Ćwiczenie nr 1

Metoda Przemieszczeń

Belka

Prowadzący: mgr inż. Anita Kacor
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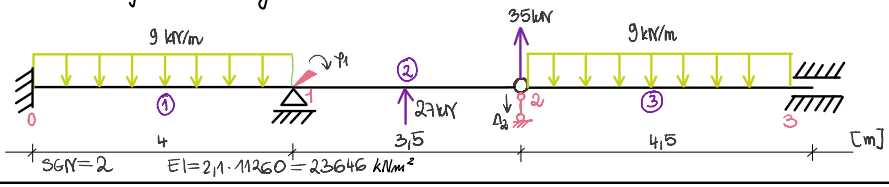
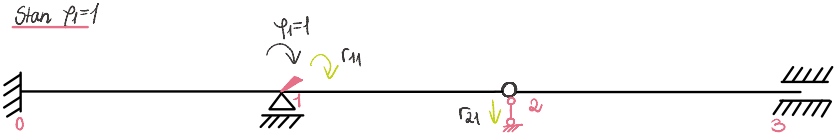
Nr indeksu: 152258
Semestr/rok: IV / 2022/2023

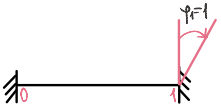
Układ podstawowy:



SGN = 2 $\quad EI = 21 \cdot 11260 = 23646\ kNm^2$

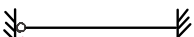
Stan $\varphi_1 = 1$





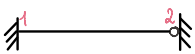
$M_{01} = \frac{2EI}{4} \cdot 1 = 0{,}5EI$

$M_{10} = \frac{2EI}{4} \cdot 2 = EI$



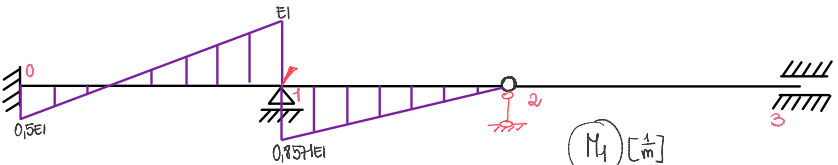
$M_{20} = 0$

$M_{32} = 0$



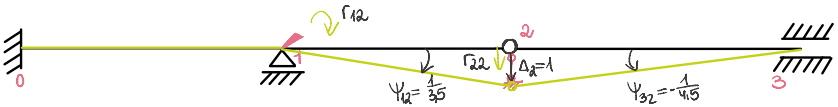
$M_{12} = \frac{3EI}{3,5} \cdot 1 = 0{,}857EI$

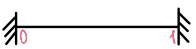
$M_{21} = 0$



M_1 $[\frac{1}{m}]$

Stan $\Delta_2 = 1$



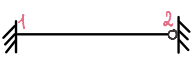


$M_{01} = 0$

$M_{10} = 0$

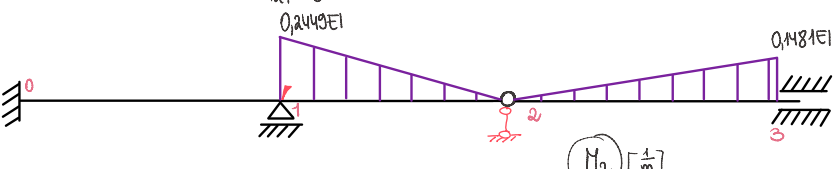
$M_{20} = 0$

$M_{32} = \frac{3EI}{4,5} \cdot \frac{1}{4,5} = 0{,}1481EI$



$M_{12} = \frac{3EI}{3,5} \cdot (-\frac{1}{3,5}) = -0{,}2449EI$

$M_{21} = 0$

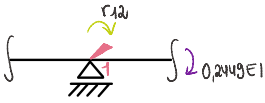


M_2 $[\frac{1}{m}]$

$$\Sigma M^{(1)} = 0$$

$$r_{11} - EI - 0{,}8571EI = 0$$

$$r_{11} = 1{,}857EI$$



$$\Sigma M^{(1)} = 0$$

$$r_{12} + 0{,}2449EI = 0$$

$$r_{12} = -0{,}2449EI$$

RPN

$$r_{21} \cdot \bar{1} + 0{,}8571EI \cdot \frac{\bar{1}}{3{,}5} = 0$$

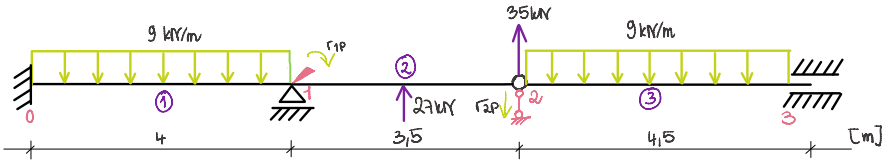
$$r_{21} = 0{,}2449EI$$

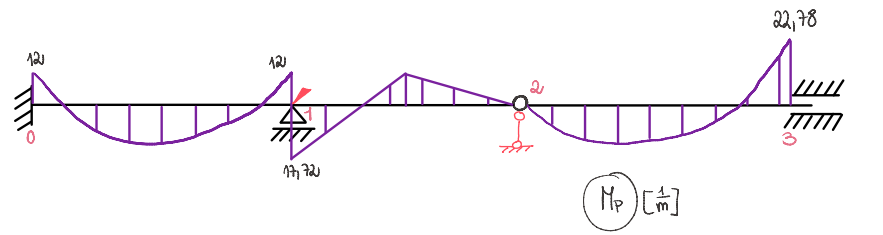
$$r_{22} \cdot \bar{1} - 0{,}2449EI \cdot \frac{\bar{1}}{3{,}5} + 0{,}1481EI \cdot \left(-\frac{\bar{1}}{4{,}5}\right) = 0$$

$$r_{22} = 0{,}1029EI$$

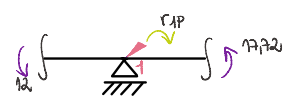
$$K = \begin{bmatrix} 1{,}857EI & -0{,}2449EI \\ -0{,}2449EI & 0{,}1029EI \end{bmatrix}$$

Stan P





M_P $[\frac{1}{m}]$



$$\Sigma M^{(1)} = 0$$

$$r_{1P} - 12 - 17{,}72 = 0$$

$$r_{1P} = 29{,}72 \text{ kNm}$$

RPN

$$r_{2P} \cdot \bar{1} + 17{,}72 \cdot \frac{\bar{1}}{3{,}5} + 22{,}78 \left(-\frac{\bar{1}}{4{,}5}\right) - 27 \cdot 0{,}\bar{5} - 35 \cdot \bar{1} + 9 \cdot 4{,}5 \cdot 0{,}\bar{5} = 0$$

$$r_{2P} = 28{,}25 \text{ kN}$$

URK

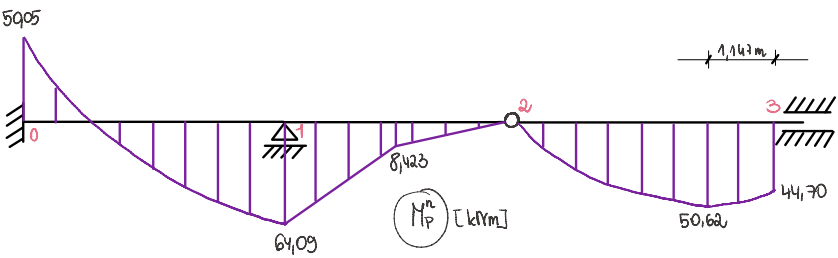
$$\begin{cases} r_{11}\varphi_1 + r_{12}\Delta_2 + r_{1p} = 0 \\ r_{21}\varphi_1 + r_{22}\Delta_2 + r_{2p} = 0 \end{cases} \qquad K = \begin{bmatrix} 1{,}857EI & -0{,}2449EI \\ -0{,}2449EI & 0{,}1029EI \end{bmatrix}$$

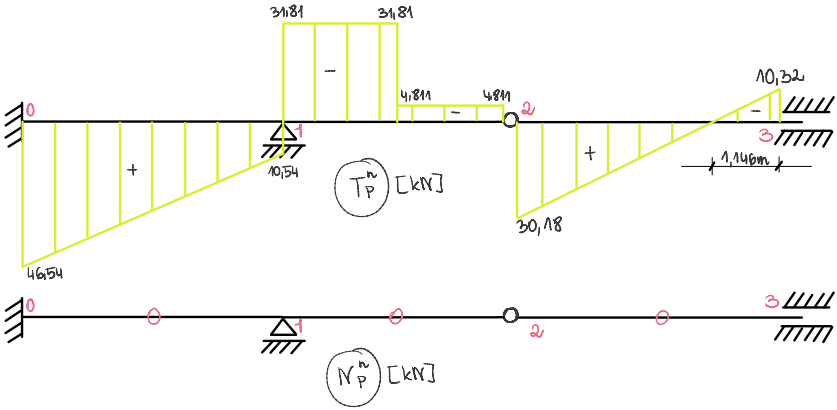
$$\begin{cases} 1{,}857\varphi_1 - 0{,}2449\Delta_2 + 29{,}72 = 0 \\ -0{,}2449\varphi_1 + 0{,}1029\Delta_2 + 28{,}25 = 0 \end{cases}$$

$$\begin{cases} \varphi_1 = -\dfrac{7609}{EI} = -0{,}1844^\circ \\ \Delta_2 = -\dfrac{465{,}64}{EI} = -0{,}01927\,m \end{cases}$$

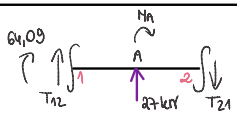
$$M_{ik}^{n} = M_{ik}^{(1)}\varphi_1 + M_{ik}^{(2)}\Delta_2 + M_{ik}^{(P)}$$

ik	$M_{ik}^{(1)}$	$M_{ik}^{(2)}$	$M_{ik}^{(P)}$	$M_{ik}^{(n)}$ [kNm]
01	0,5	0	−12	−50,09
10	1	0	12	−64,09
12	0,8571	−0,2449	17,72	64,09
32	0	0,1481	22,78	−44,70



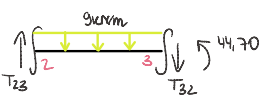


$$\Sigma M^{(0)} = 9 \cdot 4 \cdot 2 + T_{10} \cdot 4 - 64{,}09 - 50{,}05 = 0$$
$$T_{10} = 10{,}54 \text{ kN}$$
$$\Sigma M^{(1)} = -9 \cdot 4 \cdot 2 + T_{01} \, 4 - 64{,}09 - 50{,}05 = 0$$
$$T_{01} = 46{,}54 \text{ kN}$$



$$\Sigma M^{(1)} = 64{,}09 - 27 \cdot \frac{3{,}5}{2} + T_{21} \cdot 3{,}5 = 0$$
$$T_{21} = -4{,}811 \text{ kN}$$
$$\Sigma M^{(2)} = 64{,}09 + T_{12} \cdot 3{,}5 + 27 \cdot \frac{3{,}5}{2} = 0$$
$$T_{12} = -31{,}81 \text{ kN}$$

$$\Sigma M^{(A)} = 64{,}09 - 31{,}81 \cdot \frac{3{,}5}{2} + M_A = 0$$
$$M_A = -8{,}423 \text{ kNm}$$

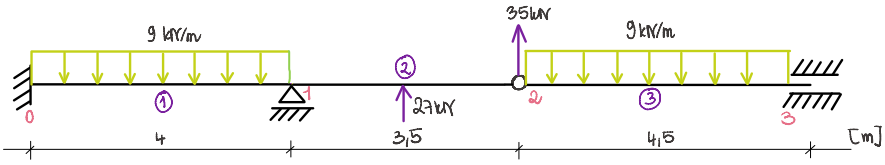


$$\Sigma M^{(2)} = 9 \cdot 4{,}5 \cdot \frac{4{,}5}{2} - 44{,}70 + T_{32} \, 4{,}5 = 0$$
$$T_{32} = -10{,}32 \text{ kN}$$
$$\Sigma M^{(3)} = -9 \, 4{,}5 \cdot \frac{4{,}5}{2} - 44{,}70 + T_{23} \cdot 4{,}5 = 0$$
$$T_{23} = 30{,}18 \text{ kN}$$

$$T(x) = -9 \cdot x + 10{,}32 = 0$$
$$x = 1{,}147 \text{ m}$$

$$\Sigma M^{(1{,}147)} = M_{EXT} + 9 \cdot 1{,}147 \, \frac{1{,}147}{2} - 44{,}70 - 10{,}32 \cdot 1{,}147 = 0$$

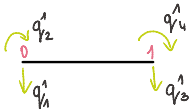
$$M_{EXT} = 50{,}62 \text{ kNm}$$



$EI = const \Rightarrow EI = 2{,}1 \cdot 11260 = 23646$

Macierze sztywności

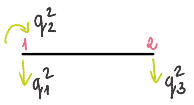
①



$$q_1 = \begin{bmatrix} q_1^1 \\ q_2^1 \\ q_3^1 \\ q_4^1 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{2EI}{l} & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} = EI \begin{bmatrix} 0{,}1875 & 0{,}3750 & -0{,}1875 & 0{,}3750 \\ 0{,}3750 & 1 & -0{,}3750 & 0{,}5 \\ -0{,}1875 & -0{,}3750 & 0{,}1875 & -0{,}3750 \\ 0{,}3750 & 0{,}5 & -0{,}3750 & 1 \end{bmatrix}$$

②



$$q_2 = \begin{bmatrix} q_1^2 \\ q_2^2 \\ q_3^2 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} \end{bmatrix} = EI \begin{bmatrix} 0{,}06997 & 0{,}2449 & -0{,}06997 \\ 0{,}2449 & 0{,}8571 & -0{,}2449 \\ -0{,}06997 & -0{,}2449 & 0{,}06997 \end{bmatrix}$$

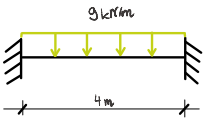
③

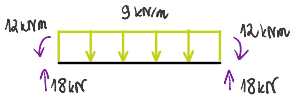
$$q_3 = \begin{bmatrix} q_1^3 \\ q_2^3 \\ q_3^3 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} \frac{3EI}{l^3} & -\frac{3EI}{l^3} & \frac{3EI}{l^2} \\ -\frac{3EI}{l^3} & \frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \frac{3EI}{l^2} & -\frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix} = EI \begin{bmatrix} 0,03292 & -0,03292 & 0,1481 \\ -0,03292 & 0,03292 & -0,1481 \\ 0,1481 & -0,1481 & 0,6667 \end{bmatrix}$$

Wektory reakcji przywęzłowych od obciążenia przęsłowego

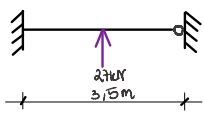
①

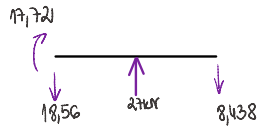




$$R_{01} = \begin{bmatrix} -18 \\ -12 \\ -18 \\ 12 \end{bmatrix}$$

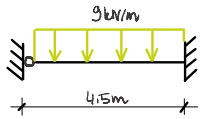
②

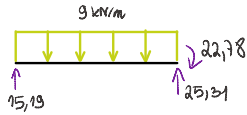




$$R_{02} = \begin{bmatrix} 18,56 \\ 17,72 \\ 8,438 \end{bmatrix}$$

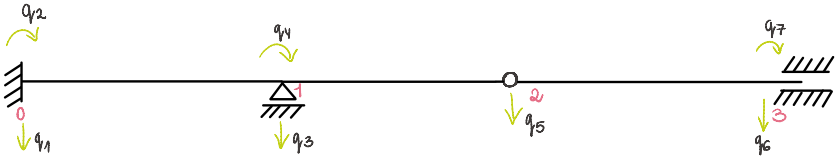
③





$$R_{03} = \begin{bmatrix} -15,19 \\ -25,31 \\ 22,78 \end{bmatrix}$$

Przemieszczenia w układzie globalnym



Tablica powiązań

Nr elementu	Numery przemieszczeń			
	1	2	3	4
1	1	2	3	4
2	3	4	5	–
3	5	6	7	–

GLOBALNE

Agregacja macierzy sztywności

$$K_1 = EI \begin{bmatrix} 0{,}1875 & 0{,}3750 & -0{,}1875 & 0{,}3750 \\ 0{,}3750 & 1 & -0{,}3750 & 0{,}5 \\ -0{,}1875 & -0{,}3750 & 0{,}1875 & -0{,}3750 \\ 0{,}3750 & 0{,}5 & -0{,}3750 & 1 \end{bmatrix}$$

$$K_2 = EI \begin{bmatrix} 0{,}06997 & 0{,}2449 & -0{,}06997 \\ 0{,}2449 & 0{,}8571 & -0{,}2449 \\ -0{,}06997 & -0{,}2449 & 0{,}06997 \end{bmatrix}$$

$$K_3 = EI \begin{bmatrix} 0{,}03292 & -0{,}03292 & 0{,}1481 \\ -0{,}03292 & 0{,}03292 & -0{,}1481 \\ 0{,}1481 & -0{,}1481 & 0{,}6667 \end{bmatrix}$$

$$K = EI \begin{bmatrix} 0{,}1875 & 0{,}3750 & -0{,}1875 & 0{,}3750 & 0 & 0 & 0 \\ 0{,}3750 & 1 & -0{,}3750 & 0{,}5 & 0 & 0 & 0 \\ -0{,}1875 & -0{,}3750 & 0{,}1875+0{,}06997 & -0{,}3750+0{,}2449 & -0{,}06997 & 0 & 0 \\ 0{,}3750 & 0{,}5 & -0{,}3750+0{,}2449 & 1+0{,}8571 & -0{,}2449 & 0 & 0 \\ 0 & 0 & -0{,}06997 & -0{,}2449 & 0{,}06997+0{,}03292 & -0{,}03292 & 0{,}1481 \\ 0 & 0 & 0 & 0 & -0{,}03292 & 0{,}03292 & -0{,}1481 \\ 0 & 0 & 0 & 0 & 0{,}1481 & -0{,}1481 & 0{,}6667 \end{bmatrix}$$

$$\underline{K} = EI \begin{bmatrix} 0{,}1875 & 0{,}3750 & -0{,}1875 & 0{,}3750 & 0 & 0 & 0 \\ 0{,}3750 & 1 & -0{,}3750 & 0{,}5 & 0 & 0 & 0 \\ -0{,}1875 & -0{,}3750 & 0{,}2575 & -0{,}1301 & -0{,}06997 & 0 & 0 \\ 0{,}3750 & 0{,}5 & -0{,}1301 & 1{,}8571 & -0{,}2449 & 0 & 0 \\ 0 & 0 & -0{,}06997 & -0{,}2449 & 0{,}1029 & -0{,}03292 & 0{,}1481 \\ 0 & 0 & 0 & 0 & -0{,}03292 & 0{,}03292 & -0{,}1481 \\ 0 & 0 & 0 & 0 & 0{,}1481 & -0{,}1481 & 0{,}6667 \end{bmatrix}$$

Agregacja wektora reakcji przeciwległych od obciążenia pasywnego

$$\underline{R_{01}} = \begin{bmatrix} -18 \\ -12 \\ -18 \\ 12 \end{bmatrix} \qquad \underline{R_{02}} = \begin{bmatrix} 18{,}56 \\ 17{,}72 \\ 8{,}438 \end{bmatrix} \qquad \underline{R_{03}} = \begin{bmatrix} -15{,}19 \\ -25{,}31 \\ 22{,}78 \end{bmatrix}$$

$$\underline{R_0} = \begin{bmatrix} -18 \\ -12 \\ -18 + 18{,}56 \\ 12 + 17{,}72 \\ 8{,}438 - 15{,}19 \\ -25{,}31 \\ 22{,}78 \end{bmatrix} = \begin{bmatrix} -18 \\ -12 \\ 0{,}56 \\ 29{,}72 \\ -6{,}752 \\ -25{,}31 \\ 22{,}78 \end{bmatrix}$$

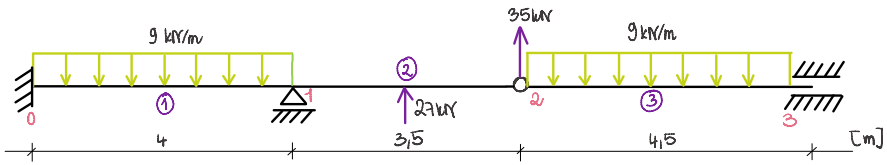
Uwzględnienie warunków brzegowych

$q_1 = 0,\ q_2 = 0,\ q_3 = 0,\ q_6 = 0,\ q_7 = 0$

$$K = EI \begin{bmatrix} 0,1875 & 0,3750 & -0,1875 & 0,3750 & 0 & 0 & 0 \\ 0,3750 & 1 & -0,3750 & 0,5 & 0 & 0 & 0 \\ -0,1875 & -0,3150 & 0,2575 & -0,1301 & 0,00997 & 0 & 0 \\ 0,3750 & 0,5 & -0,1301 & 1,1857 & -0,2449 & 0 & 0 \\ 0 & 0 & -0,00997 & -0,2449 & 0,1029 & -0,03292 & 0,1181 \\ 0 & 0 & 0 & 0 & -0,03292 & 0,03292 & -0,1181 \\ 0 & 0 & 0 & 0 & 0,1181 & -0,1181 & 0,6067 \end{bmatrix} \quad R_0 = \begin{bmatrix} -18 \\ -12 \\ -0,56 \\ 29,72 \\ -6,752 \\ -25,31 \\ 22,78 \end{bmatrix}$$

$$K_{red} = \begin{bmatrix} 1,1857 & -0,2449 \\ -0,2449 & 0,1029 \end{bmatrix} \begin{matrix} 4 \\ 5 \end{matrix} \qquad R_{0red} = \begin{bmatrix} 29,72 \\ -6,752 \end{bmatrix} \begin{matrix} 4 \\ 5 \end{matrix}$$

Wyznaczenie wektora obciążeń węzłowych



$$P_{red} = \begin{bmatrix} 0 \\ -35 \end{bmatrix} \begin{matrix} 4 \\ 5 \end{matrix}$$

Rozwiązanie równowagi

$$K_{red}\, q_{red} = P_{red} - R_{0red}$$

$$EI \begin{bmatrix} 1,1857 & -0,2449 \\ -0,2449 & 0,1029 \end{bmatrix} \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 0 \\ -35 \end{bmatrix} - \begin{bmatrix} 29,72 \\ -6,752 \end{bmatrix}$$

$$EI \begin{bmatrix} 1,1857 & -0,2449 \\ -0,2449 & 0,1029 \end{bmatrix} \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} -29,72 \\ -28,25 \end{bmatrix}$$

$$\begin{cases} 1,1857 EI q_4 - 0,2449 EI q_5 = -29,72 \\ -0,2449 EI q_4 + 0,1029 EI q_5 = -28,25 \end{cases} \qquad \begin{cases} q_4 = -\frac{7609}{EI} = -0,1844^{\circ} \\ q_5 = -\frac{465,64}{EI} = -0,01927\ m \end{cases}$$

Wektory przemieszczeń dla elementów

$$\underline{q_1'} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -76{,}09 \end{bmatrix} \qquad \underline{q_2'} = \begin{bmatrix} q_3 \\ q_4 \\ q_5 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0 \\ -76{,}09 \\ -455{,}64 \end{bmatrix} \qquad \underline{q_3'} = \begin{bmatrix} q_5 \\ q_6 \\ q_7 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -455{,}64 \\ 0 \\ 0 \end{bmatrix}$$

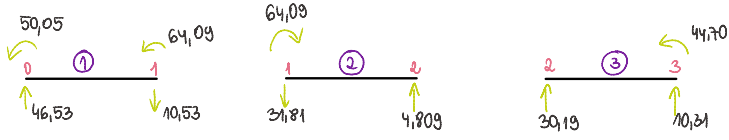
Wektory reakcji węzłowych

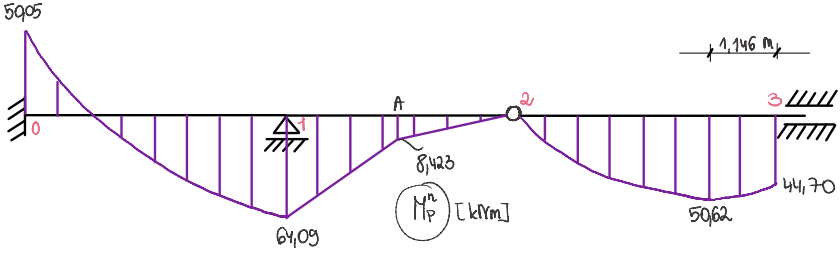
$$\underline{R_1} = \underline{K_1}\,\underline{q_1} + \underline{R_{01}} = EI\begin{bmatrix} 0{,}1875 & 0{,}3750 & -0{,}1875 & 0{,}3750 \\ 0{,}3750 & 1 & -0{,}3750 & 0{,}5 \\ -0{,}1875 & -0{,}3750 & 0{,}1875 & -0{,}3750 \\ 0{,}3750 & 0{,}5 & -0{,}3750 & 1 \end{bmatrix} \cdot \frac{1}{EI}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -76{,}09 \end{bmatrix} + \begin{bmatrix} -18 \\ -12 \\ -18 \\ 12 \end{bmatrix} = \begin{bmatrix} -46{,}53 \\ -50{,}05 \\ 10{,}153 \\ -64{,}09 \end{bmatrix}$$

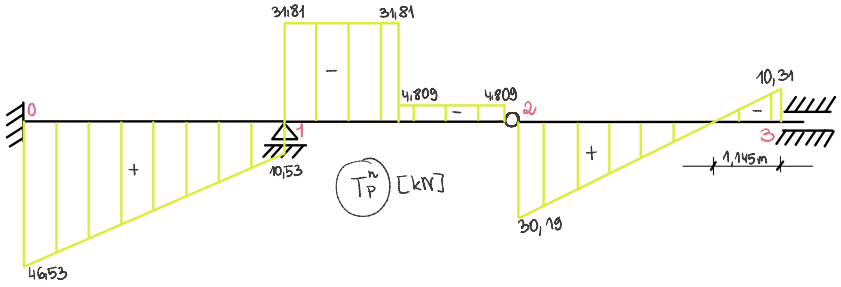
$$\underline{R_2} = \underline{K_2}\,\underline{q_2} + \underline{R_{02}} = EI\begin{bmatrix} 0{,}06997 & 0{,}2449 & -0{,}06997 \\ 0{,}2449 & 0{,}8571 & -0{,}2449 \\ -0{,}06997 & -0{,}2449 & 0{,}06997 \end{bmatrix} \cdot \frac{1}{EI}\begin{bmatrix} 0 \\ -76{,}09 \\ -455{,}64 \end{bmatrix} + \begin{bmatrix} 18{,}56 \\ 17{,}72 \\ 8{,}438 \end{bmatrix} = \begin{bmatrix} 31{,}81 \\ 64{,}09 \\ -4{,}809 \end{bmatrix}$$

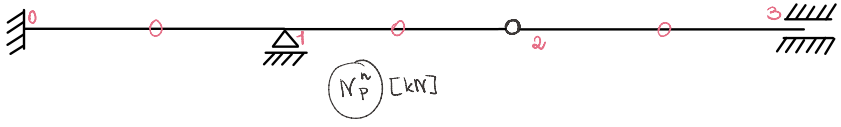
$$\underline{R_3} = \underline{K_3}\,\underline{q_3} + \underline{R_{03}} = EI\begin{bmatrix} 0{,}03292 & -0{,}03292 & 0{,}1481 \\ -0{,}03292 & 0{,}03292 & -0{,}1481 \\ 0{,}1481 & -0{,}1481 & 0{,}6667 \end{bmatrix} \cdot \frac{1}{EI}\begin{bmatrix} -455{,}64 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -15{,}19 \\ -25{,}31 \\ 22{,}78 \end{bmatrix} = \begin{bmatrix} -30{,}19 \\ -10{,}31 \\ -44{,}70 \end{bmatrix}$$

Siły węzłowe









$$T(x) = -9 \cdot x + 10{,}31 = 0$$
$$x = 1{,}146\,m$$

$$\Sigma M^{(1146)} = M_{EXT} + 9 \cdot 1{,}146 \cdot \frac{1146}{2} - 44{,}70 - 10{,}32 \cdot 1{,}146 = 0$$

$$M_{EXT} = 50{,}62\ kNm$$

$$\Sigma M^{(A)} = 64{,}09 - 31{,}81 \cdot \frac{3{,}5}{2} + M_A = 0$$
$$M_A = -8{,}423\ kNm$$