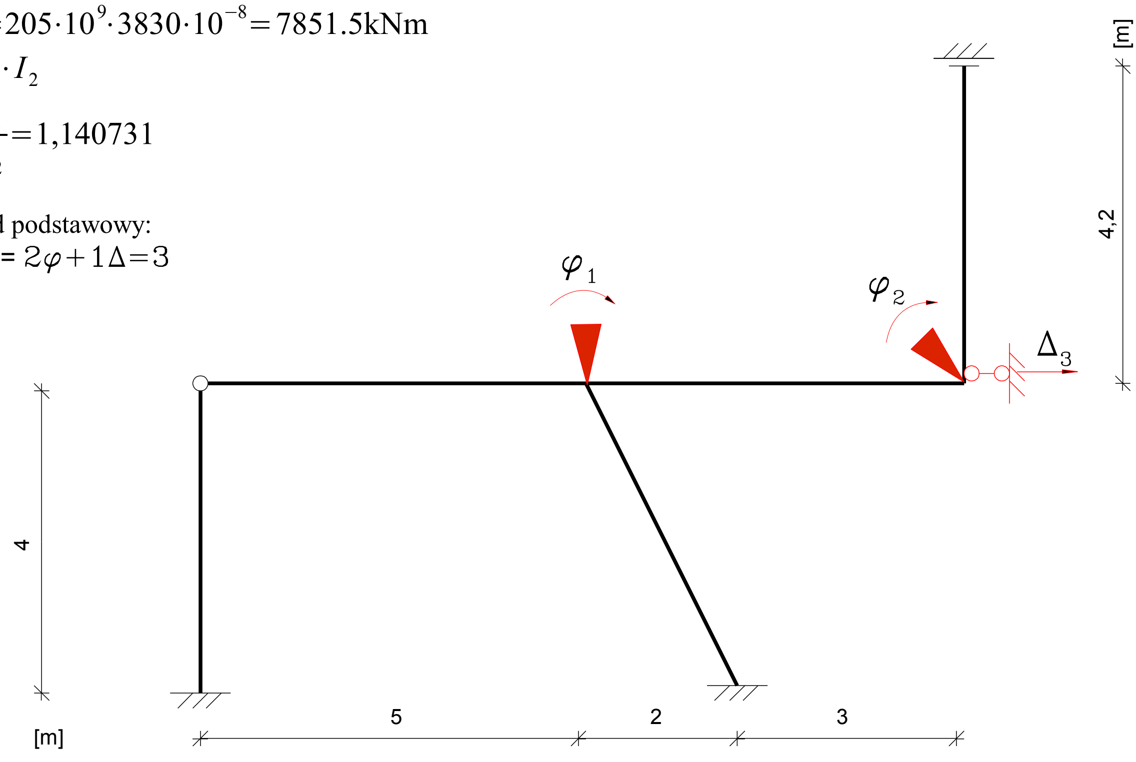
$EI_1 = 205 \cdot 10^9 \cdot 4369 \cdot 10^{-8} = 8956.45\text{kNm}$

$EI_2 = 205 \cdot 10^9 \cdot 3830 \cdot 10^{-8} = 7851.5\text{kNm}$

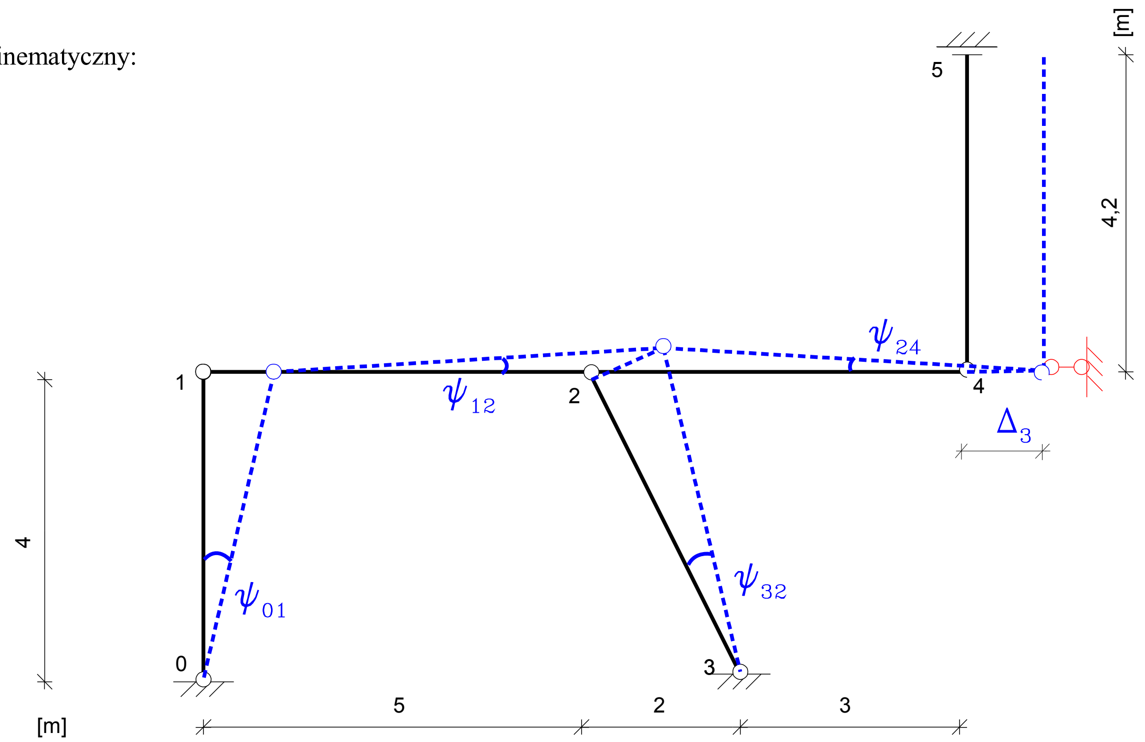
$I_1 = n \cdot I_2$

$n = \frac{I_1}{I_2} = 1{,}140731$

Układ podstawowy:
SGN = $2\varphi + 1\Delta = 3$



Łańcuch kinematyczny:



<u>0124 →</u>

$$0+\psi_{01}\cdot 4+\psi_{12}\cdot 0+\psi_{24}\cdot 0=1$$

$$\psi_{01}=\frac{1}{4}$$

<u>0123 →</u>

$$0+\psi_{01}\cdot 4+\psi_{12}\cdot 0-\psi_{32}\cdot 4=0$$

$$0+\frac{1}{4}\cdot 4+\psi_{12}\cdot 0-\psi_{32}\cdot 4=0$$

$$\psi_{32}=\frac{1}{4}$$

<u>0123 ↑</u>

$$0+\psi_{01}\cdot 0+\psi_{12}\cdot 5-\psi_{32}\cdot 2=0$$

$$0+\frac{1}{4}\cdot 0+\psi_{12}\cdot 5+\frac{1}{4}\cdot 2=0$$

$$\psi_{32}=-\frac{1}{10}$$

<u>0124 ↑</u>

$$0+\psi_{01}\cdot 0+\psi_{12}\cdot 5-\psi_{24}\cdot 5=0$$

$$0+\frac{1}{4}\cdot 0+\frac{1}{10}\cdot 5-\psi_{24}\cdot 5=0$$

$$\psi_{24}=\frac{1}{10}$$

$$\psi_{01}=\frac{1}{4}$$

$$\psi_{12}=-\frac{1}{10}$$

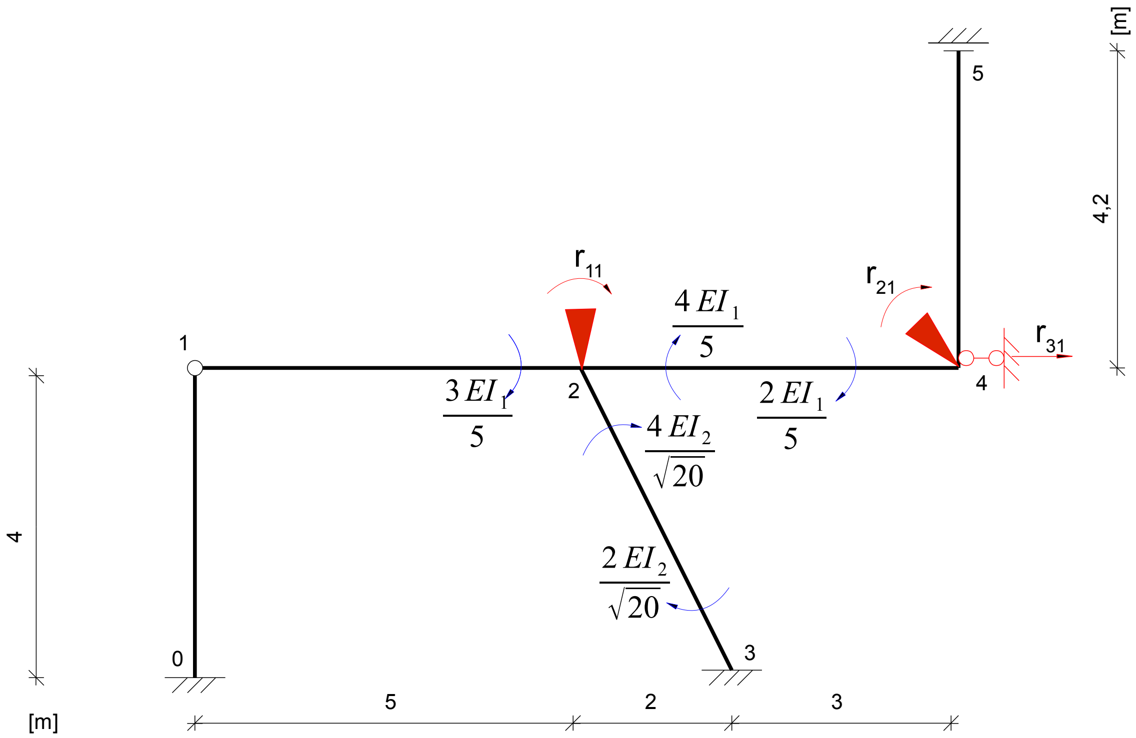
$$\psi_{32}=\frac{1}{4}$$

$$\psi_{24}=\frac{1}{10}$$

$$\psi_{45}=0$$

Stan „φ_1” :

$\varphi_1 = 1$



Momenty przęsłowe obliczono przy użyciu wzorów transformacyjnych:

$$M_{21} = \frac{3\,EI_1}{l}\cdot(\phi_1 - \psi_{12}) = \frac{3\,EI_1}{5}\cdot(1-0) = \frac{3\,EI_1}{5}$$

$$M_{23} = \frac{2\,EI_2}{l}\cdot(2\,\phi_2 + \phi_3 - 3\,\psi_{23}) = \frac{2\,EI_2}{\sqrt{20}}\cdot(2\cdot1 + 0 - 3\cdot0) = \frac{4\,EI_2}{\sqrt{20}}$$

$$M_{32} = \frac{2\,EI_2}{l}\cdot(2\,\phi_3 + \phi_2 - 3\,\psi_{23}) = \frac{2\,EI_2}{\sqrt{20}}\cdot(2\cdot0 + 1 - 3\cdot0) = \frac{2\,EI_2}{\sqrt{20}}$$

$$M_{24} = \frac{2\,EI_1}{l}\cdot(2\,\phi_2 + \phi_4 - 3\,\psi_{24}) = \frac{2\,EI_1}{5}\cdot(2\cdot1 + 0 - 3\cdot0) = \frac{4\,EI_1}{5}$$

$$M_{42} = \frac{2\,EI_1}{l}\cdot(2\,\phi_4 + \phi_2 - 3\,\psi_{24}) = \frac{2\,EI_1}{5}\cdot(2\cdot0 + 1 - 3\cdot0) = \frac{2\,EI_1}{5}$$

$$M_{01} = 0\,;\ M_{10};\ M_{12} = 0\,;\ M_{45} = 0\,;\ M_{54} = 0$$

Obliczanie reakcji r_{11}; r_{21}; r_{31}:

$$r_{11} = \frac{3EI_1}{5} + \frac{4EI_2}{\sqrt{20}} + \frac{4EI_1}{5} = \frac{3\cdot1.140731\,EI_2}{5} + \frac{4EI_2}{\sqrt{20}} + \frac{4\cdot1.140731\,EI_2}{5} = \underline{2.2491451\,EI_2}$$

$$r_{21} = \frac{2EI_1}{5} = \frac{2\cdot1.140731\,EI_2}{5} = \underline{0.456292\,EI_2}$$

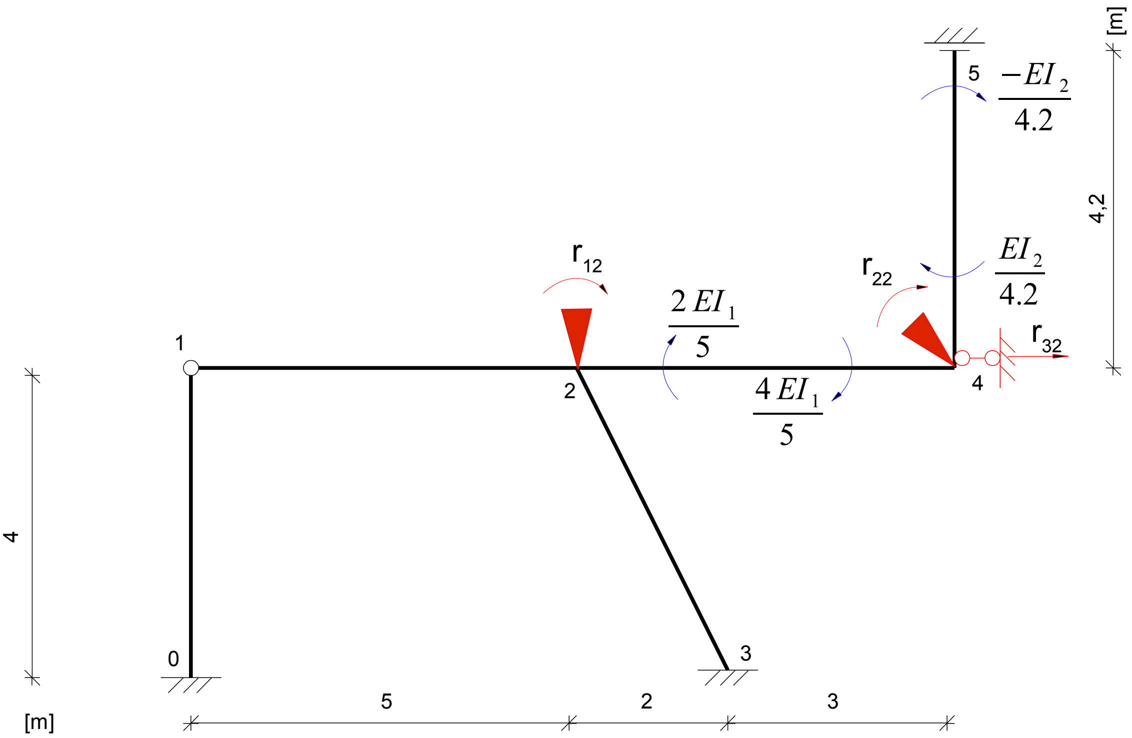
$$r_{31}\cdot\bar{1} + \frac{3\,EI_1}{5}\cdot\left(\frac{-\bar{1}}{10}\right) + \left(4\frac{EI_2}{\sqrt{20}} + 2\frac{EI_2}{\sqrt{20}}\right)\cdot\frac{\bar{1}}{4} + \left(\frac{4\,EI_1}{5} + \frac{2\,EI_1}{5}\right)\cdot\frac{\bar{1}}{10} = 0$$

$$r_{31}\cdot\bar{1} + \frac{3\cdot1.140731\,EI_2}{5}\cdot\left(\frac{-\bar{1}}{10}\right) + \left(4\frac{EI_2}{\sqrt{20}} + \frac{2\,EI_2}{\sqrt{20}}\right)\cdot\frac{\bar{1}}{4} + \left(\frac{4\cdot1.140731\,EI_2}{5} + \frac{2\cdot1.140731\,EI_2}{5}\right)\cdot\frac{\bar{1}}{10} = 0$$

$$r_{31} = \underline{-0{,}403854\,EI_2}$$

Stan „φ_2" :

$\varphi_2 = 1$



Momenty przęsłowe obliczono przy użyciu wzorów transformacyjnych:

$$M_{24} = \frac{2EI_1}{l}\cdot(2\phi_2 + \phi_4 - 3\psi_{24}) = \frac{2EI_1}{5}\cdot(2\cdot 0 + 1 - 3\cdot 0) = \frac{2EI_1}{5}$$

$$M_{24} = \frac{2EI_1}{l}\cdot(2\phi_2 + \phi_4 - 3\psi_{24}) = \frac{2EI_1}{5}\cdot(2\cdot 0 + 1 - 3\cdot 0) = \frac{2EI_1}{5}$$

$$M_{45} = \frac{EI_2}{l}\cdot(\phi_4 - \phi_5) = \frac{EI_2}{4.2}\cdot(1 - 0) = \frac{EI_2}{4.2}$$

$$M_{54} = \frac{EI_2}{l}\cdot(\phi_5 - \phi_4) = \frac{EI_2}{4.2}\cdot(0 - 1) = \frac{-EI_2}{4.2}$$

$$M_{01} = 0\,;\, M_{10} = 0;\, M_{12} = 0\,;\, M_{21} = 0\,;\, M_{23} = 0\,;\, M_{32} = 0$$

Obliczanie reakcji r_{12}; r_{22}; r_{32}:

$$r_{12} = \frac{2EI_1}{5} = \frac{2\cdot 1.140731\, EI_2}{5} = \underline{0.456292\, EI_2}$$

$$r_{22} = \frac{4EI_1}{5} + \frac{EI_2}{4.2} = \frac{4\cdot 1.140731\, EI_2}{5} + \frac{EI_2}{4.2} = \underline{1.150680\, EI_2}$$

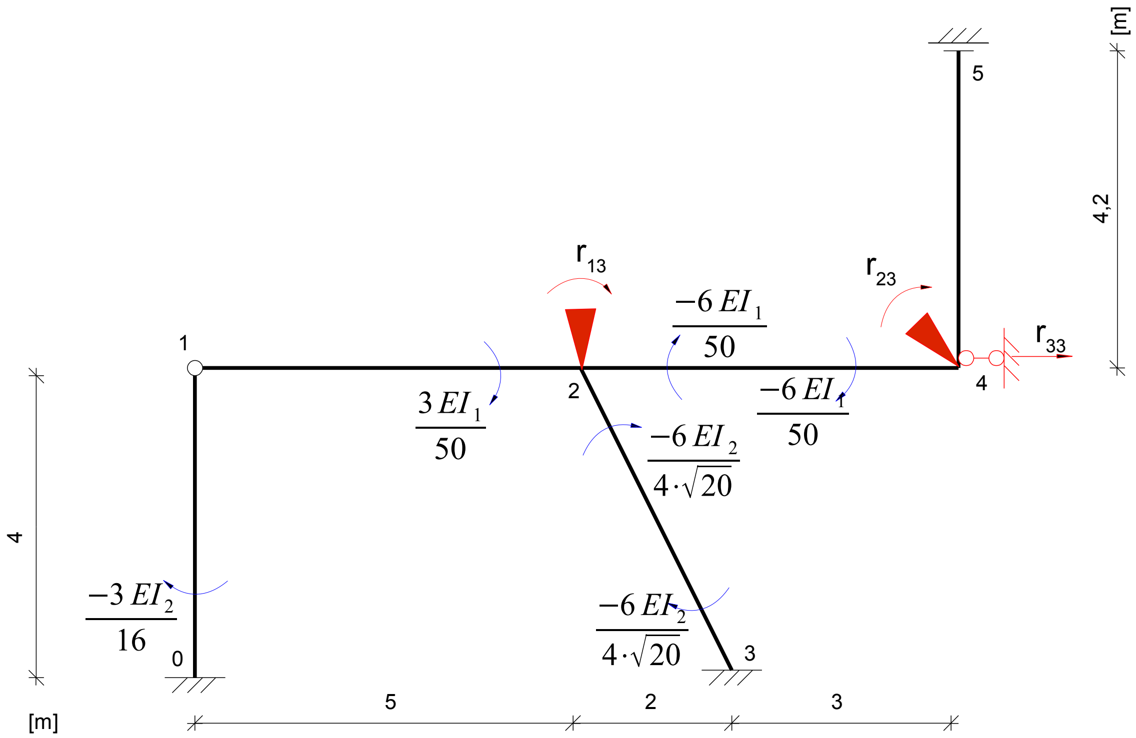
$$r_{32}\cdot\bar{1} + \left(\frac{4EI_1}{5} + \frac{2EI_1}{5}\right)\cdot\frac{\bar{1}}{10} = 0$$

$$r_{32}\cdot\bar{1} + \left(\frac{4\cdot 1.140731\, EI_2}{5} + \frac{2\cdot 1.140731\, EI_2}{5}\right)\cdot\frac{\bar{1}}{10} = 0$$

$$r_{32} = \underline{-0{,}136888\, EI_2}$$

Stan „Δ_3" :

$\Delta_3 = 1$



Momenty przęsłowe obliczono przy użyciu wzorów transformacyjnych:

$$M_{01} = \frac{3EI_2}{l} \cdot (\phi_0 - \psi_{01}) = \frac{3EI_2}{4} \cdot \left(0 - \left(\frac{1}{4}\right)\right) = \frac{-3EI_2}{16}$$

$$M_{21} = \frac{3EI_1}{l} \cdot (\phi_1 - \psi_{12}) = \frac{3EI_1}{5} \cdot \left(0 - \left(\frac{-1}{10}\right)\right) = \frac{3EI_1}{50}$$

$$M_{23} = \frac{2EI_2}{l} \cdot (2\phi_2 + \phi_3 - 3\psi_{23}) = \frac{2EI_2}{\sqrt{20}} \cdot \left(2 \cdot 0 + 0 - 3 \cdot \left(\frac{1}{4}\right)\right) = \frac{-6EI_2}{4 \cdot \sqrt{20}}$$

$$M_{32} = \frac{2EI_2}{l} \cdot (2\phi_3 + \phi_2 - 3\psi_{23}) = \frac{2EI_2}{\sqrt{20}} \cdot \left(2 \cdot 0 + 0 - 3 \cdot \left(\frac{1}{4}\right)\right) = \frac{-6EI_2}{4 \cdot \sqrt{20}}$$

$$M_{24} = \frac{2EI_1}{l} \cdot (2\phi_2 + \phi_4 - 3\psi_{24}) = \frac{2EI_1}{5} \cdot \left(2 \cdot 0 + 0 - 3 \cdot \left(\frac{1}{10}\right)\right) = \frac{-6EI_1}{50}$$

$$M_{42} = \frac{2EI_1}{l} \cdot (2\phi_4 + \phi_2 - 3\psi_{24}) = \frac{2EI_1}{5} \cdot \left(2 \cdot 0 + 0 - 3 \cdot \left(\frac{1}{10}\right)\right) = \frac{-6EI_1}{50}$$

$$M_{10};\ M_{12} = 0;\ M_{45} = 0;\ M_{54} = 0$$

Obliczanie reakcji r_{13}; r_{23}; r_{33}:

$$r_{13} = \frac{3EI_1}{50} - \frac{6EI_1}{50} - \frac{6EI_2}{4 \cdot \sqrt{20}} = \frac{3 \cdot 1.140731\, EI_2}{50} - \frac{6 \cdot 1.140731\, EI_2}{50} - \frac{6EI_2}{4 \cdot \sqrt{20}} = \underline{-0.403854\, EI_2}$$

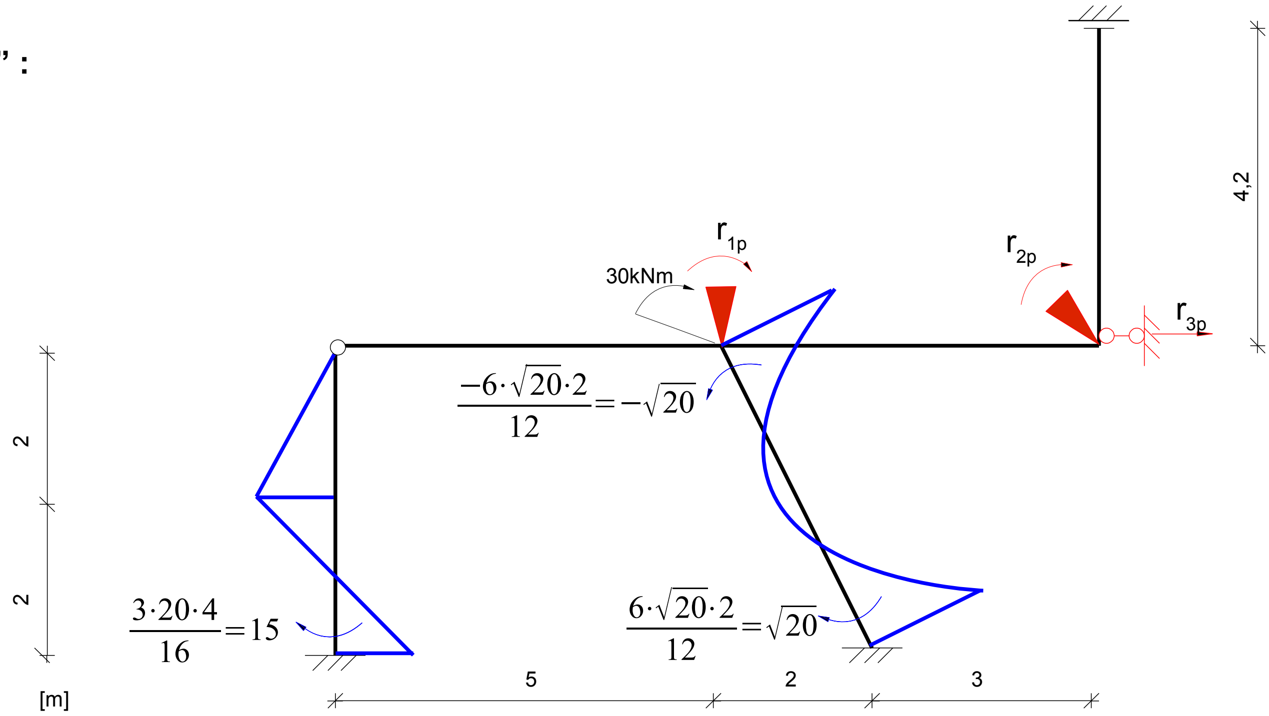
$$r_{23} = \frac{-6EI_1}{50} = \frac{-6 \cdot 1.140731\, EI_2}{50} = \underline{-0.136888\, EI_2}$$

$$r_{33} \cdot \bar{1} - 2 \cdot \frac{6EI_1}{50} \cdot \frac{\bar{1}}{10} - 2 \cdot \frac{6EI_2}{4 \cdot \sqrt{20}} \cdot \frac{\bar{1}}{4} + \frac{3EI_1}{50} \cdot \left(\frac{-\bar{1}}{10}\right) - \frac{3EI_2}{16} \cdot \frac{\bar{1}}{4} = 0$$

$$r_{33} \cdot \bar{1} - 2 \cdot \frac{6 \cdot 1.140731\, EI_2}{50} \cdot \frac{\bar{1}}{10} - 2 \cdot \frac{6EI_2}{4 \cdot \sqrt{20}} \cdot \frac{\bar{1}}{4} + \frac{3 \cdot 1.140731\, EI_2}{50} \cdot \left(\frac{-\bar{1}}{10}\right) - \frac{3EI_2}{16} \cdot \frac{\bar{1}}{4} = 0$$

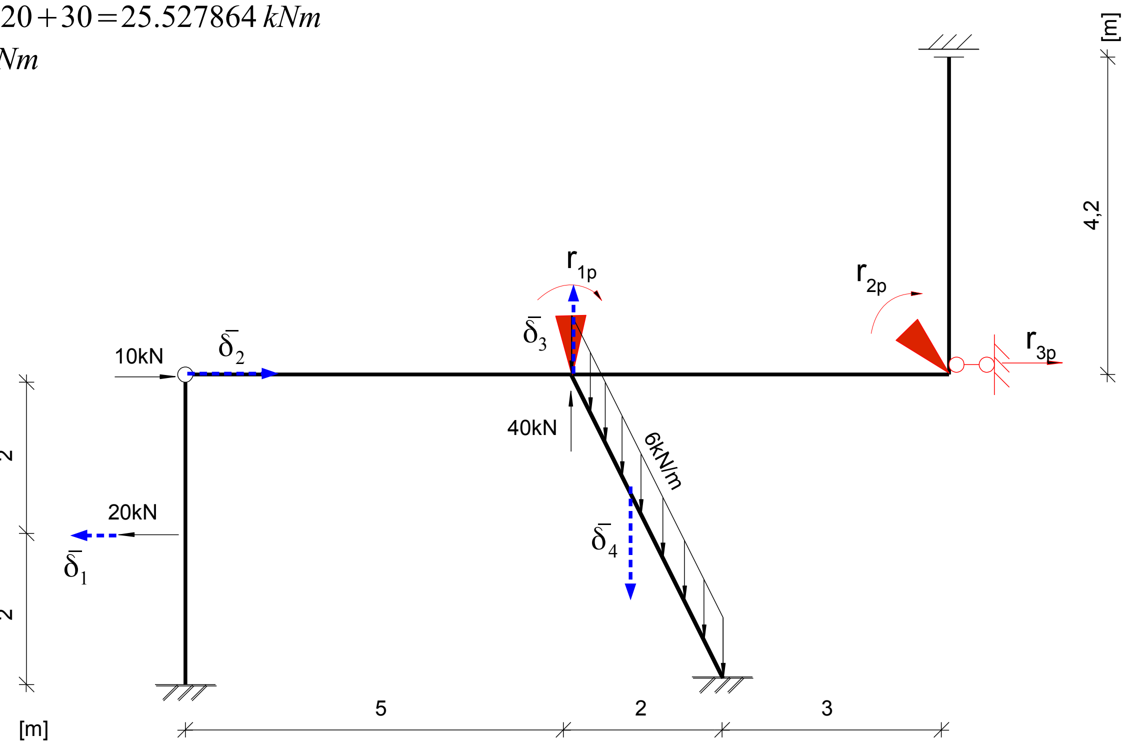
$$r_{33} = \underline{0{,}248802\, EI_2}$$

Stan **„P” :**



$r_{1p}=-\sqrt{20}+30=25.527864\ kNm$

$r_{2p}=0\ kNm$



$0+2\cdot\psi_{01}=-\bar{\delta}_1$ $\quad \bar{\delta}_2=1$ $\quad 0+2\cdot\psi_{32}=\bar{\delta}_3$ $\quad 0-1\cdot\psi_{32}=\bar{\delta}_4$

$0+\frac{2\cdot 1}{4}=-\bar{\delta}_1$ $\quad 0+\frac{2\cdot 1}{4}=\bar{\delta}_3$ $\quad 0-\frac{1\cdot 1}{4}=\bar{\delta}_4$

$\bar{\delta}_1=-\frac{1}{2}$ $\quad \bar{\delta}_3=\frac{1}{2}$ $\quad \bar{\delta}_4=-\frac{1}{4}$

$r_{3p}\cdot\bar{1}+20\cdot\bar{\delta}_1+10\cdot\bar{\delta}_2+40\cdot\bar{\delta}_3+6\cdot\sqrt{20}\cdot\bar{\delta}_4+15\cdot\bar{\psi}_{01}+(\sqrt{20}-\sqrt{20})\cdot\bar{\psi}_{23}=0$

$r_{3p}=-17.041796\ kN$

$$\begin{bmatrix} 2.291451 & 0,456292 & -0,403854 \\ 0,456292 & 1,150680 & -0,136888 \\ -0,403854 & -0,136888 & 0,248802 \end{bmatrix} \cdot EI_2 \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \Delta_3 \end{bmatrix} + \begin{bmatrix} -34,472136 \\ 0 \\ -17,041796 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2.291451\, EI_2\, \phi_1 + 0,456292\, EI_2\, \phi_2 - 0,403854\, EI_2\, \Delta_3 - 34,472136 = 0$$
$$0,456292\, EI_2\, \phi_1 + 1,150680\, EI_2\, \phi_2 - 0,136888\, EI_2\, \Delta_3 + 0 = 0$$
$$-0,403854\, EI_2\, \phi_1 + -0,136888\, EI_2\, \phi_2 + 0,248802\, EI_2\, \Delta_3 - 17,041796 = 0$$

Po obliczeniu układu równań uzyskujemy:

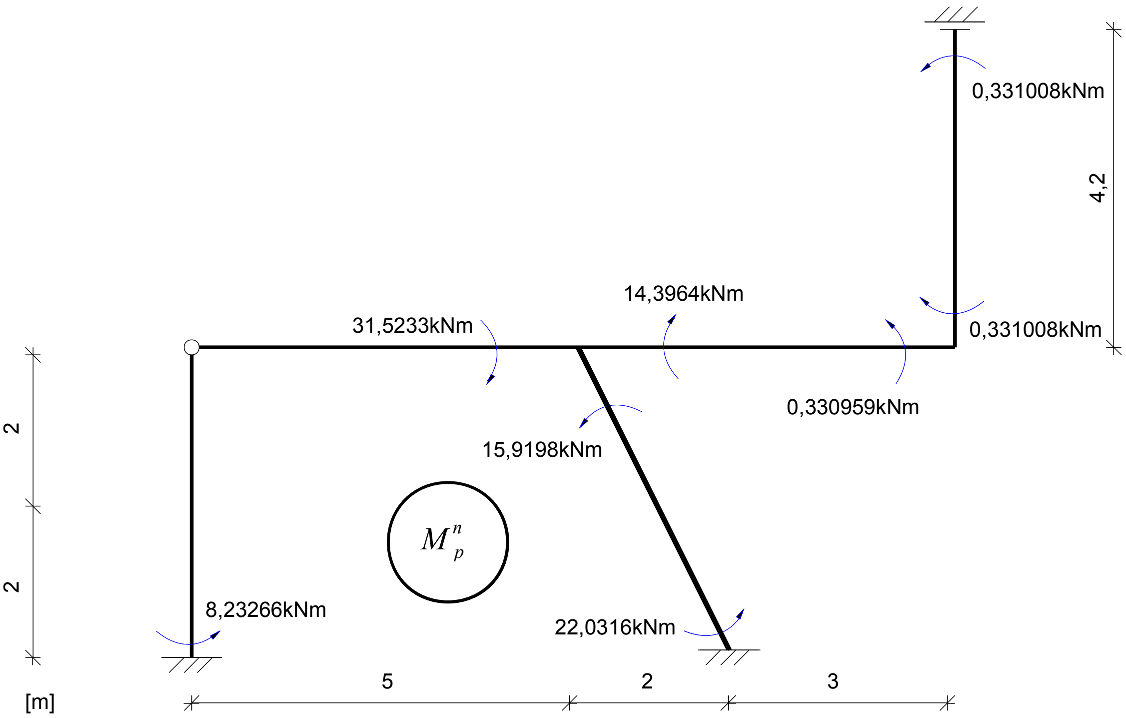
$$\phi_1 = \frac{33,666475}{EI_2}$$
$$\phi_2 = \frac{1,3902315}{EI_2}$$
$$\Delta_3 = \frac{123,90753}{EI_2}$$

Korzystając z zasady superpozycji obliczymy wartość momentów:

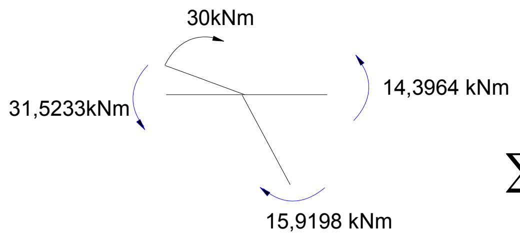
$$M_p^n = M_1 \cdot \phi_1 + M_2 \cdot \phi_2 + M_3 \cdot u_3 + M_p^0$$
$$M_{01}^n = -\frac{3 \cdot EI_2}{16} \cdot \frac{123,90753}{EI_2} + 15 = -8,23266\, kNm$$
$$M_{21}^n = \frac{3 \cdot 1,140731 \cdot EI_2}{5} \cdot \frac{33,666475}{EI_2} + \frac{3 \cdot 1,140731 \cdot EI_2}{50} \cdot \frac{123,90753}{EI_2} = 31,5233\, kNm$$
$$M_{23}^n = \frac{4EI_2}{\sqrt{20}} \cdot \frac{33,666475}{EI_2} - \frac{6EI_2}{4 \cdot \sqrt{20}} \cdot \frac{123,90753}{EI_2} - 4,472136 = -15,9198\, kNm$$
$$M_{32}^n = \frac{2EI_2}{\sqrt{20}} \cdot \frac{33,666475}{EI_2} - \frac{6EI_2}{4 \cdot \sqrt{20}} \cdot \frac{123,90753}{EI_2} + 4,472136 = -22,0316\, kNm$$
$$M_{24}^n = \frac{4 \cdot 1,140731\, EI_2}{5} \cdot \frac{33,666475}{EI_2} + \frac{2 \cdot 1,140731\, EI_2}{5} \cdot \frac{1,3902315}{EI_2} - \frac{6 \cdot 1,140731\, EI_2}{50} \cdot \frac{123,90753}{EI_2} = 14,3964\, kNm$$
$$M_{42}^n = \frac{2 \cdot 1,140731\, EI_2}{5} \cdot \frac{33,666475}{EI_2} + \frac{4 \cdot 1,140731\, EI_2}{5} \cdot \frac{1,3902315}{EI_2} - \frac{6 \cdot 1,140731\, EI_2}{50} \cdot \frac{123,90753}{EI_2} = -0,330959\, kNm$$
$$M_{45}^n = \frac{EI_2}{4,2} \cdot \frac{1,3902315}{EI_2} = 0,0331008\, kNm$$
$$M_{54}^n = -\frac{EI_2}{4,2} \cdot \frac{1,3902315}{EI_2} = -0,331008\, kNm$$

Podsumowując:

$$M_{01}^n = -8,23266\, kNm$$
$$M_{21}^n = 31,5233\, kNm$$
$$M_{23}^n = -15,9198\, kNm$$
$$M_{32}^n = -22,0316\, kNm$$
$$M_{24}^n = 14,3964\, kNm$$
$$M_{42}^n = -0,330959\, kNm$$
$$M_{45}^n = 0,331008\, kNm$$
$$M_{54}^n = -0,331008\, kNm$$

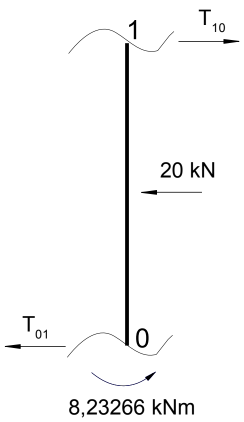


Sprawdzenie węzła:



$$\sum M_2 = 30 + 15{,}9198 - 31{,}5233 - 14{,}3964 = 0{,}0001 \approx 0$$

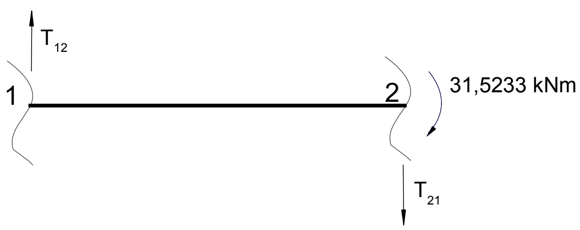
WYZNACZANIE TNĄCYCH:



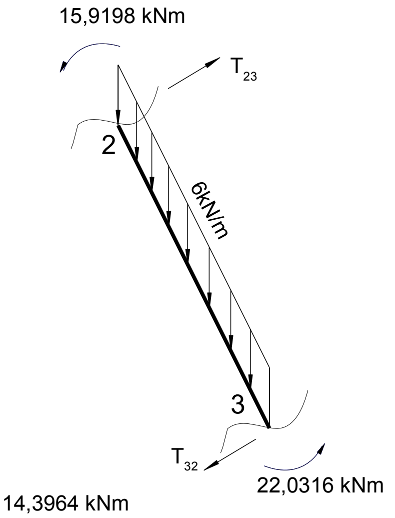
$$\sum M_0 = 4T_{10} - 20 \cdot 2 - 8{,}23266 = 0$$
$$T_{10} = 12{,}0582\,kN$$
$$\sum M_1 = T_{01} \cdot 4 + 20 \cdot 2 - 8{,}23266 = 0$$
$$T_{01} = -7{,}94184\,kN$$

Wartość momentu zginającego pod siłą:

$$M = T_{10} \cdot 2 = 12{,}0582 \cdot 2 = 24{,}11634\,kNm$$

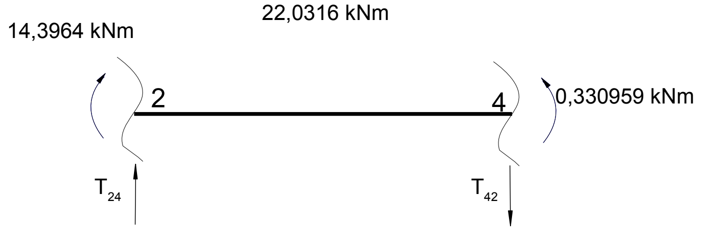


$$\sum M_1 = T_{21} \cdot 5 + 31{,}5233$$
$$T_{21} = T_{12} = -6{,}30466\,kN$$

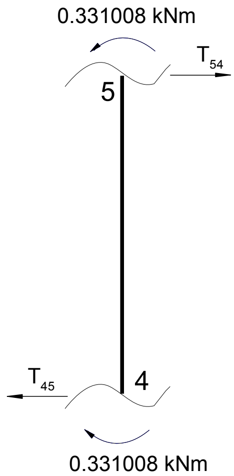


$$\sum M_3 = -15{,}9198 + T_{23} \cdot \sqrt{20} - 6 \cdot \sqrt{20} \cdot 1 - 22{,}0316 = 0$$
$$T_{23} = 14{,}4862\ kN$$
$$\sum M_2 = -15{,}9198 + 6 \cdot \sqrt{20} \cdot 1 + T_{32} \cdot \sqrt{20} - 22{,}0316 = 0$$
$$T_{32} = 2{,}48619\ kN$$

Siła tnąca nie zmienia znaku → nie ma ekstremum momentów zginających

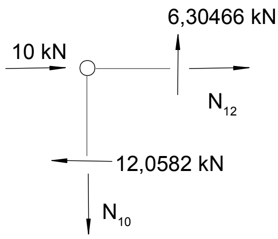


$$\sum M_2 = 14{,}3964 - 0{,}330959 + T_{42} \cdot 5 = 0$$
$$T_{42} = T_{24} = -2{,}81309\ kN$$

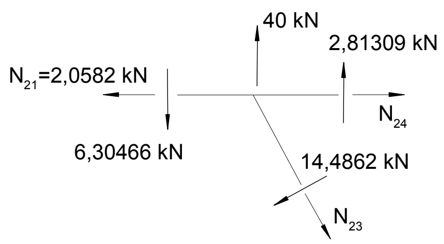


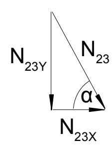
$$T_{54} = T_{45} = 0$$

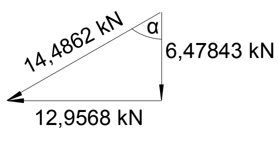
WYZNACZANIE NORMALNYCH:



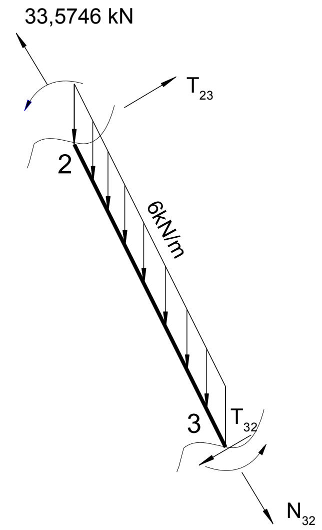
$$N_{10} = N_{01} = 6{,}30466\ kN$$
$$N_{12} = N_{21} = 2{,}0582\ kN$$



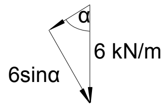


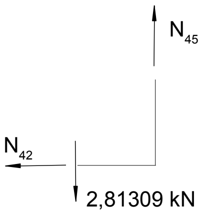


$$\sin\alpha = 0{,}894427$$
$$\cos\alpha = 0{,}447214$$
$$\sum Y = -6{,}30466 + 40 + 2{,}81309 - 6{,}47843 - N_{23} \cdot \sin\alpha = 0$$
$$N_{23} = 33{,}5746\ kN$$
$$\sum X = -2{,}0582 + N_{24} - 12{,}9568 + N_{23} \cdot \cos\alpha = 0$$
$$N_{24} = N_{42} = 0$$

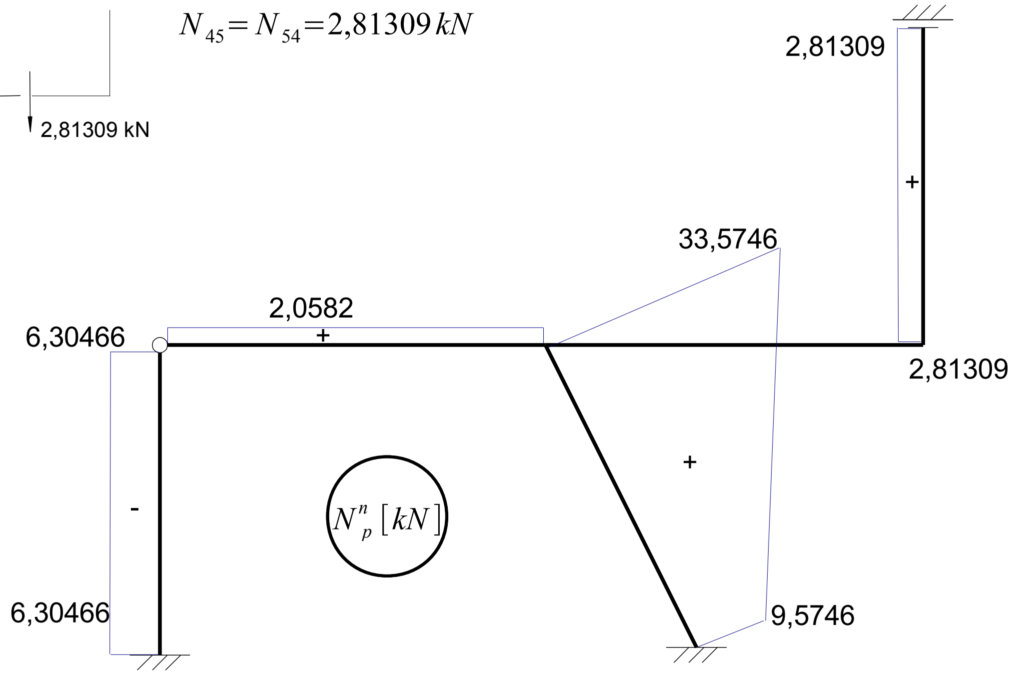


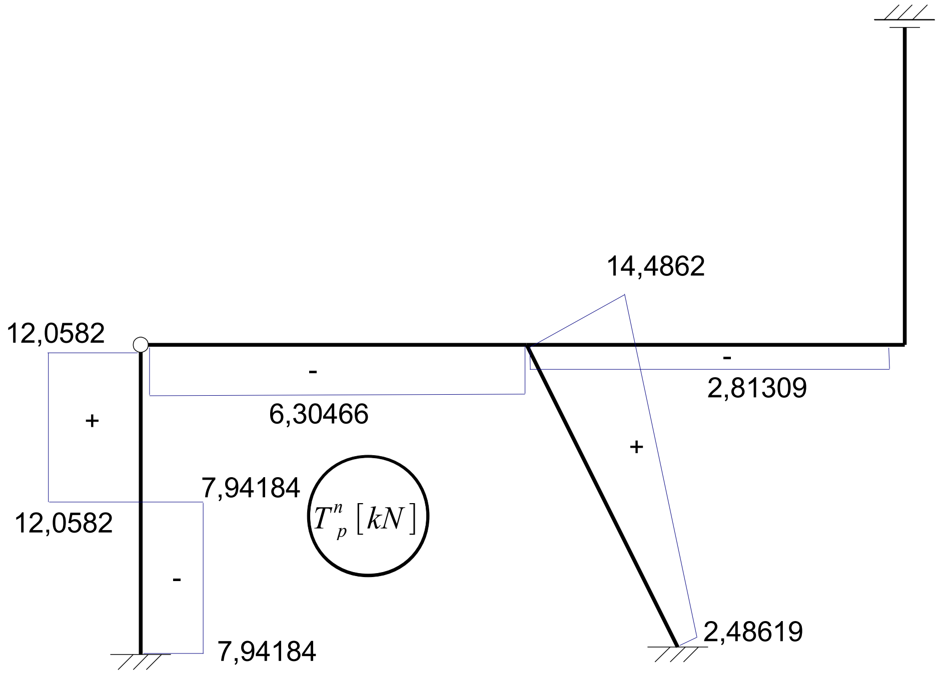
$$N_{32}=33{,}5746-6\cdot\sin\alpha\sqrt{20}=9{,}5746\,kN$$

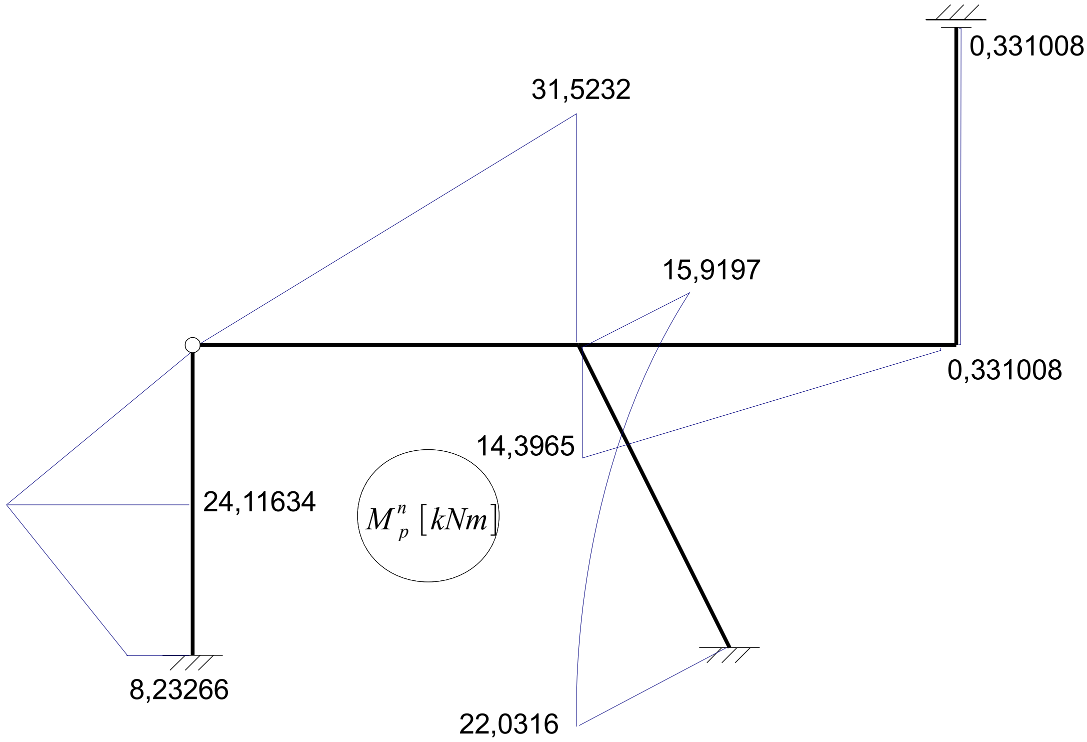




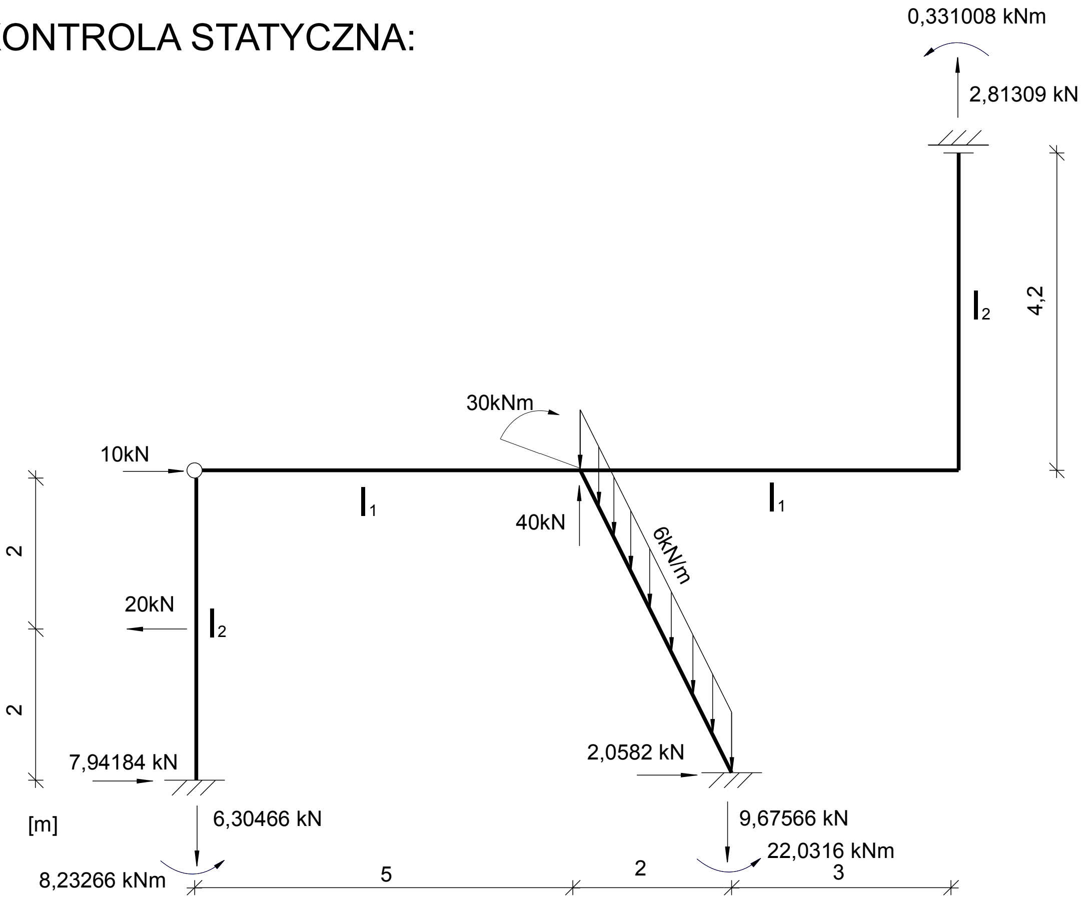
$$N_{45}=N_{54}=2{,}81309\,kN$$







KONTROLA STATYCZNA:



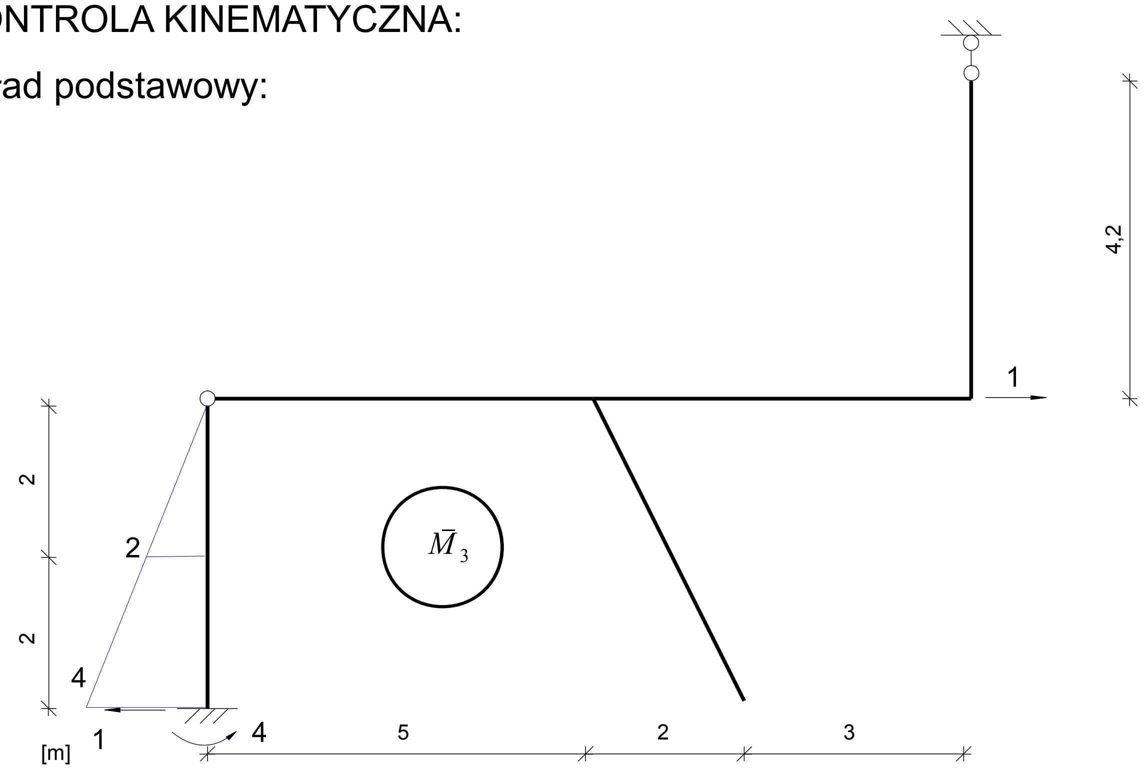
$$\sum X = 7{,}94184 - 20 + 10 + 2{,}0582 = 0{,}0004 \approx 0$$

$$\sum Y = -6{,}30466 + 40 - 6\cdot\sqrt{20} - 9{,}67566 + 2{,}81309 = -0{,}00004 \approx 0$$

$$\sum M_1 = 20\cdot 2 - 7{,}94184\cdot 4 - 8{,}23266 - 40\cdot 5 + 30 + 6\cdot\sqrt{20}\cdot 6 - 2{,}0582\cdot 4 + 9{,}67566\cdot 7 - 22{,}0316 - 0{,}331008 +$$
$$-2{,}81309\cdot 10 = 0{,}00011 \approx 0$$

KONTROLA KINEMATYCZNA:

Układ podstawowy:



$$\bar{1}\cdot u_3 = \sum \int \frac{M_p^n \cdot \bar{M}^0}{EI} ds$$

$$\bar{1}\cdot u_3 = \frac{1}{EI_2}\cdot\left[\frac{1}{2}\cdot 4\cdot 2\cdot\left(\frac{2}{3}\cdot 8{,}23266+\frac{1}{3}\cdot 24{,}11634\right)+\frac{1}{2}\cdot 2\cdot 2\cdot\left(\frac{2}{3}\cdot 24{,}11634+\frac{1}{3}\cdot 8{,}23266\right)+\frac{1}{2}\cdot 24{,}11634\cdot 2\cdot\left(\frac{2}{3}\cdot 2\right)\right]=\frac{123{,}9076}{EI_2}$$

$$\frac{123{,}9076}{EI_2}=\frac{123{,}90753}{EI_2}$$

Sprawdzenie naprężeń normalnych wywołanych obciążeniami w obu grupach prętów:

$Dla\ I_1 = 4369\ cm^4$

$M_{y1}^{ext} = 31{,}5232\ kNm = 3152{,}32\ kNcm$

$Dla\ I_2 = 3830\ cm^4$

$M_{y2}^{ext} = 24{,}1164\ kNm = 2411{,}64\ kNcm$

$Wytrzymałość\ materiału: R = 215\text{MPa} = 21{,}5\ \frac{kN}{cm^2}$

$\sigma = \frac{M}{W}$

$Dla\ IPE\ 240 \rightarrow W_{y1} = 361\text{cm}^3$

$Dla\ HEB\ 180 \rightarrow W_{y2} = 426\text{cm}^3$

Obliczamy:

$$\sigma_1 = \frac{M_{y1}^{ext}}{W_{y1}} = \frac{3152{,}32}{361} = 8{,}7321884\frac{kN}{cm^2} = 87{,}321884\ MPa \qquad < 215\ MPa$$

$$\sigma_2 = \frac{M_{y2}^{ext}}{W_{y2}} = \frac{2411{,}64}{426} = 5{,}6611268\frac{kN}{cm^2} = 56{,}611268\ MPa \qquad < 215\ MPa$$

Żeby optymalnie zaprojektować kontstrukcję należałoby zmienić przekroje. Zmiana przekrojów spowoduje zmianę współczynnika n , od którego zależy rozkład momentów zginających. Obliczenia należałoby powtórzyć.