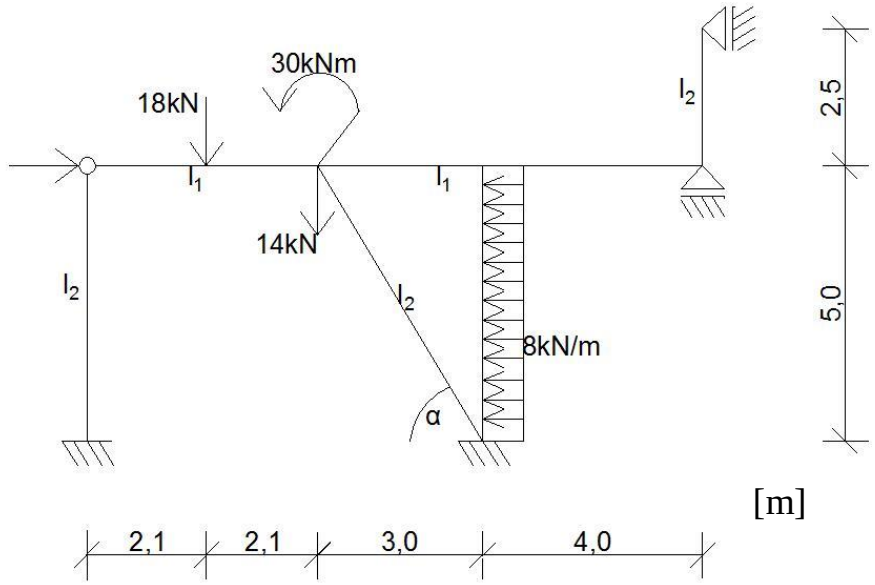
METODA PRZEMIESZCZEŃ

Wykonały:
Katarzyna Tomczyszyn
Katarzyna Winiecka

SCHEMAT KONSTRUKCJI



1. Przyjąć przekroje I1 i I2 z profili dwuteowych (IN, IPE, HEB, HEA):

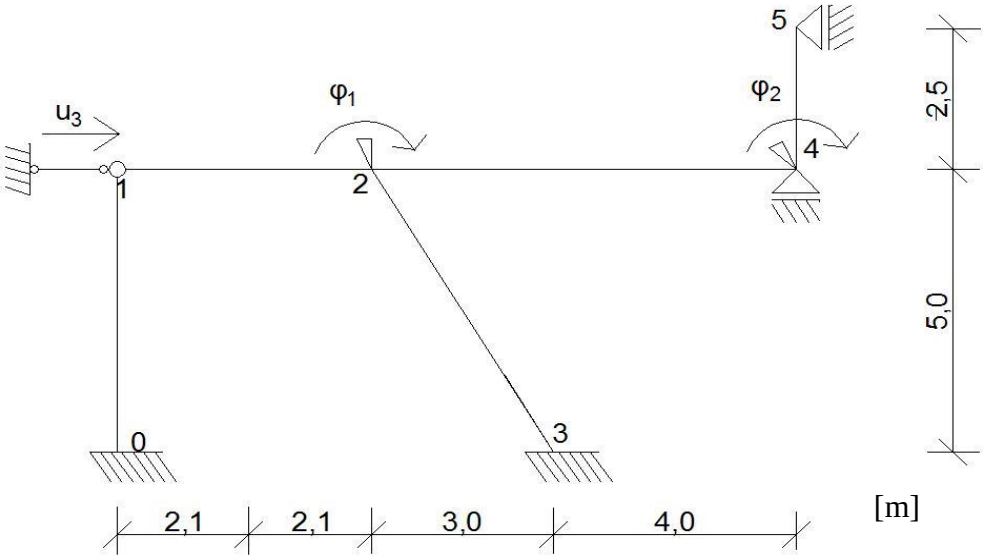
Przyjęto przekroje:

I 240 PE $I_{y1} = 3890\ cm^4$ $\underline{EI_1} = 7974{,}50\ kNm^2$

I 200 PE $I_{y2} = 1940\ cm^4$ $\underline{EI_2} = 3977\ kNm^2$

2. Korzystając z metody przemieszczeń obliczyć siły przekrojowe (M, T, N) od zadanego obciążenia oraz wykonać kontrolę kinematyczną i kontrolę statyczną:

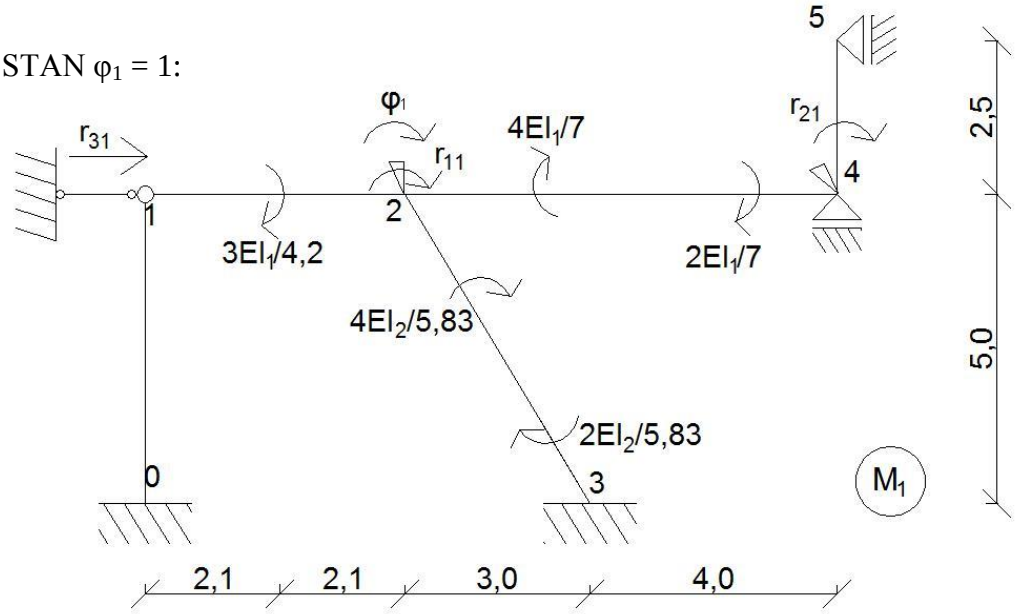
UKŁAD PODSTAWOWY: Stopień kinematycznej niewyznaczalności : **SKN = 3**



UKŁAD RÓWNAŃ KANONICZNYCH:

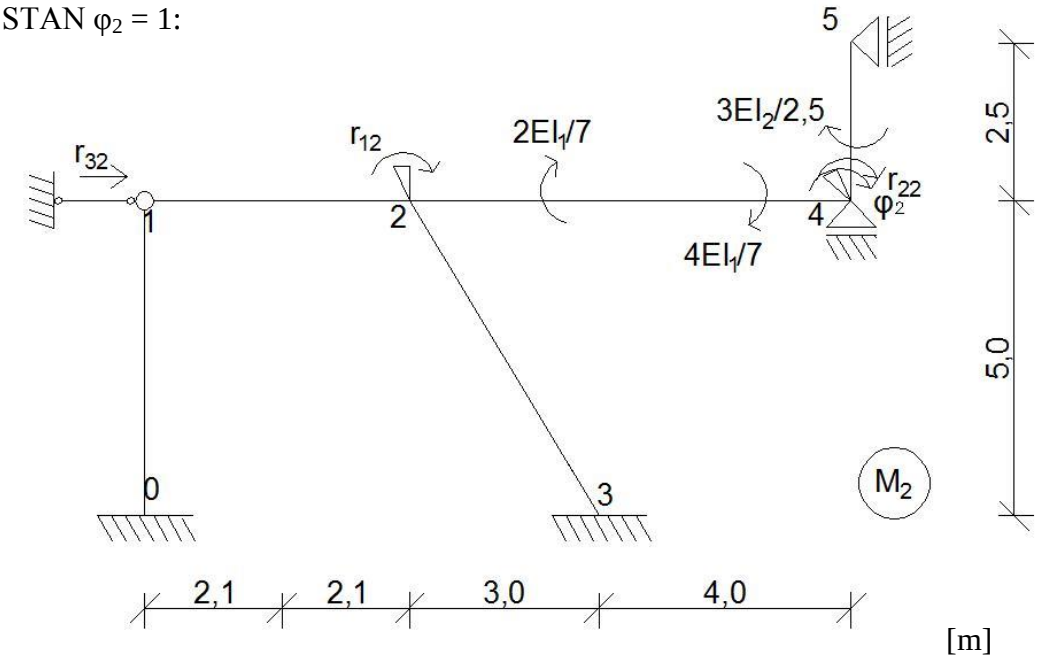
$$\begin{cases} r_{11} \cdot \varphi_1 + r_{12} \cdot \varphi_2 + r_{13} \cdot u_3 + r_{1p} = 0 \\ r_{21} \cdot \varphi_1 + r_{22} \cdot \varphi_2 + r_{23} \cdot u_3 + r_{2p} = 0 \\ r_{31} \cdot \varphi_1 + r_{32} \cdot \varphi_2 + r_{33} \cdot u_3 + r_{3p} = 0 \end{cases}$$

STAN $\varphi_1 = 1$:



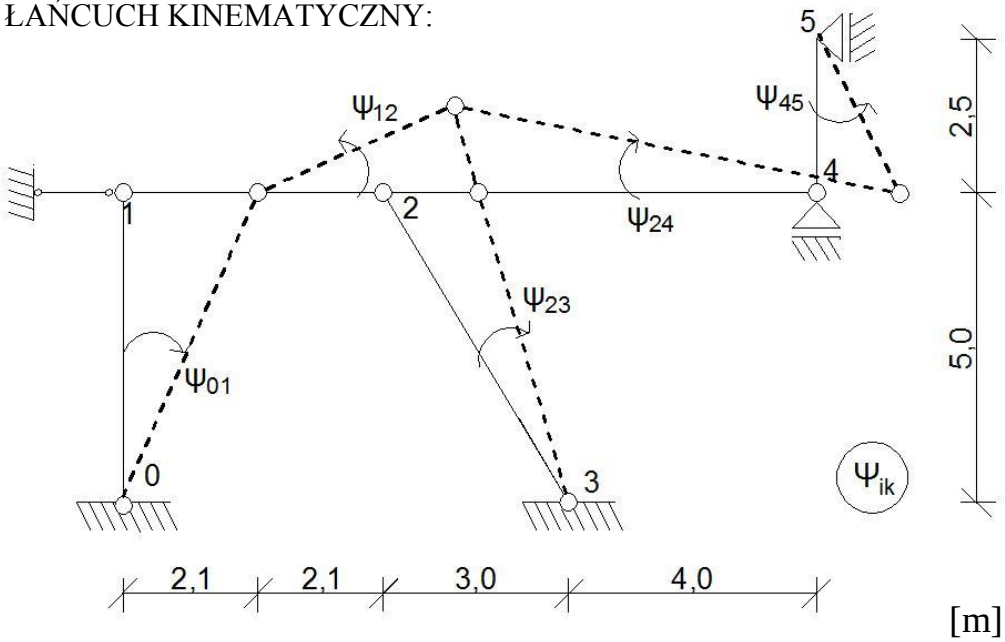
[m]

STAN $\varphi_2 = 1$:



[m]

ŁAŃCUCH KINEMATYCZNY:

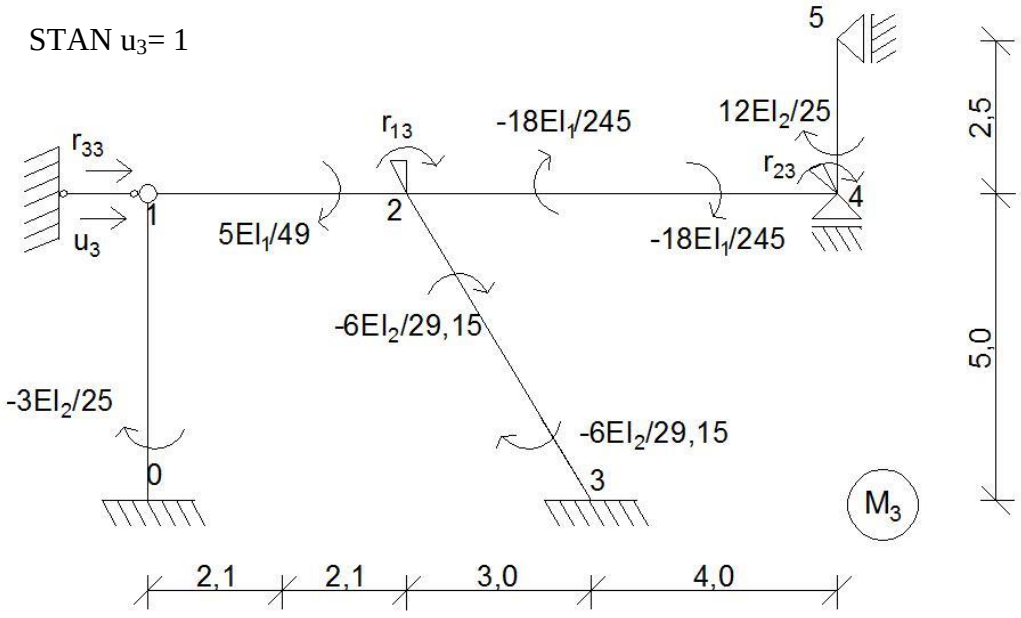


[m]

sin α = 0,85763

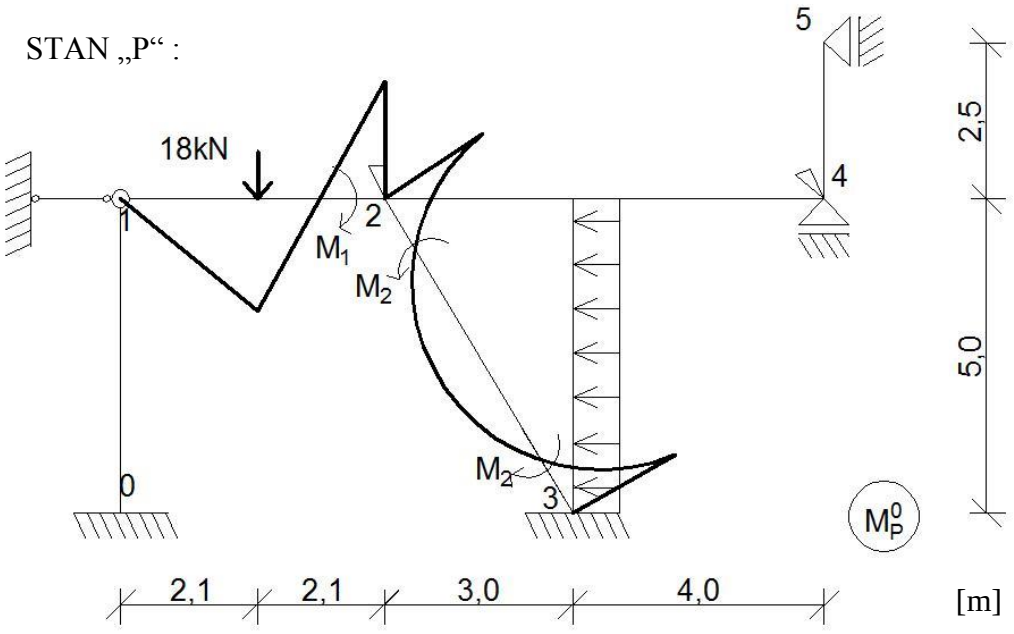
cos α = 0,51458

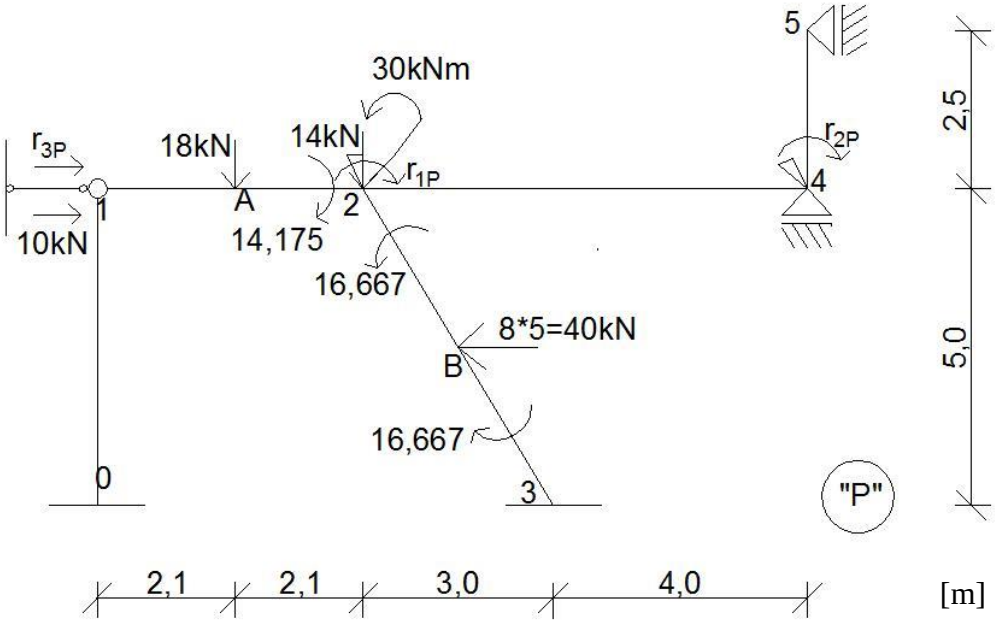
STAN u_3= 1



[m]

STAN „P“ :





RÓWNANIA ŁAŃCUCHA KINEMATYCZNEGO:

↓ 01A $0 + 0 + 2{,}1\ \Psi_{12} = \delta_A^V$

$\delta_A^V = -0{,}3$

↓ 012 $0 + 0 + 4{,}2\ \Psi_{12} = \delta_2^V$

$\delta_2^V = -0{,}6$

→ 3B $0 + 2{,}5\ \Psi_{32} = \delta_B^H$

$\delta_B^H = 0{,}3$

$r_{11} = 1{,}285714\ EI_1 + 0{,}686106\ EI_2 = 12995.29198$

$r_{12} = r_{21} = 2278{,}428571$

$r_{31} \cdot \overline{1} + \left(\frac{4EI_2}{5{,}83} + \frac{2EI_2}{5{,}83}\right) \cdot \overline{0{,}2} + \frac{3EI_1}{4{,}2} \cdot \left(\overline{-0{,}142857}\right) + \left(\frac{4EI_1}{7} + \frac{2EI_1}{7}\right) \cdot \left(\overline{0{,}085714}\right) = 0$

$r_{13} = r_{31} = -0{,}205832\ EI_2 + 0{,}028571 EI_1 = -590{,}754043$

$r_{12} = 0{,}285714\ EI_1 = 2278{,}428571$

$r_{22} = 1{,}2\ EI_2 + 0{,}571429\ EI_1 = 9329{,}257143$

$r_{32} \cdot \overline{1} + \left(\frac{2EI_1}{7} + \frac{4EI_1}{7}\right) \cdot \left(\overline{0{,}085714}\right) + \frac{3EI_2}{2{,}5} \cdot \left(\overline{-0{,}4}\right) = 0$

$r_{23} = r_{32} = 0{,}48\ EI_2 - 0{,}073469\ EI_1 = 1323{,}08146$

$r_{13} = r_{31} = -0{,}205832 EI_2 + 0{,}10204 EI_1 - 0{,}073469 EI_1 = -590{,}75442$

$r_{23} = r_{23} = 0{,}48\ EI_2 - 0{,}073469\ EI_1 = 1323{,}08146$

$r_{33} \cdot \overline{1} + (-0{,}12\ EI_2) \cdot \overline{0{,}2} + 0{,}10204\ EI_1 \cdot \left(\overline{-0{,}142857}\right) + (-0{,}205832\ EI_2 \cdot 2) \cdot \left(\overline{0{,}2}\right) + (0{,}48\ EI_2) \cdot \left(\overline{-0{,}4}\right) + (-0{,}073469\ EI_1 \cdot 2) \cdot \left(\overline{0{,}085714}\right) = 0$

$r_{33} = 1403{,}152507$

$r_{3P} \cdot \overline{1} + 10 \cdot \overline{1} + 18 \cdot \overline{-0{,}3} + 14 \cdot \overline{-0{,}6} - 40 \cdot \overline{0{,}5} + 14{,}175 \cdot \overline{-0{,}142857} + (16{,}667 - 16{,}667) \cdot \overline{0{,}2} - 30 \cdot 0 = 0$

$r_{3P} = 25{,}825$ kN

$r_{2P} = 0$

$r_{1P} = -16{,}667 + 30{,}0 + 14{,}175 = 27{,}508$ kNm

ROZWIĄZANIE UKŁADU RÓWNAŃ KANONICZNYCH:

$$\begin{bmatrix} 12985{,}291 & 2278{,}315 & -590{,}754 \\ 2278{,}315 & 9329{,}029 & 1322{,}834 \\ -590{,}754 & 1322{,}834 & 1403{,}152 \end{bmatrix} \cdot \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ u_3 \end{Bmatrix} + \begin{Bmatrix} 27{,}508 \\ 0 \\ 25{,}825 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

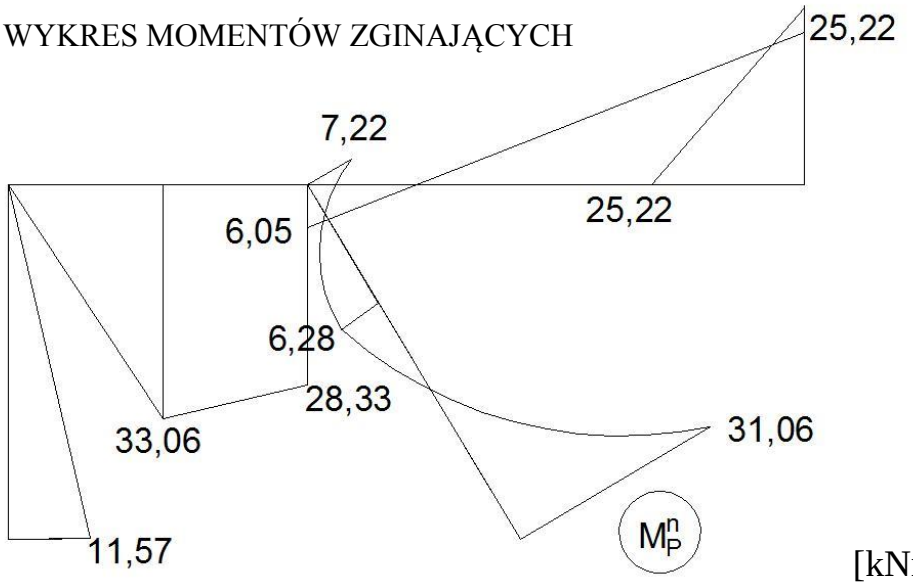
φ_1= -0,003997 rad
φ_2= 0,004419 rad
u_3= -0,02425 m

WYZNACZENIE M_p^n :

$$M_p^{(n)} = M_1 \cdot \varphi_1 + M_2 \cdot \varphi_2 + M_3 \cdot \Delta_3 + M_p^0$$

ik	M_1	M_2	M_3	M_p^0	M_{ik}^n [kNm]
01	0	0	-477,24	0	11,5734
21	5696,0714	0	813,7245	14,175	-28,32843
23	2728,6449	0	-818,5935	-16,667	-7,72299
32	1364,3225	0	-818,5935	16,667	31,064286
24	4556,8571	2278,4286	-585,8816	0	6,05104
42	2278,4286	4556,8571	-585,8816	0	25,21809
45	0	4772,4	1908,96	0	-25,22395

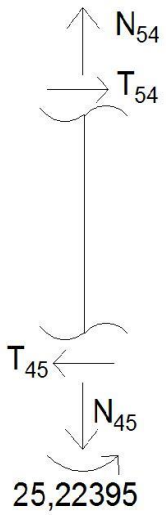
WYKRES MOMENTÓW ZGINAJĄCYCH



[kNm]

WYZNACZENIE WARTOŚCI SIŁ TNĄCYCH:

PRĘT 45:



$\Sigma M_5 = 0$

$-25{,}22395 + T_{45} \bullet 2{,}5 = 0$

$T_{45} = 10{,}08958$ kN

$\Sigma X = 0$

$T_{54} - T_{45} = 0$

$T_{54} = 10{,}08958$ kN

PRĘT 42 :

$\Sigma M_2 = 0$

$6{,}05104 + 25{,}21809 + T_{42} \bullet 7 = 0$

$T_{42} = -4{,}46702$ kN

$\Sigma Y = 0$

$T_{24} - T_{42} = 0$

$T_{24} = -4{,}46702$ kN

PRĘT 21:

$\Sigma M_1 = 0$

$-28{,}32843 + 18 \bullet 2{,}1 + T_{21} \bullet 4{,}2 = 0$

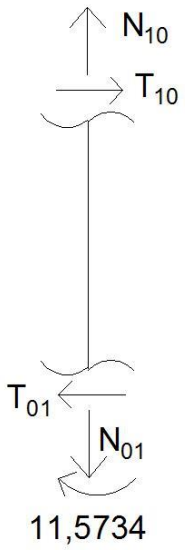
$T_{21} = -2{,}25514$ kN

$\Sigma Y = 0$

$2{,}25514 - 18 + T_{12} = 0$

$T_{12} = 15{,}74486$ kN

PRĘT 10:

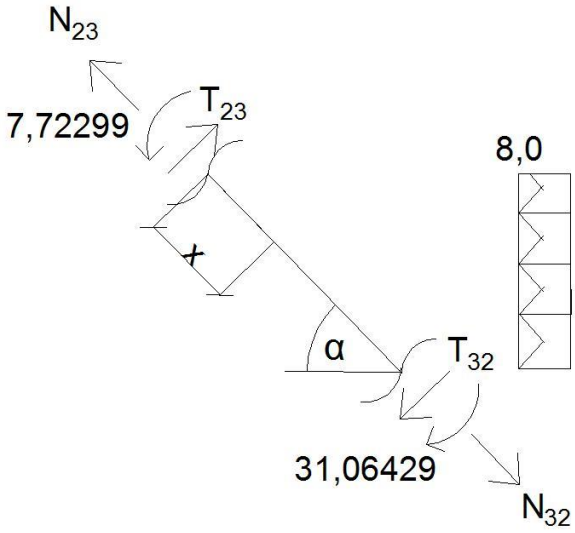


$\Sigma M_1 = 0$

$11{,}5734 + T_{01} \bullet 5 = 0$ $\quad T_{01} = -2{,}31468$ kN

$\Sigma X = 0 T_{10} - T_{01} = 0$ $\quad T_{10} = -2{,}31468$ kN

PRĘT 23:



$\Sigma M_2 = 0$

$31{,}064286 - 7{,}72299 + 8 \cdot 5 \cdot 2{,}5 + T_{32} \cdot 5{,}83 = 0$

$T_{32} = -21{,}156312$ kN

$\Sigma M_3 = 0$

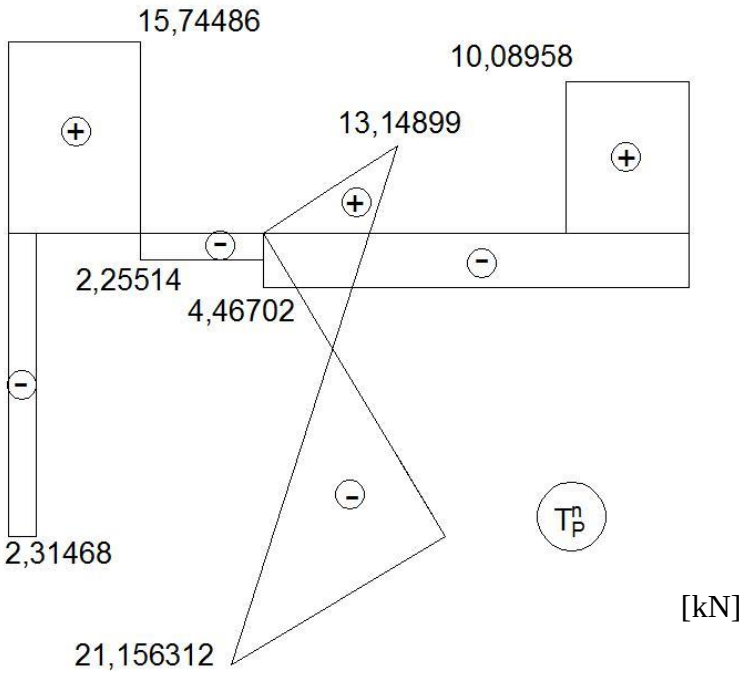
$31{,}064286 - 7{,}72299 - 8 \cdot 5 \cdot 2{,}5 + T_{23} \cdot 5{,}83 = 0$

$T_{23} = 13{,}14899$ kN

$$x = \frac{T^L}{q^+} = \frac{13{,}14899}{8 \cdot sin^2\alpha} = 2{,}23\ m$$

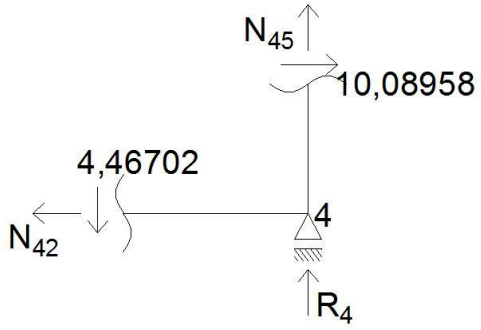
$M_e = 6{,}28$ kNm

WYKRES SIŁ TNĄCYCH:



WYZNACZENIE WARTOŚCI SIŁ NORMALNYCH:

WĘZEŁ 4:



$N_{45} = 0$

$\Sigma Y = 0$

$-4{,}46702 + 0 + R_4 = 0$

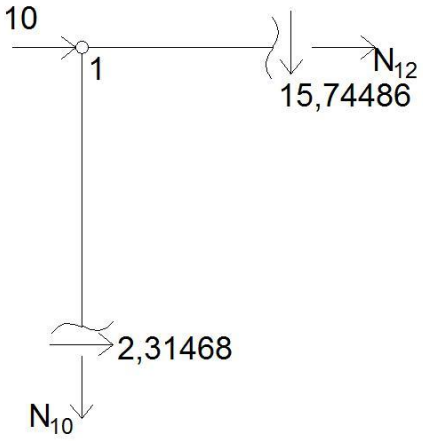
$R_4 = 4{,}46702$ kN

$\Sigma X = 0$

$N_{42} - 10{,}08958 = 0$

$N_{42} = 10{,}08958$ kN

WĘZEŁ 1:



$\Sigma X = 0$

$10 + 2{,}31468 + N_{12} = 0$

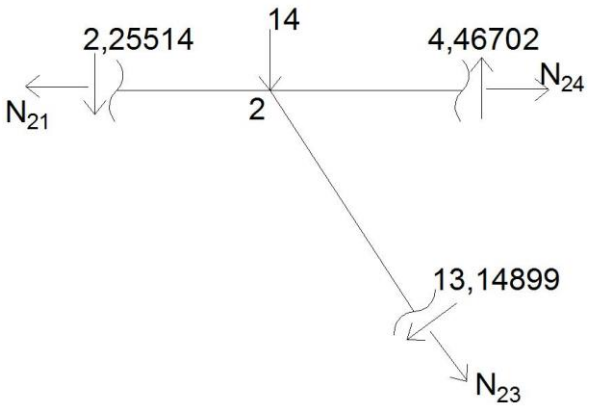
$N_{12} = -12{,}31468$ kN

$\Sigma Y = 0$

$-15{,}74486 - N_{10} = 0$

$N_{10} = -15{,}74486$ kN

WĘZEŁ 2:



$N_{21} = N_{12} = -12{,}31468$ kN

$N_{24} = N_{42} = 10{,}08958$ kN

$\Sigma Y = 0$

$2{,}25514 + 14 - 4{,}46702 + N_{23} \cdot \sin\alpha + 13{,}14899 \cdot \cos\alpha = 0$

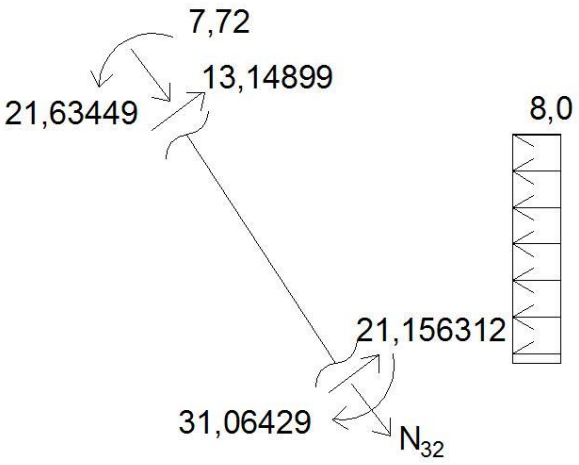
$N_{23} = -21{,}634429$ kN

SPRAWDZENIE: Σ X = 0

12,31468 + 10,08958 + N_{23} • cosα – 13,14899 • sinα = 0

N_{23} = -21,624002 kN

PRĘT 23:

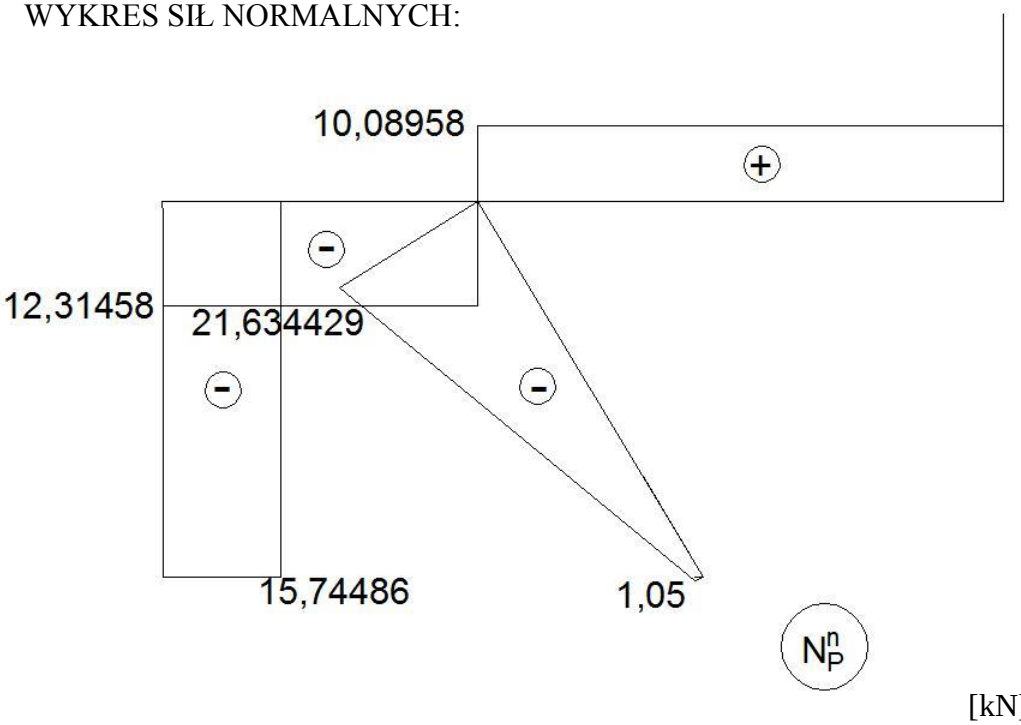


Σ l pręta = 0

21,63449 + N_{32} – 8 • 5 cosα = 0

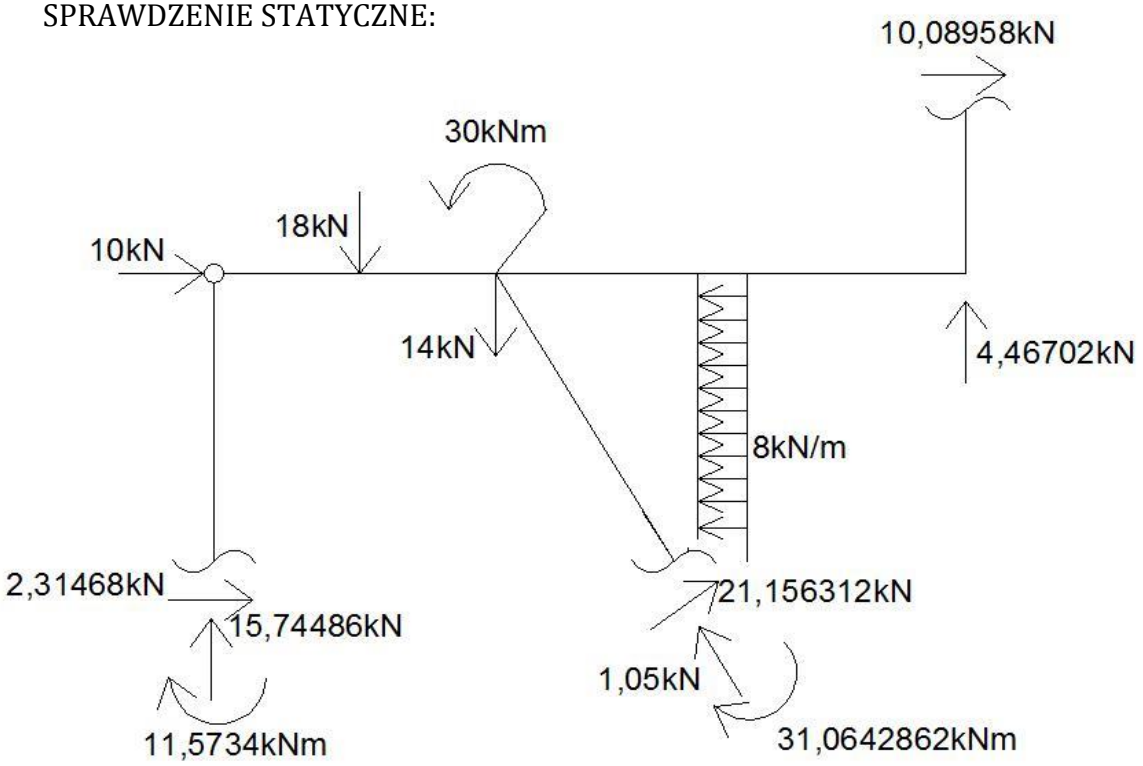
N_{32} = -1,05 kN

WYKRES SIŁ NORMALNYCH:



[kN]

SPRAWDZENIE STATYCZNE:

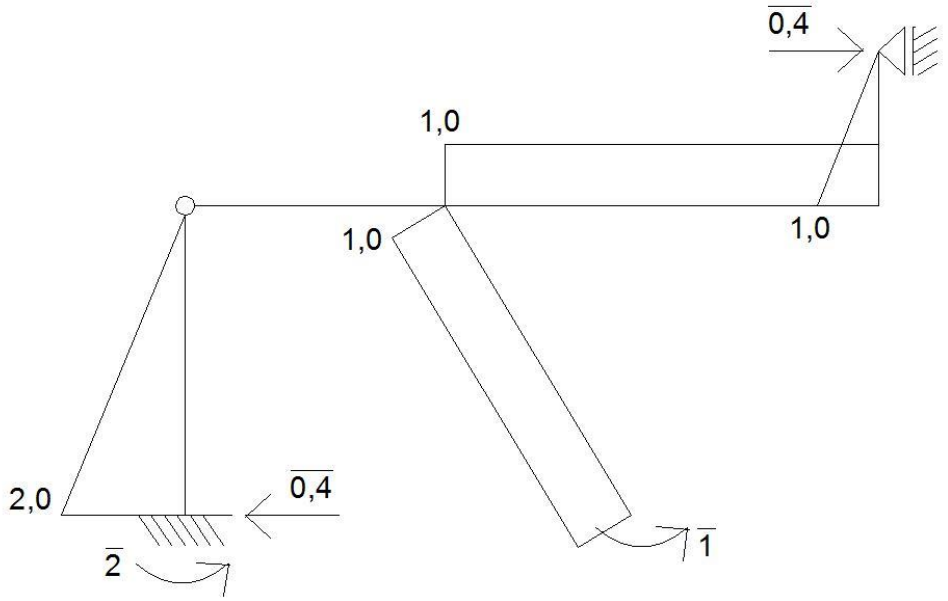


$\Sigma X = 0$
$2{,}31468 + 10 - 8 \cdot 5 + 10{,}08958 + 21{,}156312 \sin\alpha - 1{,}05 \cos\alpha = 0{,}0082 \text{ kN} \approx 0$

$\Sigma Y = 0$
$15{,}74486 - 18 - 14 + 4{,}46702 + 21{,}156312 \cos\alpha + 1{,}05 \sin\alpha = 0{,}00099 \text{ kN} \approx 0$

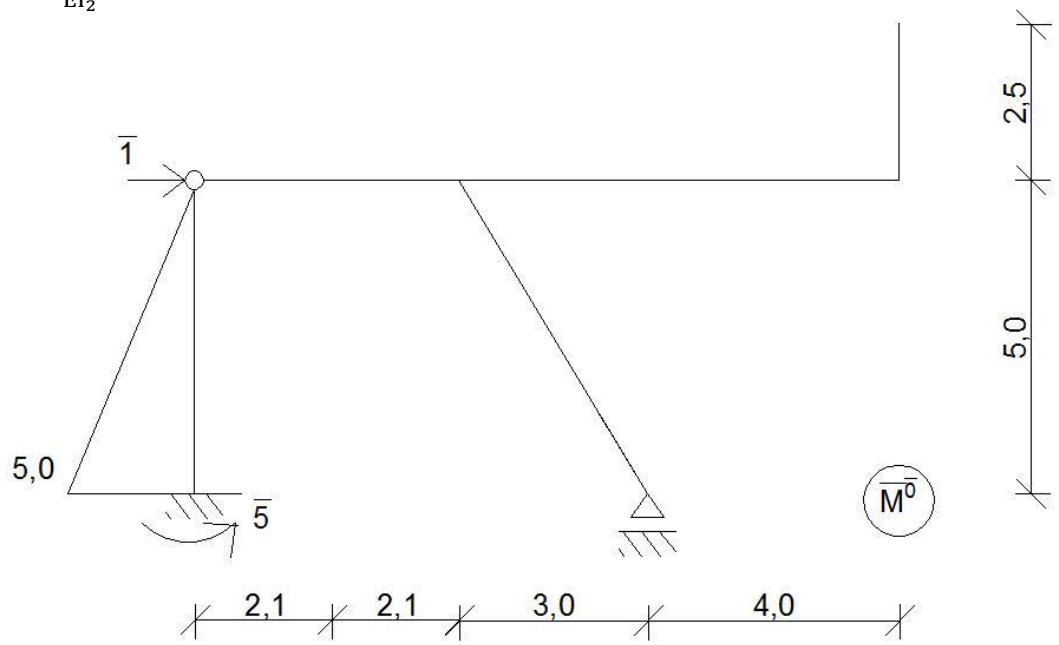
$\Sigma M_2 = 0$
$15{,}74486 \cdot 4{,}2 + 11{,}5734 - 2{,}31468 \cdot 5 - 18 \cdot 2{,}1 - 30 + 31{,}0642862 - 21{,}156312 \cdot 5{,}83 +$
$10{,}08958 \cdot 2{,}5 - 4{,}46702 \cdot 7 + 8 \cdot 5 \cdot 2{,}5 = 0{,}006 \text{ kNm} \approx 0$

SPRAWDZENIE KINEMATYCZNE:



$$\bar{1} \cdot \varphi_3 = \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 16{,}667 \cdot 5{,}83 \cdot 1 - \frac{1}{2}\cdot 16{,}667 \cdot 5{,}83 \cdot 1 + \frac{2}{3}\cdot\frac{8\cdot 5^2}{8}\cdot 5{,}83 \cdot 1\right] =$$

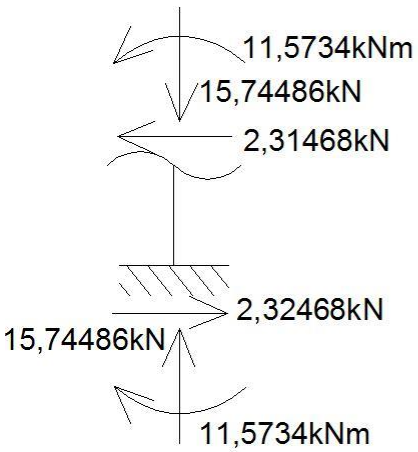
$$-\frac{0{,}001943}{EI_2} = -4{,}886 \cdot 10^{-7}\ \text{rad} \approx 0$$

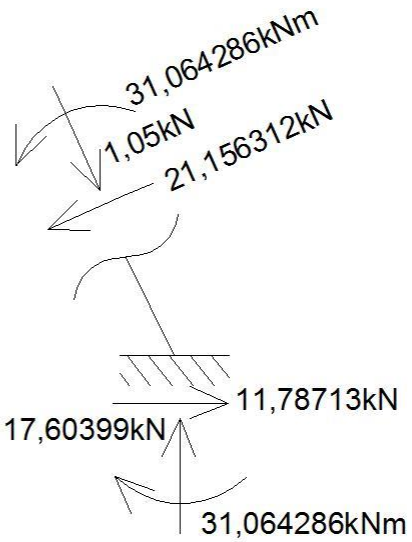


$$\bar{1} \cdot u = \Sigma \int \frac{M_P^n \cdot \overline{M^0}}{EI}\, dx$$

$$\bar{1} \cdot u = \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 11{,}57340 \cdot 5 \cdot \frac{2}{3}\cdot 5\right] = -\frac{96{,}445}{EI_2} = -0{,}02418\ \text{m} \approx u_3$$

REAKCJE PODPOROWE:





$$\begin{cases} V_0 = 15{,}74486\ kN \\ H_0 = 2{,}314680\ kN \\ M_0 = 11{,}5734\ kNm \end{cases}$$

$$\begin{cases} V_3 = 11{,}78713\ kN \\ H_3 = 17{,}60399\ kN \\ M_3 = 31{,}064286\ kNm \end{cases}$$

SPRAWDZENIE NAPRĘŻEŃ:

$$\frac{M_{max}}{W} \leq \sigma_{dop} = 21{,}5\ \frac{kN}{cm^2}$$

Dla przekroju I_1 maksymalny moment wynosi: 33,06 kNm

Wskaźnik wytrzymałości W dla tego przekroju wynosi: 324 cm^3

$$\sigma = \frac{3306kNcm}{324cm^3} = 10{,}2037\ \frac{kN}{cm^2} < \sigma_{dop}$$

Dla przekroju I_2 maksymalny moment wynosi: 31,06 kNm

Wskaźnik wytrzymałości W dla tego przekroju wynosi: 194 cm^3

$$\sigma = \frac{3106kNcm}{194cm^3} = 16{,}0103\ \frac{kN}{cm^2} < \sigma_{dop}$$

Oba przekroje spełniają warunek wytrwałościowy.