

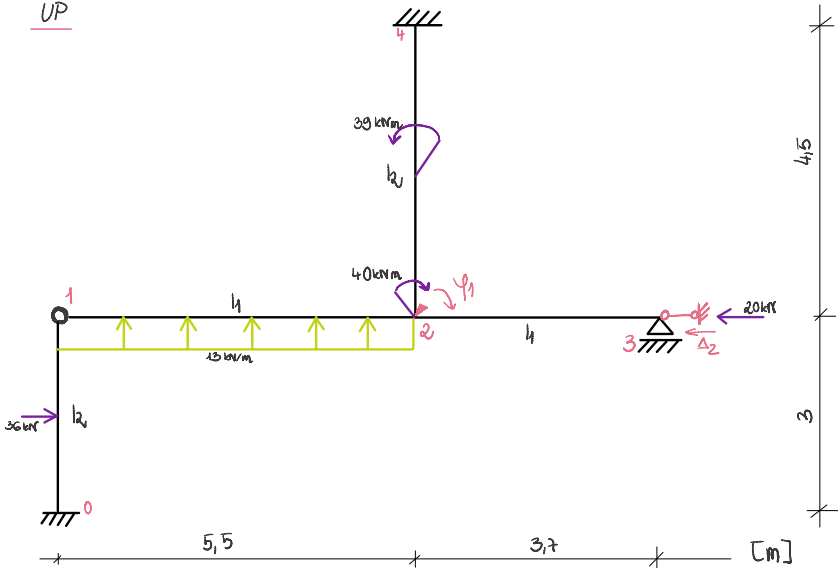


Ćwiczenie nr 1
Metoda Przemieszczeń
Rama

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Nr indeksu: 152258
Semestr/rok: IV / 2022/2023

Sily zewnętrzne

UP



$SGR=2$

Układ równań kanonicznych:

$l_1 \Rightarrow 1160 \text{ HEB}$

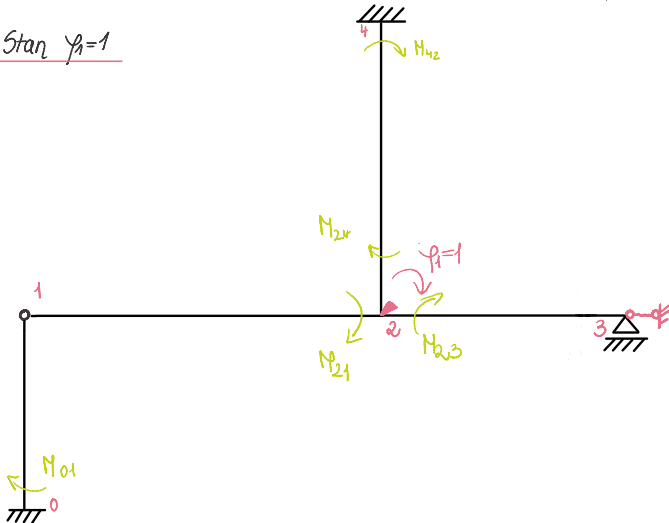
$l_2 \Rightarrow 1140 \text{ PE}$

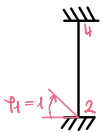
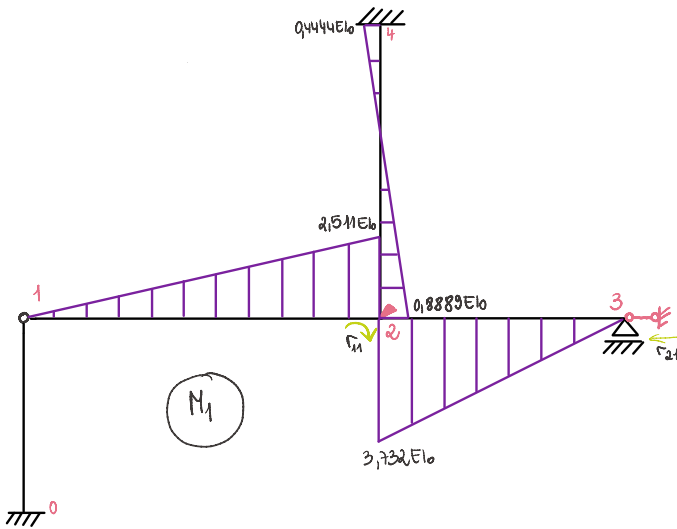
$EI_1 = 2,1 \cdot 2490 = 5229 \text{ kNm}^2 \quad EI_1 = 4,603 EI_0$

$EI_2 = 2,1 \cdot 541 = 1136,1 \text{ kNm}^2 \quad EI_2 = EI_0$

$$\begin{cases} r_{11} \varphi_1 + r_{12} \Delta_2 + r_{1p} = 0 \\ r_{21} \varphi_1 + r_{22} \Delta_2 + r_{2p} = 0 \end{cases}$$

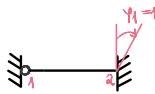
Stan $\varphi_1=1$





$$M_{12} = \frac{2 \cdot E I_2}{4,5} \cdot \frac{2 \cdot E I_0}{4,5} = 0,4444 E I_0$$

$$M_{21} = \frac{2 \cdot E I_0}{4,5} \cdot 2 \cdot 1 = \frac{4 E I_0}{4,5} = 0,8889 E I_0$$



$$M_{12} = 0$$

$$M_{21} = \frac{3 E I_1}{5,5} \cdot 1 = \frac{13,81 E I_0}{5,5} = 2,511 E I_0$$



$$M_{23} = \frac{3 \cdot E I_1}{3,7} \cdot 1 = 3,732 E I_0$$

$$M_{32} = 0$$



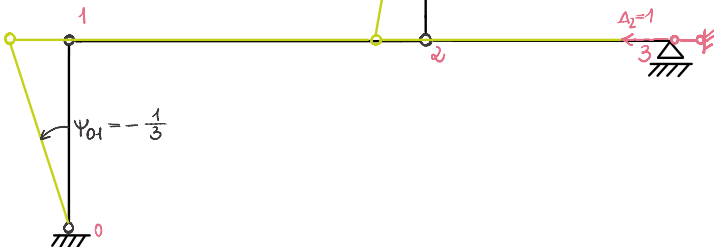
$$M_{10} = 0$$

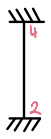
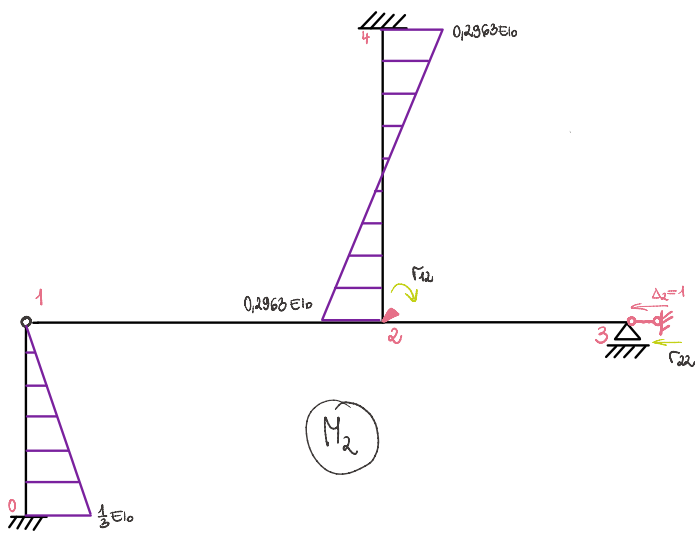
$$M_{01} = 0$$



$$\psi_{42} = \frac{1}{4,5}$$

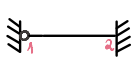
Stan $\Delta_2 = 1$





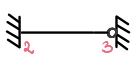
$$M_{12} = \frac{2EI_2}{45} \cdot \left(-3 \cdot \frac{1}{45}\right) = -\frac{6E10}{415^2} = -0.2963E10$$

$$M_{21} = -\frac{6E10}{415^2} = -0.2963E10$$



$$M_{24} = 0$$

$$M_{42} = \frac{3EI_1}{5.5} \cdot 0 = 0$$



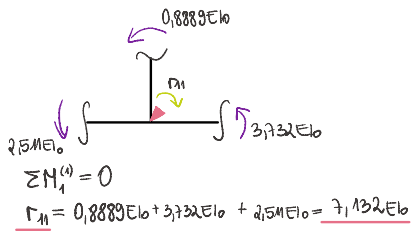
$$M_{23} = \frac{3EI_1}{3.7} \cdot 0 = 0$$

$$M_{32} = 0$$



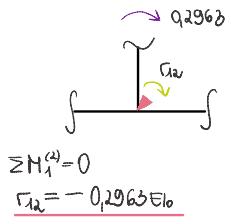
$$M_{10} = 0$$

$$M_{01} = \frac{2EI_2}{3} \cdot \frac{1}{3} = \frac{1}{3}E10$$



$$\sum M_1^{(2)} = 0$$

$$r_{11} = 0.8889E10 + 3.732E10 + 2.154E10 = \underline{7.132E10}$$



$$\sum M_2^{(4)} = 0$$

$$r_{22} = -0.2963E10$$

RPW

$$r_{21} \cdot \bar{1} + (0.4444E10 + 0.8889E10) \cdot \frac{1}{45} = 0$$

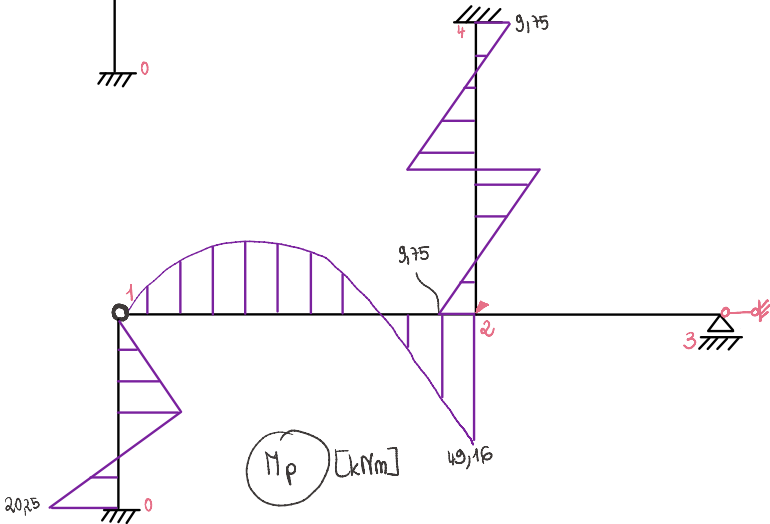
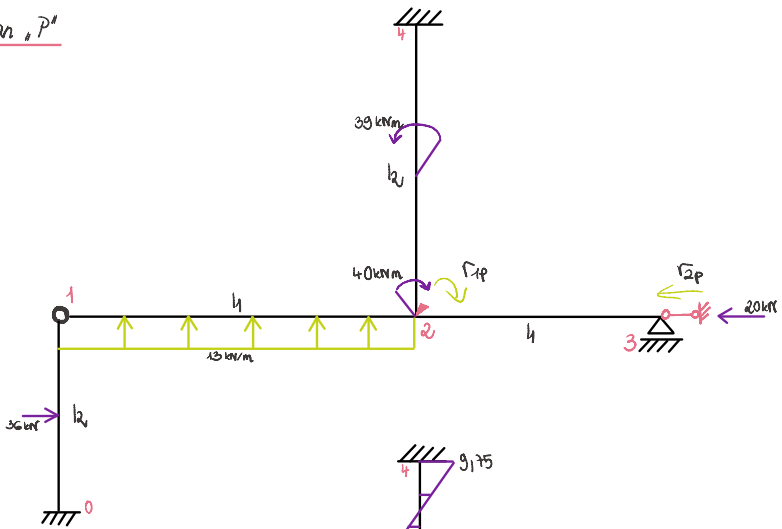
$$r_{21} = -0.2963E10$$

$$r_{22} \cdot \bar{1} + (2 \cdot (-0.2963E10)) \cdot \frac{1}{45} + \frac{E10}{3} \cdot \left(-\frac{1}{3}\right) = 0$$

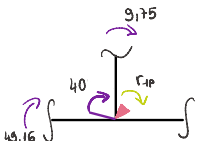
$$r_{22} = 0.1317E10 + 0.1141E10 = \underline{0.2428E10}$$

$$K = \begin{bmatrix} 7.132E10 & -0.2963E10 \\ -0.2963E10 & 0.2428E10 \end{bmatrix}$$

Stan, P'



M_p [kNm]



$$\sum M_P^{(2)} = 0$$

$$R_{2P} = -9.75 - 49.16 - 40 = -98.91 \text{ kNm}$$

RPN

$$R_{2P} \cdot 1 + 2 \cdot (-9.75) \cdot \frac{1}{4.5} + (-49.16) \cdot 0 - 20.25 \cdot (-\frac{1}{3}) + 20 \cdot 1 + 40 \cdot 0 + 13 \cdot 9.9 \cdot 0 + 36 \cdot (-\frac{1}{3}) - 39 \cdot (\frac{1}{4.5}) = 0$$

$$R_{2P} = 4.25 \text{ kN}$$

VRK

$$\begin{cases} r_{11}\varphi_1 + r_{12}A_2 + r_{1p} = 0 \\ r_{21}\varphi_1 + r_{22}A_2 + r_{2p} = 0 \end{cases}$$

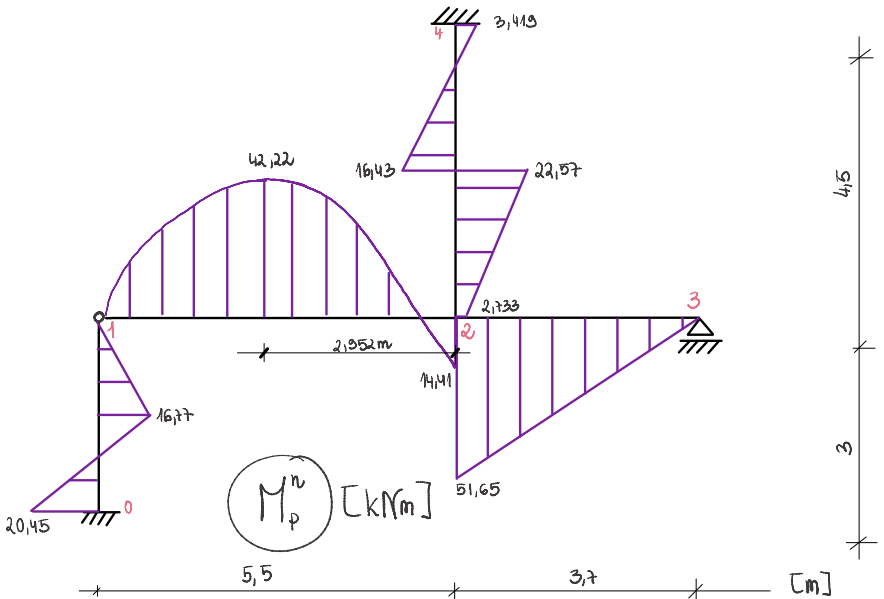
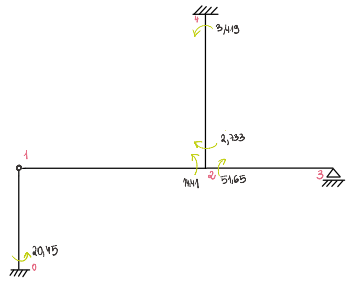
$$K = \begin{bmatrix} 7,132E10 & -0,2963E10 \\ -0,2963E10 & 0,24428E10 \end{bmatrix}$$

$$\begin{cases} 7,132E10\varphi_1 - 0,2963E10A_2 - 98,91 = 0 \\ -0,2963E10\varphi_1 + 0,24428E10A_2 + 4,25 = 0 \end{cases}$$

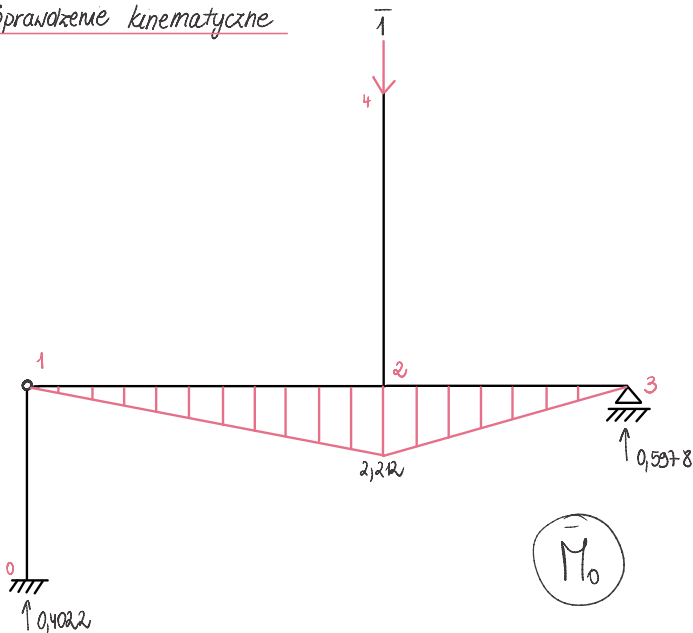
$$\begin{cases} \varphi_1 = \frac{13,84}{E10} = 0,01218 \text{ rad} = 0,698^\circ \\ A_2 = -\frac{96107}{E10} = -0,0005375 \text{ m} \end{cases}$$

$$M_{ik}^n = M_{ik}^{(1)}\varphi_1 + M_{ik}^{(2)}A_2 + M_{ik}^{(a)}$$

ik	$M_{ik}^{(1)}$	$M_{ik}^{(2)}$	$M_{ik}^{(p)}$	$M_{ik}^{(a)}$ [kNm]
01	0	$\frac{1}{3}$	-20,25	-20,45
21	2,541	0	-49,16	-14,41
23	3,732	0	0	51,65
32	0	0	0	0
24	0,8889	-0,2963	-9,75	2,733
42	0,4444	-0,2963	-9,75	-3,413



Sprawdzenie kinematyczne



$$\delta^{(4)} = \frac{1}{EI_1} \left[\frac{1}{2} \cdot 51,65 \cdot 3,7 \cdot \frac{2}{3} \cdot 2,212 + \frac{1}{2} \cdot 14,41 \cdot 5,5 \cdot \frac{2}{3} \cdot 2,212 - \frac{2}{3} \cdot \frac{13 \cdot 5,5^2}{8} \cdot 5,5 \cdot \frac{1}{2} \cdot 2,212 \right] =$$

$$= \frac{0,00046}{4,603 EI_0} = \frac{0,00009993}{EI_0} < \frac{1}{EI_0}$$

Sprawdzenie naprężeń

$$EI_1 \quad M_{\max} = 51,65 \text{ kNm} \quad N = 20,0 \text{ kN}$$

$$\sigma_{\max} = \frac{51,65}{311} + \frac{20}{54,3} = 16,98 \frac{\text{kN}}{\text{cm}^2} = 169,8 \text{ MPa} < f_y = 235 \text{ MPa}$$

$$EI_2 \quad M_{\max} = 22,57 \text{ kNm} \quad N = 52,33 \text{ kN}$$

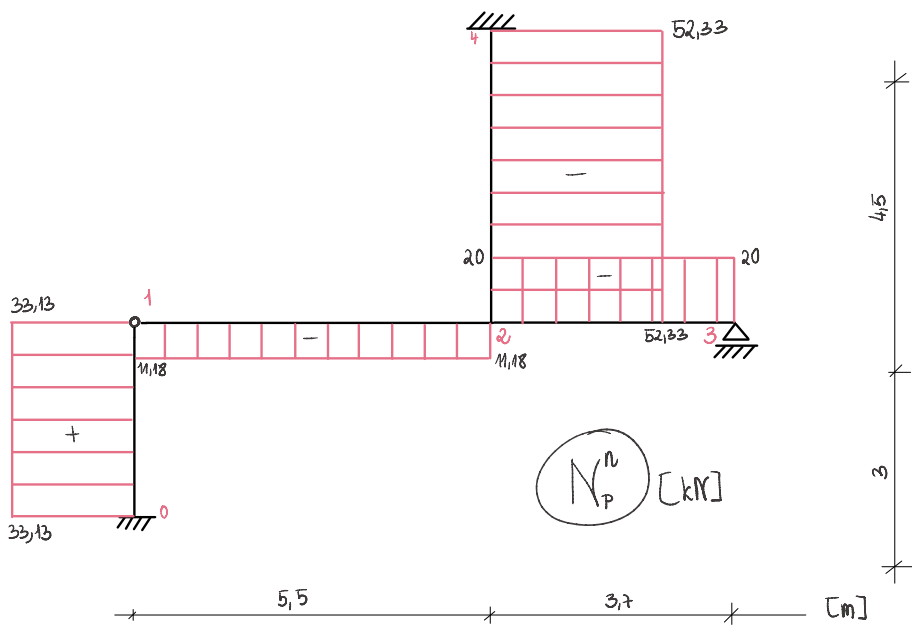
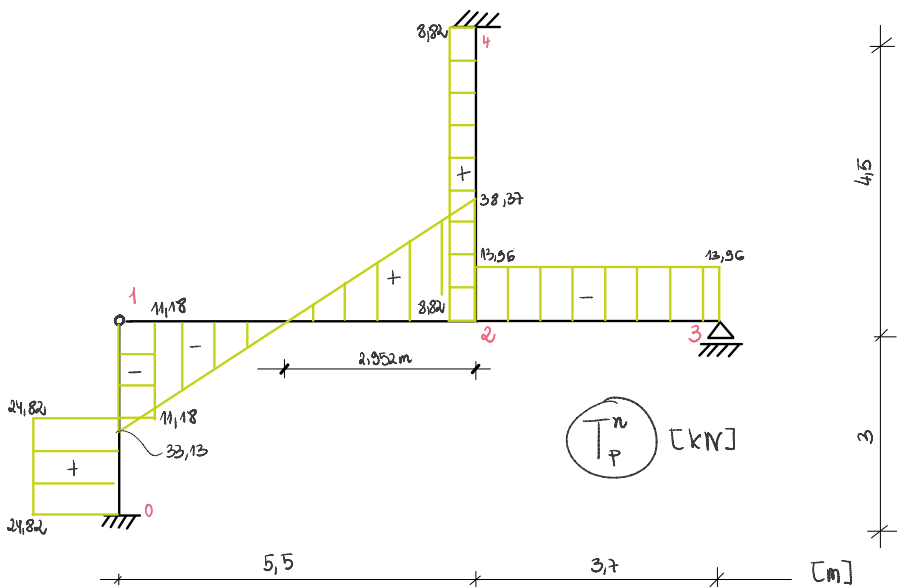
$$\sigma_{\max} = \frac{22,57}{77,3} + \frac{52,33}{16,4} = 32,39 \frac{\text{kN}}{\text{cm}^2} = 323,9 \text{ MPa} > f_y = 235 \text{ MPa}$$

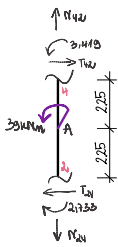
Wnioski:

- Naprężenia tylko dla grupy prętów I_1 są mniejsze od dopuszczalnych.
- Niekorzystano przekroju I_1 to 72%.
- Należy zmienić rozmiar przekroju I_2 na większy, wyznaczyć na nowo stosunek I_1/I_2 oraz wykonać obliczenia ponownie.

siły normalne wyznaczone poniżej

$I 160 \text{ HEB}$	$W_{x1} = 941 \text{ cm}^3$	$A_1 = 54,3 \text{ cm}^2$
$I 140 \text{ PE}$	$W_{x2} = 77,3 \text{ cm}^3$	$A_2 = 16,4 \text{ cm}^2$





$$\sum M^{(A)} = 0$$

$$\sum M^{(B)} = -3,419 - 39 + 2,733 + T_{24} \cdot 4,5 = 0$$

$$-T_{24} \cdot 4,5 = -39,63$$

$$T_{24} = 8,82 \text{ kN}$$

$$\sum N^{(A)} = 0$$

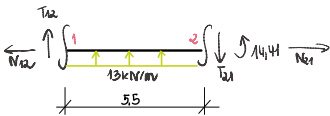
$$\sum M^{(A)} = -3,419 - 39 + 2,733 + T_{42} \cdot 4,5 = 0$$

$$T_{42} = 8,82 \text{ kN}$$

$$\sum M_A^{(A)} = M_A - 3,419 + 8,82 \cdot 2,25 = 0$$

$$M_A = 3,419 - 19,85 = -16,43 \text{ kNm}$$

$$-16,43 + 39 = 22,57 \text{ kNm}$$



$$\sum M^{(A)} = -14,41 + T_{21} \cdot 5,5 - 13 \cdot 5,5 \cdot \frac{5,5}{2} = 0$$

$$-T_{21} \cdot 5,5 = -14,41 - 196,63$$

$$T_{21} = 38,37 \text{ kN}$$

$$\sum M^{(B)} = T_{12} \cdot 5,5 + 13 \cdot 5,5 \cdot \frac{5,5}{2} - 14,41 = 0$$

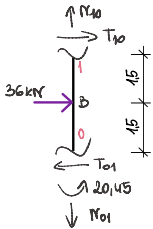
$$-T_{12} \cdot 5,5 = 196,63 - 14,41$$

$$T_{12} = -33,13 \text{ kN}$$

$$T(x) = -38,37 + 13 \cdot x = 0$$

$$x = 2,952 \text{ m}$$

$$M^{EXT} = -14,41 + 38,37 \cdot 2,952 - 13 \cdot 2,952 \cdot \frac{2,952}{2} = 42,22 \text{ kNm}$$



$$\sum M^{(A)} = -36 \cdot 1,5 + T_{01} \cdot 3 - 20,45 = 0$$

$$-T_{01} \cdot 3 = -54 - 20,45$$

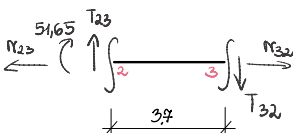
$$T_{01} = 24,82 \text{ kN}$$

$$\sum M^{(B)} = 36 \cdot 1,5 - 20,45 + T_{10} \cdot 3 = 0$$

$$-T_{10} \cdot 3 = 54 - 20,45$$

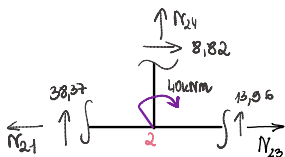
$$T_{10} = -11,48 \text{ kN}$$

$$M^{(B)} = 11,48 \cdot 1,5 = 16,77 \text{ kNm}$$



$$\sum M^{(A)} = T_{23} \cdot 3,7 + 51,65 = 0$$

$$T_{23} = -13,96 \text{ kN}$$



$$\sum X = 8,82 - 20 - N_{2,1} = 0$$

$$N_{2,1} = -11,18 \text{ kN}$$

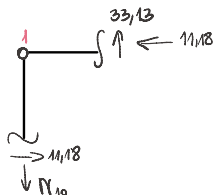
$$N_{2,3} = N_{3,2}$$

$$N_{3,2} = -20 \text{ kN}$$

$$\sum Y = 13,96 + N_{2,4} + 38,37 = 0$$

$$N_{2,4} = -52,33 \text{ kN}$$

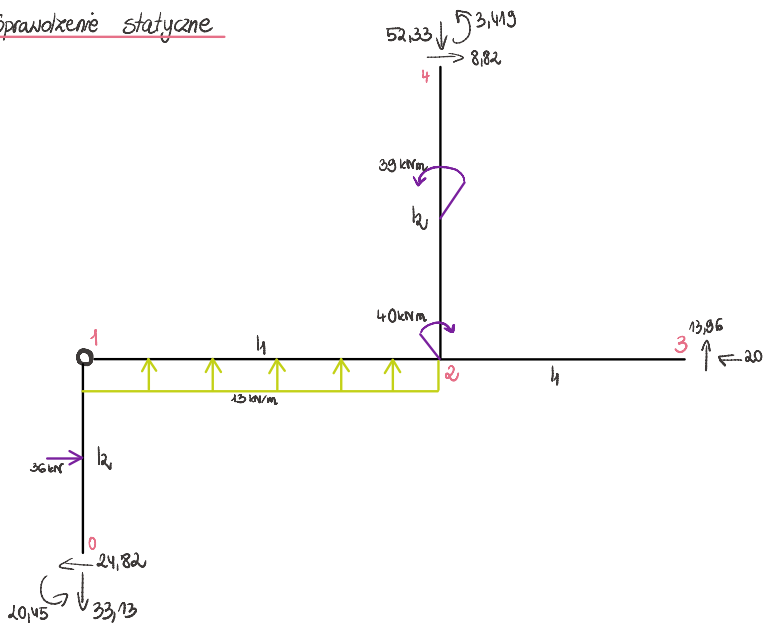
↑
z równowagi
węzła nr 3



$$\sum Y = 33,13 - N_{1,10} = 0$$

$$N_{1,10} = 33,13 \text{ kN}$$

Sprawdzenie statyczne



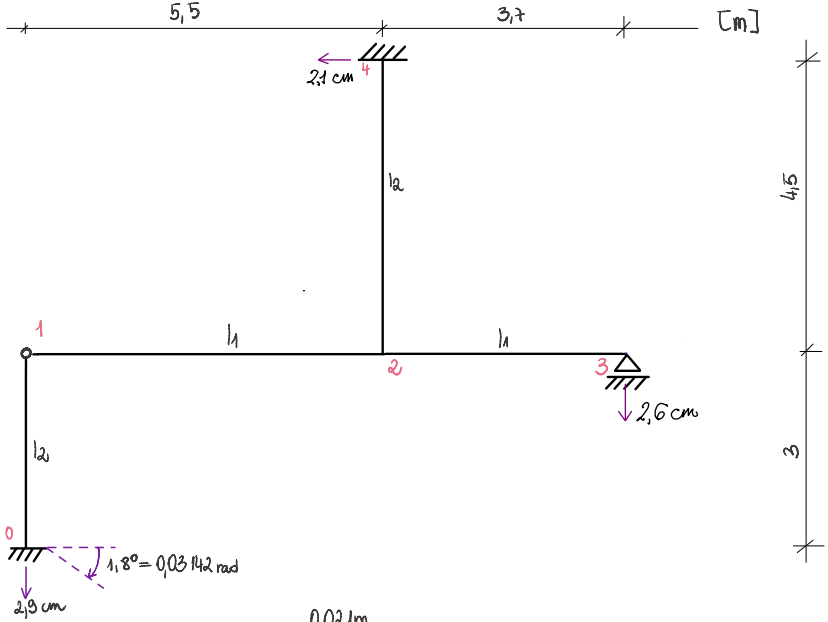
$$\left\{ \begin{array}{l} \sum X = 36 - 24,82 - 20 + 8,82 = 0 \\ \sum Y = -33,13 + 13 \cdot 5,5 + 13,96 - 52,33 = 0 \\ \sum M^{(3)} = -20,45 - 33,13 \cdot 2,2 + 24,82 \cdot 3 - 36 \cdot 1,5 + 13,96 \cdot (2,2^2 + 3,7) + 40 + 8,82 \cdot 4,5 - 52,33 \cdot 3,7 - 3,419 \cdot 3,7 - 3,419 \cdot 3,7 = 0,009 \approx 0 \end{array} \right.$$

Osiadanie popłsu

$$K = \begin{bmatrix} 7,132E_{10} & -0,2963E_{10} \\ -0,2963E_{10} & 0,2428E_{10} \end{bmatrix}$$

$$E_1 = 4,603E_{10} \quad E_0 = 1136,1 \text{ kNm}^2$$

$$E_2 = E_0$$

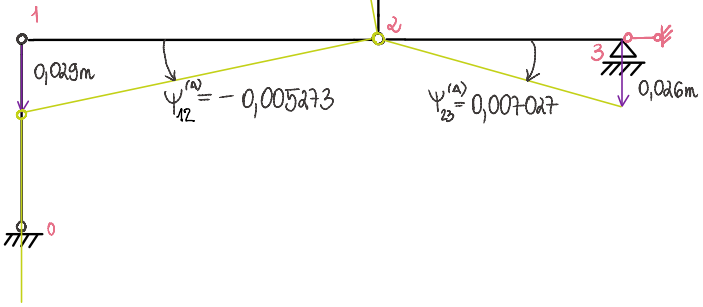


Stan Δ

$$\Psi_{12}^{(\Delta)} = -\frac{0,029}{5,5} = -0,005273$$

$$\Psi_{24}^{(\Delta)} = -\frac{0,021}{4,5} = -0,004667$$

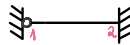
$$\Psi_{23}^{(\Delta)} = \frac{0,026}{3,7} = 0,007027$$





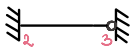
$$M_{42} = \frac{2EI_2}{4,5} \cdot 3 \cdot 0,004667 = 0,006223EI_0$$

$$M_{24} = 0,006223EI_0$$



$$M_{12} = 0$$

$$M_{21} = \frac{3EI_1}{5,5} \cdot 0,005273 = 0,01324EI_0$$



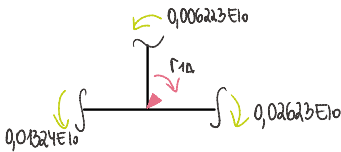
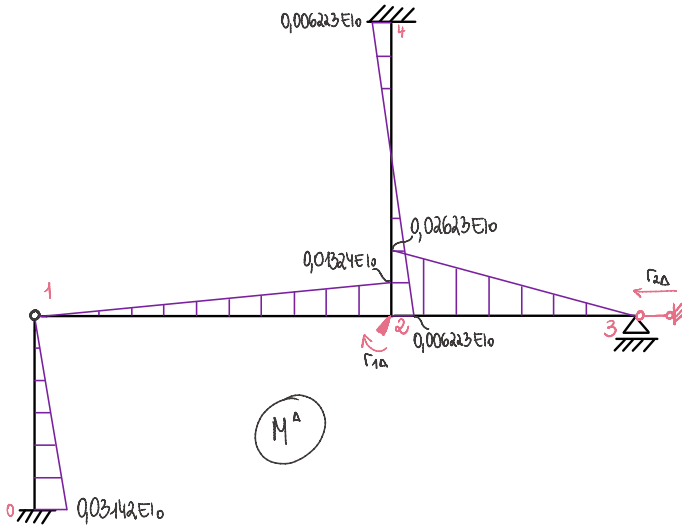
$$M_{23} = \frac{3EI_1}{3,7} \cdot (-0,007027) = -0,02623EI_0$$

$$M_{32} = 0$$



$$M_{10} = 0$$

$$M_{01} = \frac{3EI_2}{3} \cdot 0,03142 = 0,03142EI_0$$



$$\sum M = 0$$

$$r_{1A} = -0,02623EI_0 + 0,006223EI_0 + 0,01324EI_0 = \underline{\underline{-0,006767EI_0}}$$

RPN

$$r_{2A} \cdot 1 + (-0,02623EI_0) \cdot 0 + 2 \cdot 0,006223 \cdot \frac{1}{4,5} + 0,01324EI_0 \cdot 0 + 0,03142EI_0 \cdot (-\frac{1}{3}) = 0$$

$$r_{2A} = -0,002766EI_0 + 0,01047EI_0 = \underline{\underline{0,007704EI_0}}$$

URK

$$\begin{cases} r_{11}\varphi_1 + r_{12}A_2 + r_{1n} = 0 \\ r_{21}\varphi_1 + r_{22}A_2 + r_{2n} = 0 \end{cases}$$

$$\begin{cases} 7132E_{10}\varphi_1 - 0,2963E_{10}A_2 - 9006767E_{10} = 0 \\ -0,2963E_{10}\varphi_1 + 0,2428E_{10}A_2 + 0,007704E_{10} = 0 \end{cases}$$

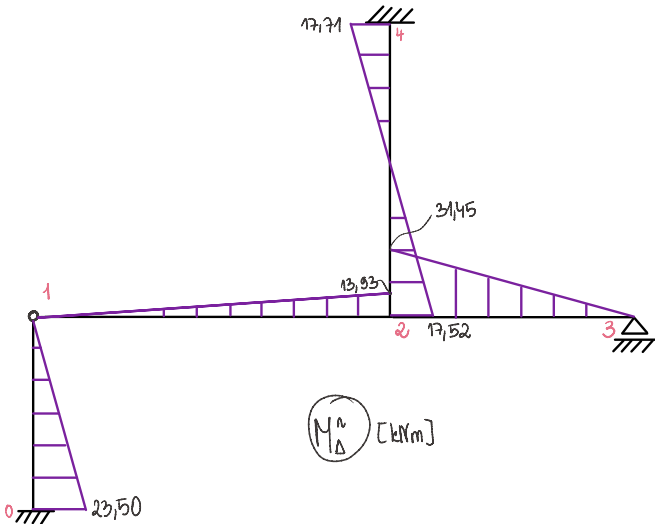
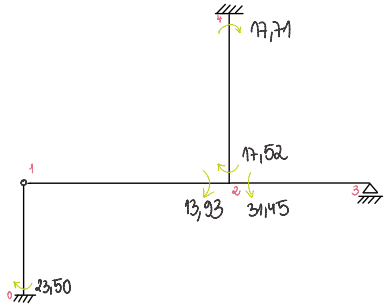
$$\varphi_1 = -0,0003891 \text{ rad}$$

$$A_2 = -0,032205 \text{ m}$$

$$M_{ik}^n = M_{ik}^{(1)}\varphi_1 + M_{ik}^{(2)}A_2 + M_{ik}^{(n)}$$

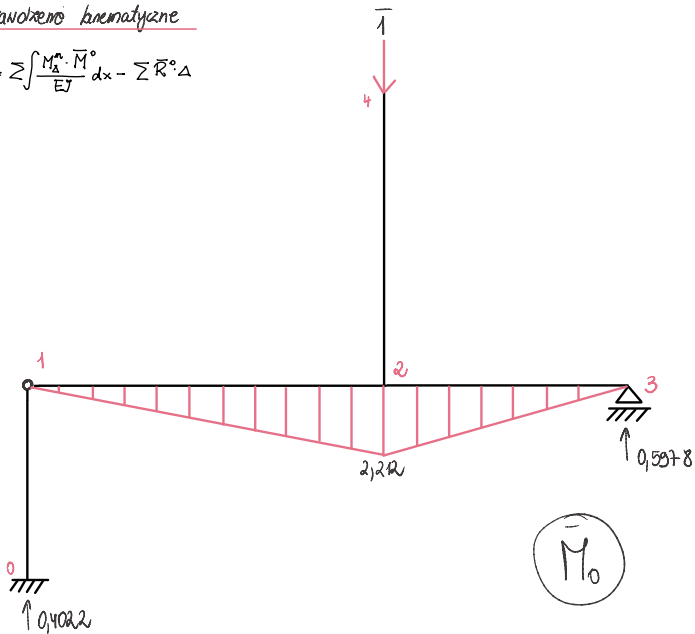
ik	$M_{ik}^{(1)}$	$M_{ik}^{(2)}$	$M_{ik}^{(n)}$	$M_{ik}^{(n)}$ [kNm]
01	0	$\frac{1}{3}$	0,03112	23,50
21	2,511	0	0,01324	13,93
23	3,732	0	-0,01623	-31,45
32	0	0	0	0
24	0,8889	-0,2963	0,006223	17,52
42	0,4444	-0,2963	0,006223	17,71

$\times E_{10} \Rightarrow \times 1136,1$

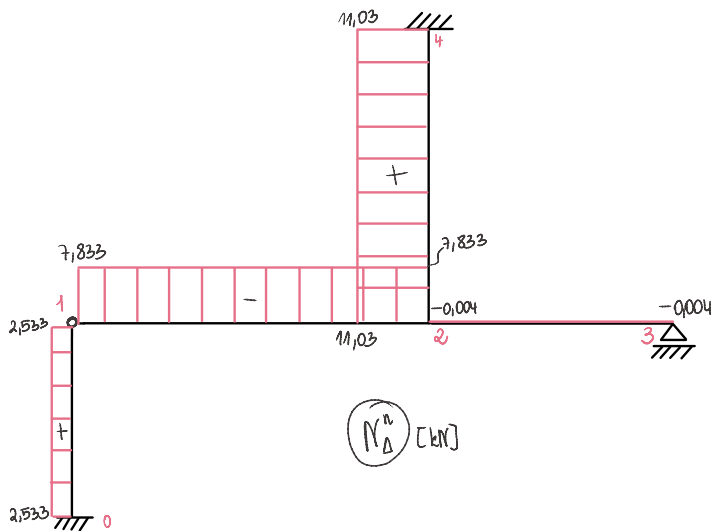
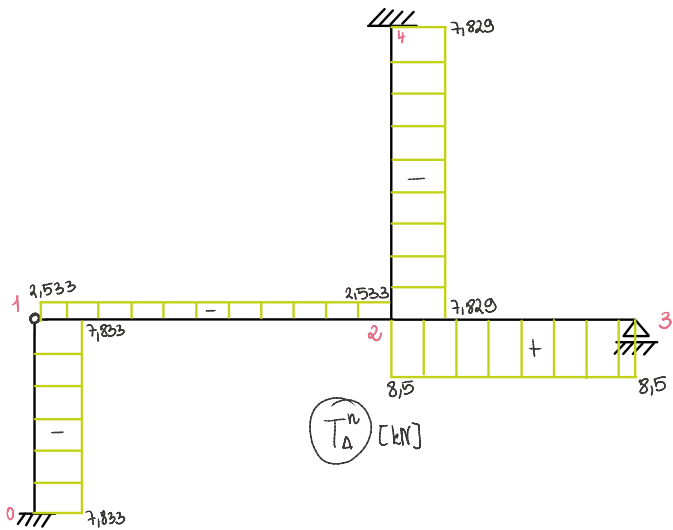


Spanoveno kinematizane

$$h^{(4)} = \sum \int \frac{M_A \cdot \bar{M}^{\circ}}{EI} dx - \sum R^{\circ} \Delta$$



$$h^{(4)} = \frac{1}{EI_1} \left[-\frac{1}{2} \cdot 2,22 \cdot \frac{2}{3} \cdot 13,93 \cdot 5,5 - \frac{1}{2} \cdot 2,22 \cdot \frac{2}{3} \cdot 31,45 \cdot 3,7 \right] - [0,026 \cdot 0,5978 - 0,029 \cdot 0,4022] = -\frac{30,91}{EI_0} + \frac{30,91}{EI_0} = \frac{0}{EI_0} < \frac{1}{EI_0}$$

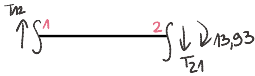




$$\sum M^{(1)} = 23,50 + T_{01} \cdot 3 = 0$$

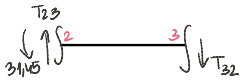
$$T_{01} = -7,833 \text{ kN}$$

$$T_{10} = -7,833 \text{ kN}$$



$$\sum M^{(1)} = T_{21} \cdot 5,5 + 13,93 = 0$$

$$T_{21} = -2,533 \text{ kN} = T_{12}$$



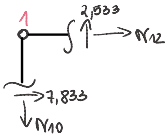
$$\sum M^{(2)} = T_{23} \cdot 3,7 - 31,15 = 0$$

$$T_{23} = 8,5 \text{ kN} = T_{32}$$



$$\sum M^{(4)} = T_{24} \cdot 4,5 + 17,52 + 17,71 = 0$$

$$T_{24} = -7,829 \text{ kN} = T_{42}$$

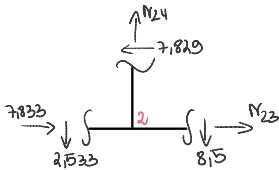


$$\sum X = N_{12} + 7,833 = 0$$

$$N_{12} = -7,833 \text{ kN}$$

$$\sum Y = 7,833 - N_{10} = 0$$

$$N_{10} = 7,833 \text{ kN}$$



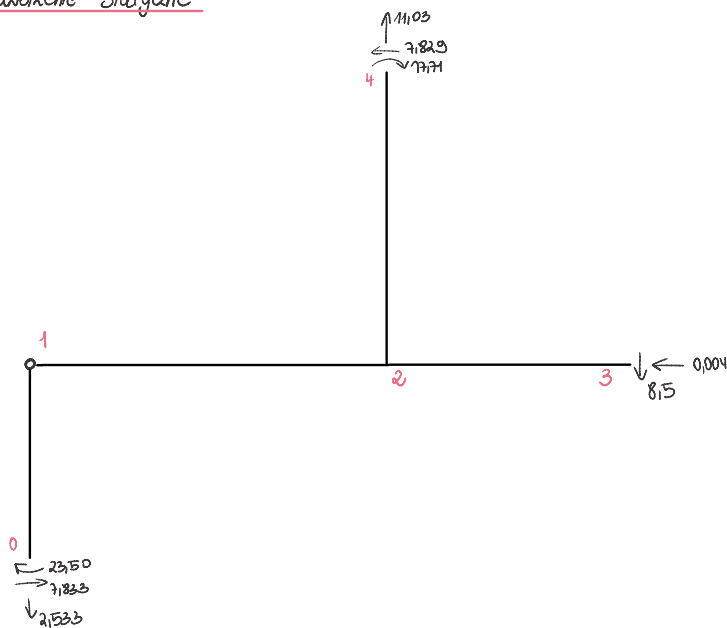
$$\sum X = -7,829 + N_{23} + 7,833 = 0$$

$$N_{23} = -0,004 \text{ kN}$$

$$\sum Y = N_{24} - 8,5 - 7,833 = 0$$

$$N_{24} = 16,333 \text{ kN}$$

Sprawozdanie statyczne



$$\begin{cases} \sum X = 7,833 - 7,829 - 0,004 = 0 \\ \sum Y = -2,533 - 8,5 + 11,03 = -0,003 \approx 0 \\ \sum H^{(0)} = 23,50 + 17,71 - 7,829 \cdot 7,5 - 11,03 \cdot 5,5 + 8,5 \cdot 9,2 + 0,004 \cdot 3 = 0,0395 \approx 0 \end{cases}$$