Politechnika Poznańska
Wydział Inżynierii Lądowej i Transportu
Instytut Analizy Konstrukcji
Zakład Mechaniki Budowli

Nazwisko i imię studenta: **Reichel Julia**
Nr albumu: **161335**
Rok akad. / Rodzaj studiów: **2024/25 BS-I**
Grupa ćw. / Grupa lab.: **C4 / L8**

MECHANIKA BUDOWLI / SEM.4 / ĆWICZENIE NR 1

METODA PRZEMIESZCZEŃ

Termin oddania: 06.05.2025

Data	Uwagi sprawdzającego	Podpis

~~ opracowanie: mgr inż. Anita Kaczor ~~ https://anita.kaczor.pracownik.put.poznan.pl ~~

Dla zadanej **RAMY:** **/ Nr schematu: 12**

1. Korzystając z metody przemieszczeń obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego obciążenia, wykonać kontrolę kinematyczną oraz statyczną.
2. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2, porównać je z wartościami dopuszczalnymi naprężeń i sformułować wnioski (w przypadku niespełnienia warunku nośności, obliczeń nie trzeba powtarzać).
3. Korzystając z metody przemieszczeń obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego osiadania podpór, wykonać kontrolę kinematyczną oraz statyczną.

Dla zadanej **BELKI**: **/ Nr schematu: 4**

1. Korzystając z metody przemieszczeń obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego obciążenia, wykonać kontrolę kinematyczną oraz statyczną.
2. Korzystając z metody przemieszczeń w ujęciu macierzowym obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego obciążenia.

W obliczeniach przyjąć: E = 210 GPa, f_y = 235 MPa.

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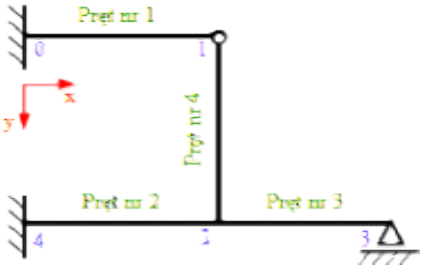
Rok akademicki: **2024/25** **BS-I**
Grupa / Grupa projektowa: **C4 / L8**

DANE DLA RAMY Nr schematu: 12

Wymiary i obciążenie przęsłowe:

Pręt	L [m]	Przekrój	q [kN/m]	P [kN]	M [kNm]
1	**5**	**I 450 HEB**	**13**	**0**	**0**
2	**5**	**I 450 HEB**	**0**	**-18**	**0**
3	**4**	**I 220**	**0**	**0**	**0**
4	**5**	**I 220**	**0**	**0**	**-36**



Obciążenie węzłowe:

siła skupiona pozioma w węźle 3	**-20 kN**
moment skupiony w węźle 2	**25 kNm**

Osiadanie podpór:

Kierunek osiadania	Nr węzła	Wartość
kątowe [º]	0	**3,2**
pionowe [cm]	4	**1,6**
pionowe [cm]	0	**0,8**
poziome [cm]	0	**2,5**

DANE DLA BELKI Nr schematu: 4

Wymiary i obciążenie przęsłowe:

Pręt	L [m]	Przekrój	q [kN/m]	P [kN]
1	**3,5**	**I 240**	**8**	**0**
2	**3**	**I 360 HEA**	**0**	**36**
3	**5,5**	**I 240**	**13**	**0**

Obciążenie węzłowe:

siła skupiona pionowa w węźle 1	**-20 kN**
moment skupiony w węźle 2	**35 kNm**

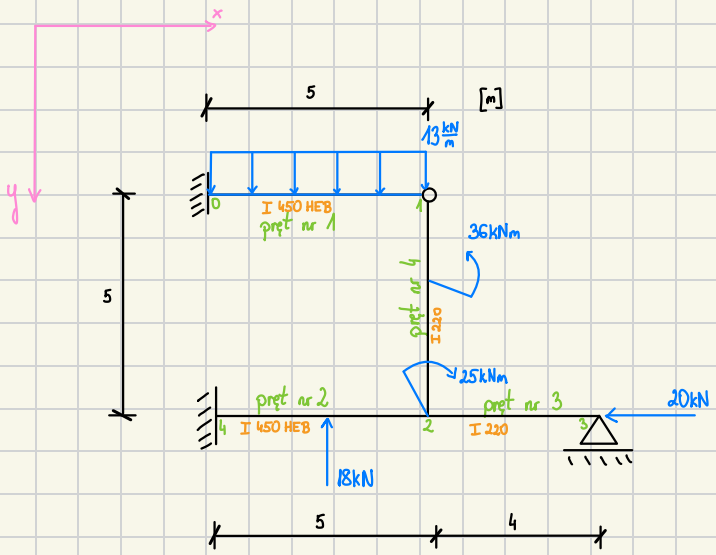
LEGENDA

Obciążenie przęsłowe:

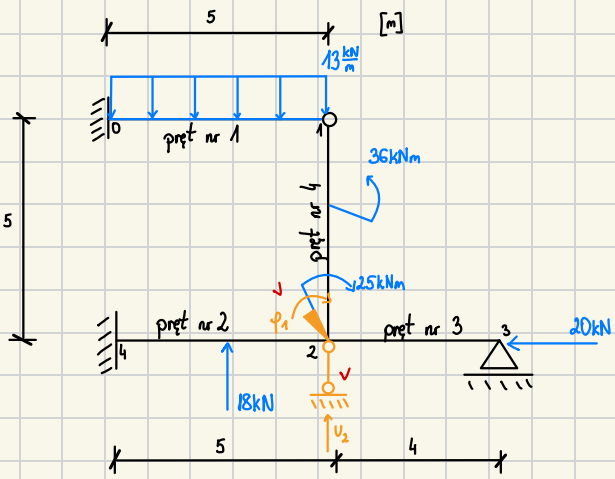
- obciążenie równomiernie rozłożone q działa na całej długości pręta, prostopadle do osi pręta;
- siła P prostopadła do osi pręta, przyłożona w połowie długości pręta;
- moment skupiony M przyłożony w połowie długości pręta.

Obciążenie przęsłowe i węzłowe – zwroty:

- siły skupione, obciążenie równomiernie rozłożone oraz liniowe osiadania podpór – wartość dodatnia oznacza zwrot zgodny z osią układu współrzędnych, który jest zaznaczony na schemacie, ujemna – przeciwny;
- momenty skupione oraz osiadania kątowe – wartość dodatnia oznacza zwrot w prawo, ujemna – w lewo.



układ podstawowy



wstępne przyjęcie przekrojów:

pręty 1,2 - przekrój: I 450 HEB
$J_1 = 79890 cm^4$

pręty 3,4 - przekrój: I 220
$J_2 = 3060 cm^4$

$\Sigma \varphi = 1$
$\Sigma u = 1$

$SGN = 1 + 1 - 2$

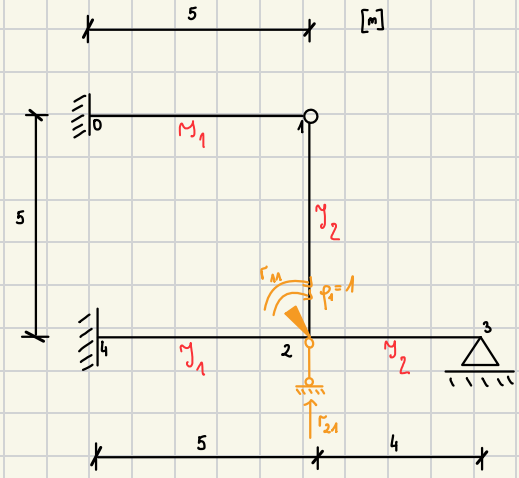
$$\frac{J_1}{J_2} = \frac{79890}{3060} = 26,11$$

$$J_1 = 26,11 J_2$$

Układ równań kanonicznych metody przemieszczeń:

$$\begin{cases} r_{11}\varphi_1 + r_{12}u_2 + r_{1P} = 0 \\ r_{21}\varphi_1 + r_{22}u_2 + r_{2P} = 0 \end{cases}$$

stan $\varphi_1 = 1$



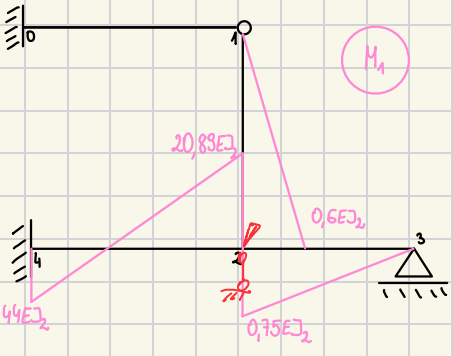
$$M_{01}^{(1)} = 0$$

$$M_{21}^{(1)} = \frac{3EJ_2}{l_{12}} = \frac{3EJ_2}{5} = 0,6EJ_2$$

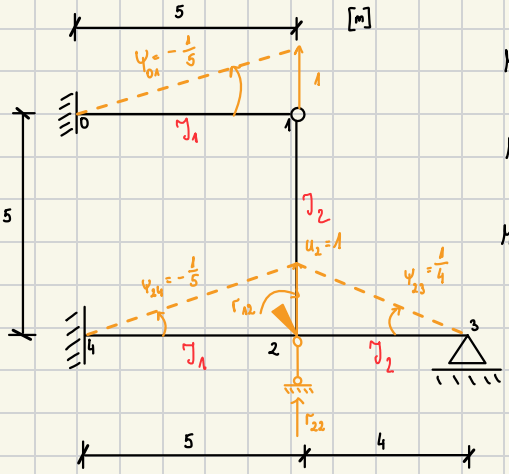
$$M_{23}^{(1)} = \frac{3EJ}{l_{23}} = \frac{3EJ_2}{4} = 0,75EJ_2$$

$$M_{24}^{(1)} = \frac{4EJ}{l_{24}} = \frac{4EJ_1}{5} = \frac{4E \cdot 26,11J_2}{5} = 20,89EJ_2$$

$$M_{42}^{(1)} = \frac{2EJ_1}{l_{24}} = \frac{2E \cdot 26,11J_2}{5} = 10,44EJ_2$$



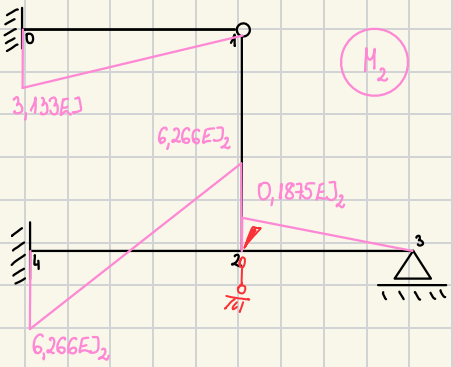
stan $u_2 = 1$



$$M_{01}^{(2)} = -\frac{3EJ_1}{l_{01}} \cdot \psi_{01} = -\frac{3E \cdot 26,11J_2}{5} \cdot \left(-\frac{1}{5}\right) = 3,133EJ_2$$

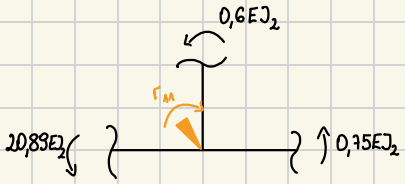
$$M_{23}^{(2)} = -\frac{3EJ_2}{l_{23}} \cdot \psi_{23} = -\frac{3EJ_2}{4} \cdot \left(\frac{1}{4}\right) = -0,1875EJ_2$$

$$M_{24}^{(2)} = -\frac{6EJ_1}{l_{24}} \cdot \psi_{24} = -\frac{6 \cdot 26,11EJ_2}{5} \cdot \left(-\frac{1}{5}\right) = 6,266EJ_2 = M_{42}^{(2)}$$



wyznaczanie r_{ik}

równowaga momentów w węźle 2 w stanie $\varphi_1 = 1$



$$\sum M_2^{(1)} = 0$$

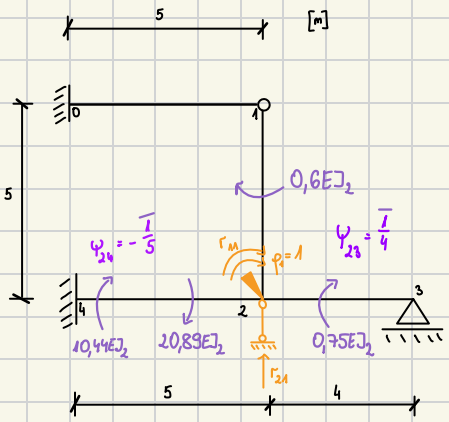
$r_{11} = 22,24EJ_2$ ✓

równowaga momentów w węźle 2 w stanie $u_1 = 1$

$$\sum M_2^{(2)} = 0$$

$r_{12} = 6,079EJ_2$ ✓

wyznaczenie reakcji r_{21}



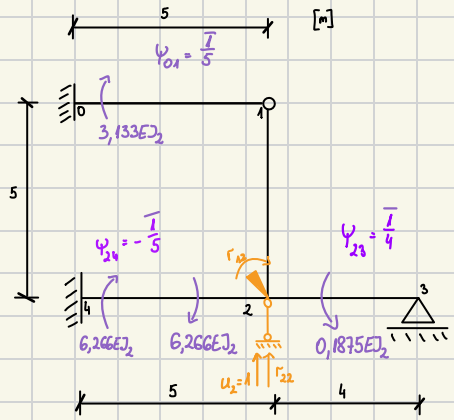
$F^{(1)}$

$$\sum_j R_j^{(1)} \cdot \bar{\sigma}_j + \sum_n M_{ik}^{(1)} \cdot \bar{\psi}_{ik} = 0$$

$$r_{21} \cdot \bar{1} + 0,6EJ_2 \cdot \bar{0} + (10,44EJ_2 + 20,89EJ_2) \cdot \left(-\frac{\bar{1}}{5}\right) + 0,75EJ_2 \cdot \frac{\bar{1}}{4} = 0$$

$r_{21} = 6,079EJ_2 = r_{12}$ ✓

Wyznaczenie reakcji r_{22}



$F^{(2)}$

$$\sum_j R_j^{(2)} \cdot \bar{\sigma}_j + \sum_n M_{ik}^{(2)} \cdot \bar{\psi}_{ik} = 0$$

$$r_{22} \cdot \bar{1} + (6,266EJ_2 \cdot 2) \cdot \left(-\tfrac{1}{5}\right) + (-0,1875EJ_2) \cdot \tfrac{1}{4} + 3,133EJ_2 \cdot \left(-\tfrac{1}{5}\right) = 0$$

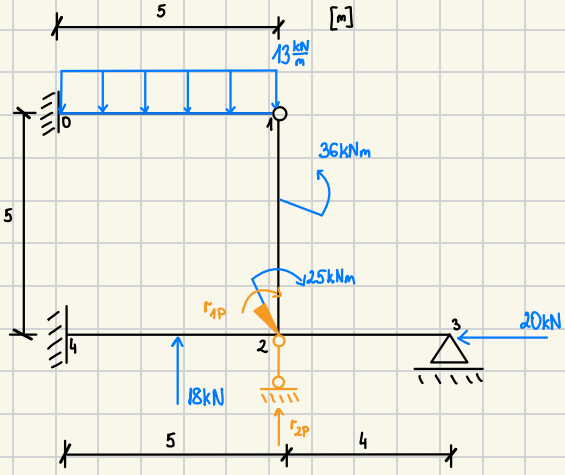
$r_{22} = 3,180EJ_2$ ✓

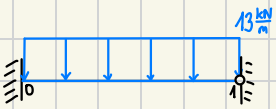
macierz sztywności

$$K = \begin{bmatrix} 22,24EJ_2 & 6,079EJ_2 \\ 6,079EJ_2 & 3,180EJ_2 \end{bmatrix}$$

Wyznaczanie reakcji r_{iP}

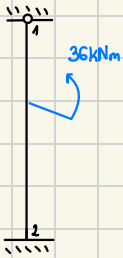
Wyznaczanie reakcji r_{1P}





$$\frac{ql^2}{8} = \frac{13 \cdot 5^2}{8} = 40,63$$

$$\frac{9}{128} \cdot ql^2 = \frac{9}{128}(13 \cdot 5^2) = 22,85$$

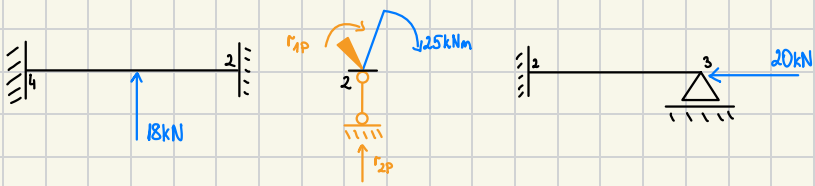


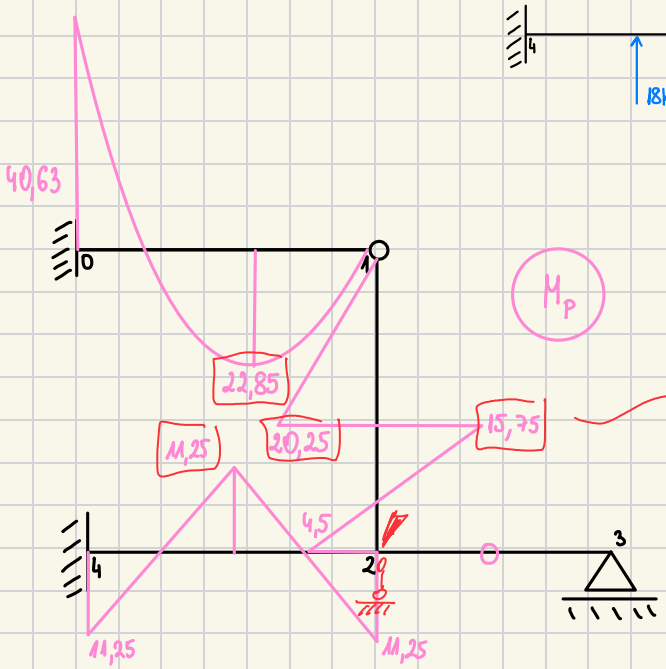
$$\frac{9}{16} \cdot H = 20,25$$

$$\frac{7}{16} \cdot H = 15,75$$

$$\frac{1}{8} \cdot H = 4,5$$

$\frac{Pl}{8} = \frac{18 \cdot 5}{8} = 11,25$ $\frac{Pl}{8} = 11,25$ $\frac{Pl}{8} = 11,25$

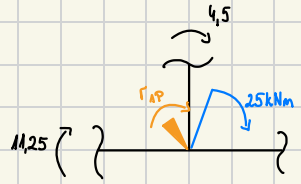




M_P

→ tych wartości nie trzeba wyznaczać, nie będziemy z nich korzystać w dalszych obliczeniach

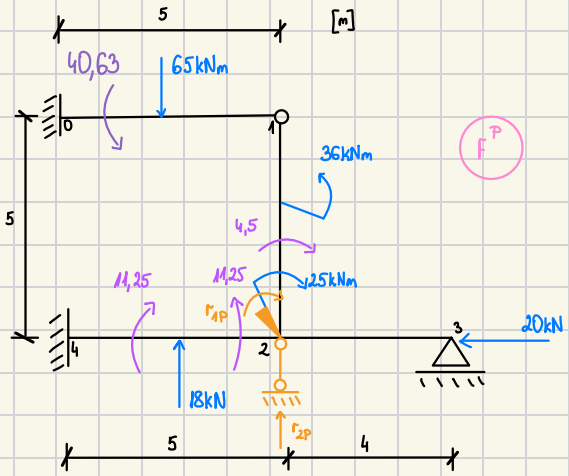
równowaga momentów w węźle 2

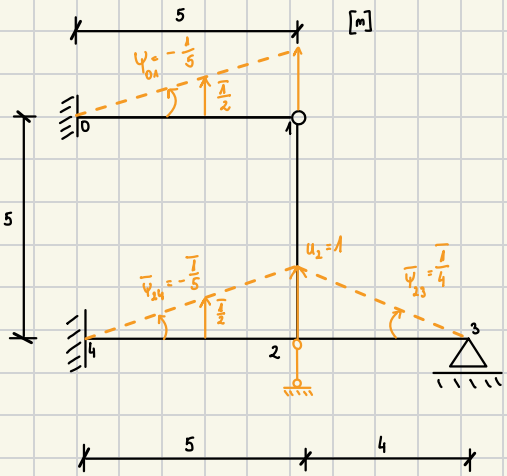


$$\Sigma M_2^{(P)} = 0$$

$$r_{1P} = -40{,}75\ \text{kNm}$$

wyznaczanie reakcji r_{2P}





m. przesunięcie przęsełka

$$\sum_i R_j^{(P)} \cdot \bar{\sigma}_j + \sum_k P_k^{(P)} \cdot \bar{\sigma}_k + \sum_n M_{ik}^{(P)} \cdot \bar{\psi}_{ik} = 0$$

$$r_{2P} \cdot \bar{1} + 65 \cdot \left(-\frac{\bar{1}}{2}\right) - 40{,}63 \cdot \left(-\frac{\bar{1}}{5}\right) + \frac{\bar{1}}{2} \cdot 18 + 11{,}25 \cdot \left(-\frac{\bar{1}}{5}\right) + (-11{,}25) \cdot \left(\frac{\bar{1}}{5}\right) + 25 \cdot \bar{0} = 0$$

$$r_{2P} = 15{,}37\ \text{kN}$$

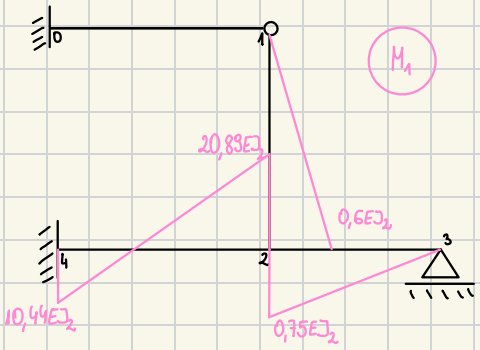
wyznaczanie niewiadomych φ_1 oraz u_2

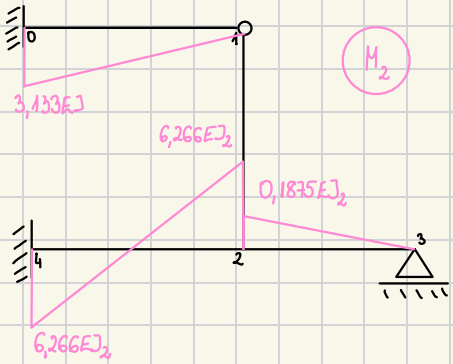
$$\begin{cases} 22{,}24 EJ_2 \varphi_1 + 6{,}079 EJ_2 u_2 - 40{,}75 = 0 \\ 6{,}079 EJ_2 \varphi_1 + 3{,}180 EJ_2 u_2 + 15{,}37 = 0 \end{cases}$$

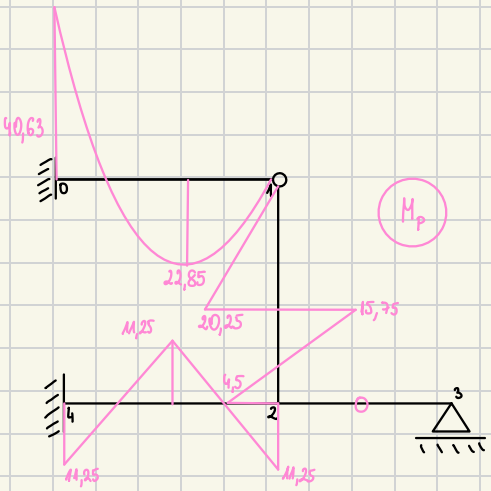
$$\begin{cases} \varphi_1 = \dfrac{6{,}604}{EJ_2} \\ u_2 = -\dfrac{17{,}46}{EJ_2} \end{cases}$$

wyznaczanie rozkładu momentów zginających

$$M_{ik}^{n} = M_{ik}^{(1)} \cdot \varphi_1 + M_{ik}^{(2)} \cdot u_2 + M_{ik}^{(P)}$$







$M_{01}^n = 3,133EJ_2 \cdot \left(-\frac{17,46}{EJ_2}\right) - 40,63 = -95,33 kNm$

$M_{10}^n = 0 kNm$

$M_{12}^n = 0 kNm$

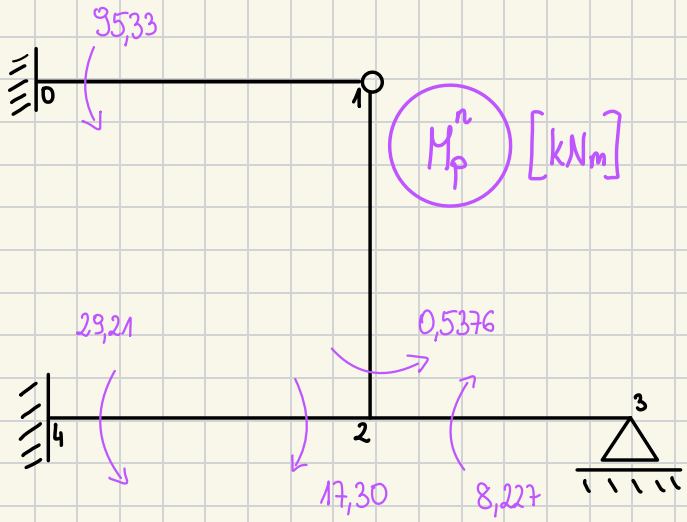
$M_{21}^n = 0,6EJ_2 \cdot \frac{6,604}{EJ_2} - 4,5 = -0,5376 kNm$

$M_{24}^n = 20,89EJ_2 \cdot \frac{6,604}{EJ_2} + 6,266EJ_2 \cdot \left(-\frac{17,46}{EJ_2}\right) - 11,25 = 17,30 kNm$

$M_{42}^n = 10,44EJ_2 \cdot \frac{6,604}{EJ_2} + 6,266EJ_2 \cdot \left(-\frac{17,46}{EJ_2}\right) + 11,25 = -29,21 kNm$

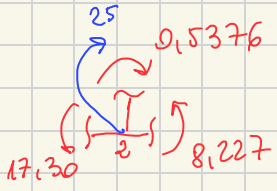
$M_{23}^n = 0,75EJ_2 \cdot \frac{6,604}{EJ_2} - 0,1875EJ_2 \cdot \left(-\frac{17,46}{EJ_2}\right) = 8,227 kNm$

$M_{32}^n = 0 kNm$



równowaga węzła 2:

$25 - 17,30 + 0,5376 - 8,227 = 0,0106 \approx 0$

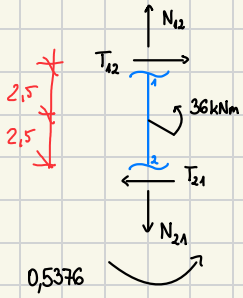


wyznaczanie rozkładu sił poprzecznych

$\Sigma M_3 = 0$

$8,227 + T_{23} \cdot 4 = 0$

$T_{23} = -2,067\ kN$



$\Sigma M_1 = 0$

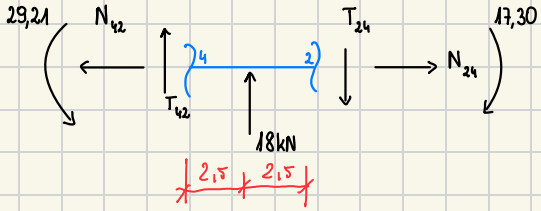
$-0,5376 + T_{21} \cdot 5 - 36 = 0$

$T_{21} = 7,308\ kN$

$\Sigma M_2 = 0$

$-0,5376 + T_{12} \cdot 5 - 36 = 0$

$T_{12} = 7,308\ kN$



$\Sigma M_4 = 0$

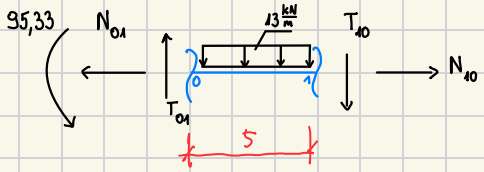
$-29,21 - 18 \cdot 2,5 + T_{24} \cdot 5 + 17,30 = 0$

$T_{24} = 11,38\ kN$

$\Sigma M_2 = 0$

$-29,21 + 18 \cdot 2,5 + 17,30 + T_{42} \cdot 5 = 0$

$T_{42} = -6,618\ kN$



$\Sigma M_0 = 0$

$-95,33 + 13 \cdot 5 \cdot 2,5 + T_{10} \cdot 5 = 0$

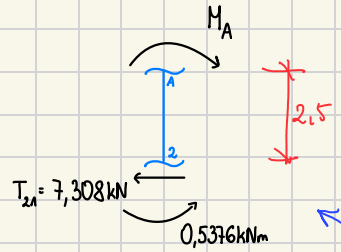
$T_{10} = -13,43\ kN$

$\Sigma M_1 = 0$

$-95,33 - 13 \cdot 5 \cdot 2,5 + T_{01} \cdot 5 = 0$

$T_{01} = 51,57\ kN$

hamujące momenty

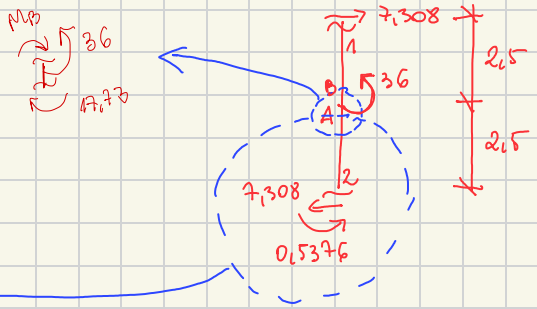


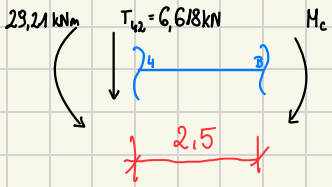
$\Sigma M_A = 0$

$7,308 \cdot 2,5 + M_A - 0,5376 = 0$

$M_A = -17,73\ kNm$

$M_B = 36 - 17,73 = 18,27\ kNm$



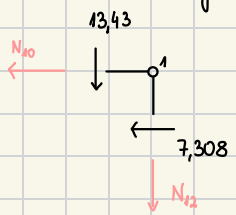


$\Sigma M_c = 0$

$M_c - 6,618 \cdot 2,5 - 29,21 = 0$

$M_c = 45,76\ kNm$

wyznaczanie sił normalnych

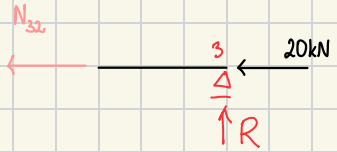


$\Sigma X = 0$

$N_{10} = -7,308\,kN$

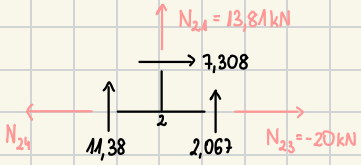
$\Sigma Y = 0$

$N_{12} = -13,43\,kN = N_{21}$



$\Sigma X = 0$

$N_{32} = -20\,kN = N_{23}$

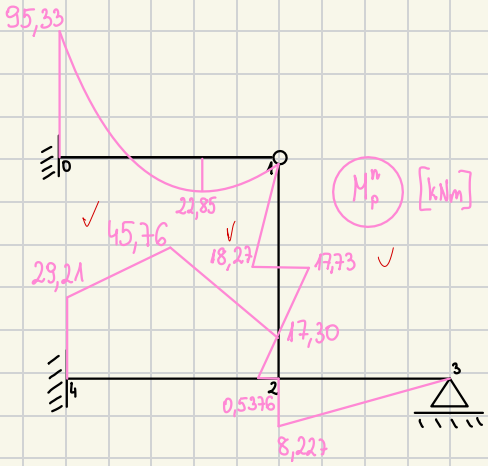


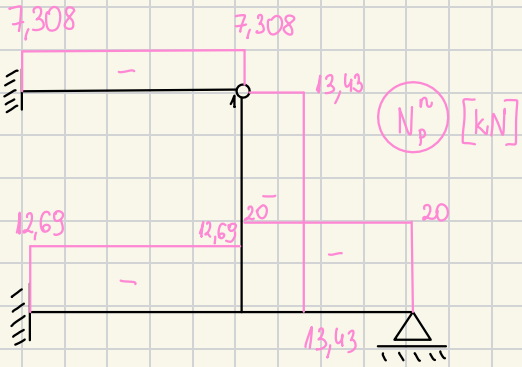
$\Sigma X = 0$

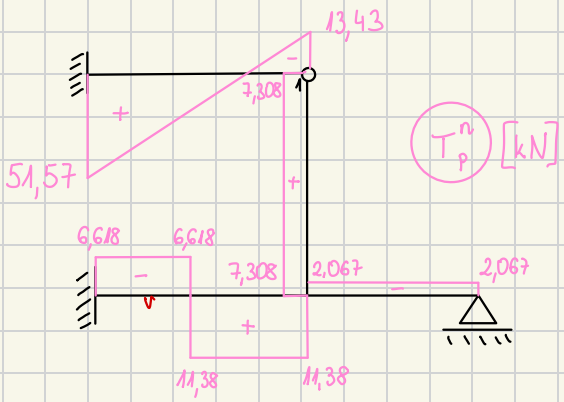
$N_{23} + 7,308 - N_{24} = 0$

$-20 + 7,308 - N_{24} = 0$

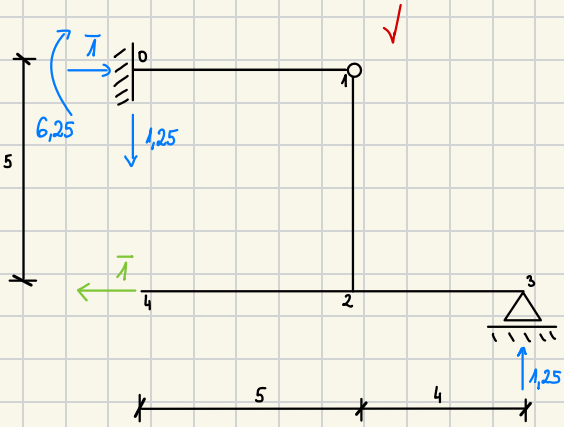
$N_{24} = -12,69\,kN$

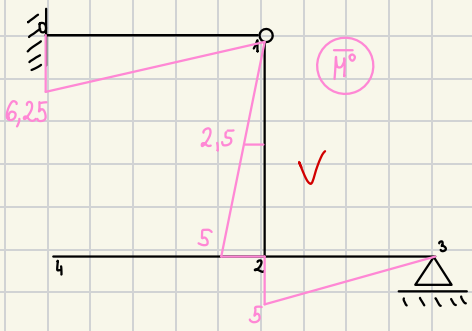


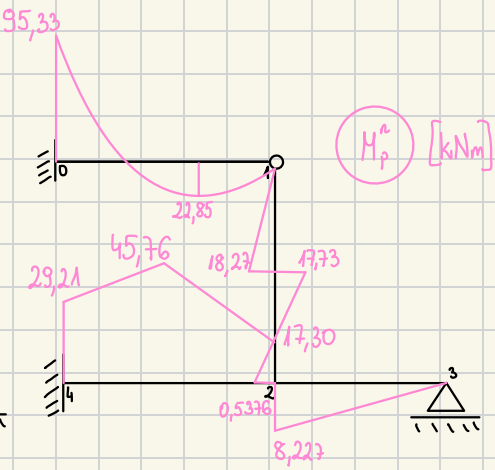




kontrola kinematyczna - sprawdzenie przemieszczenia poziomego w węźle 0 / 4





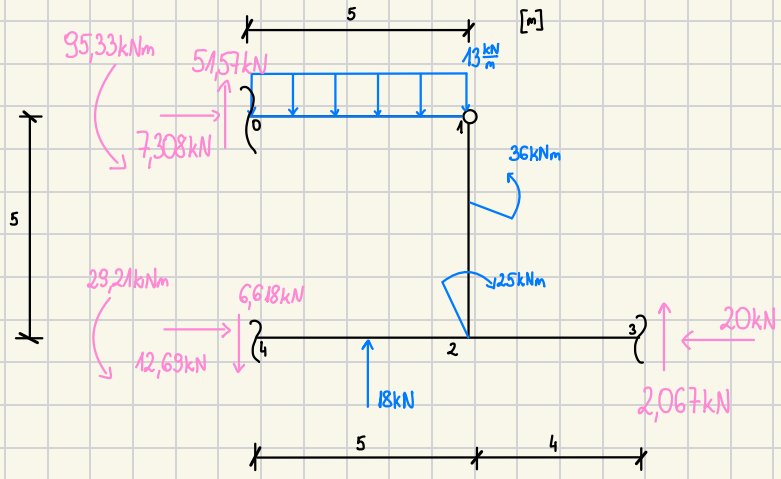


$$H_0 = \int \frac{M_p^n \cdot \overline{H}^0}{EJ}\,dx = \frac{1}{EJ_1}\left[-\frac{1}{2}\cdot 95{,}33\cdot 5\cdot\frac{2}{3}\cdot 6{,}25 + \frac{2}{3}\cdot\frac{13\cdot 5^2}{8}\cdot 5\cdot\frac{1}{2}\cdot 6{,}25\right] + \frac{1}{EJ_2}\left[\frac{1}{2}\cdot 18{,}27\cdot 2{,}5\cdot\frac{2}{3}\cdot 2{,}5 + \frac{1}{2}\cdot 0{,}5376\cdot 2{,}5\left(\frac{2}{3}\cdot 5+\frac{1}{3}\cdot 2{,}5\right) - \frac{1}{2}\cdot 17{,}73\cdot 2{,}5\left(\frac{2}{3}\cdot 2{,}5+\frac{1}{3}\cdot 5\right)\right.$$

$$\left. + \frac{1}{2}\cdot 5\cdot 4\cdot\frac{2}{3}\cdot 8{,}227\right] = \frac{-563{,}8}{26{,}11\,EJ_2} + \frac{21{,}83}{EJ_2} = \frac{0{,}01}{EJ_2}$$

$$|H_0| < \frac{1}{EJ_2} \quad \Rightarrow \quad H_0 \cong 0$$

kontrola statyczna



$$\Sigma X = 0 \qquad 12{,}69 + 7{,}308 - 20 = -0{,}002 \approx 0$$

$$\Sigma Y = 0 \qquad 51{,}57 - 6{,}618 + 2{,}067 - 13\cdot 5 + 18 = 0{,}019 \approx 0$$

$$\Sigma M = 0 \qquad 7{,}308 + 12{,}69 - 20 = -0{,}002 \approx 0$$

sprawdzenie naprężeń normalnych wywołanych momentem zginającym

przekrój dla prętów 1,2: I 450 HEB
$J_1 = 79890\,cm^4$
$W_y = 3550\,cm^3$
$A = 218\,cm^2$

$M_{max} = M_{01}$
$M_{max} = |95,33|\,kNm$ $\quad N_{01} = -7,308\,kN$

$\sigma_{max} = \frac{9533}{3550} = 26,85\,MPa < \sigma_{dop}$

$\sigma = \frac{|M|}{W} + \frac{|N|}{A} = \frac{9533}{3550} + \frac{7,308}{218} = 2,719\,\frac{kN}{cm^2} = 27,19\,MPa < \sigma_{dop}$

przekrój dla prętów 3,4: I 220
$J_2 = 3060\,cm^4$
$W_y = 278\,cm^3$
$A = 39,6\,cm^2$

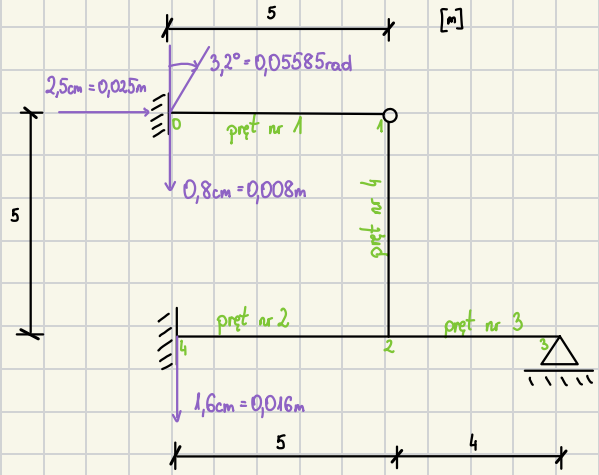
$M_{max} = M_B$
$M_{max} = |18,27|\,kNm$ $\quad N_B = N_{32} = -13,43\,kN$

$\sigma_{max} = \frac{1827}{278} = 65,72\,MPa < \sigma_{dop}$

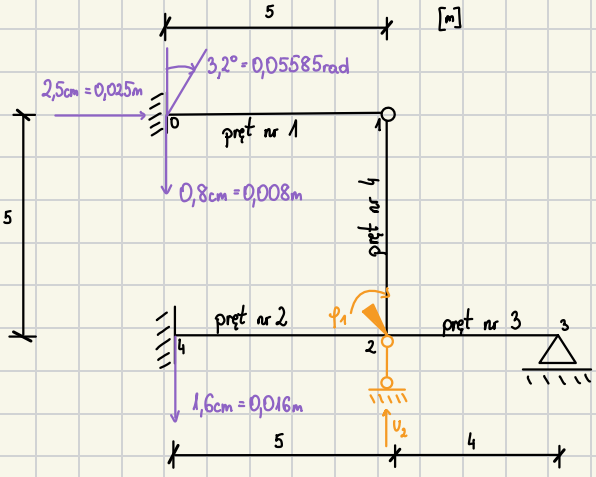
$\sigma = \frac{1827}{278} + \frac{13,43}{39,6} = 6,911\,\frac{kN}{cm^2} = 69,11\,MPa < \sigma_{dop}$

Rama - osiadanie podpór

układ rzeczywisty

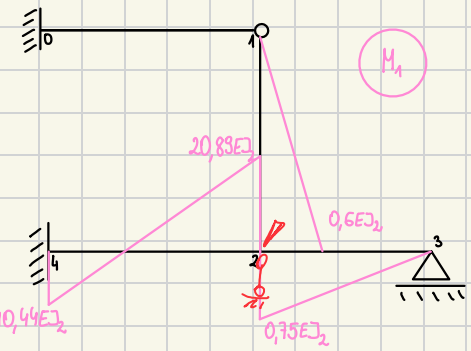


układ podstawowy

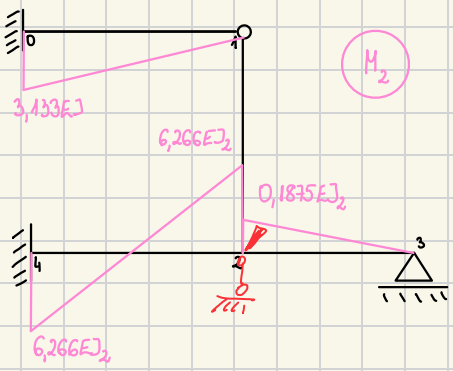


$$\begin{cases} r_{11}\varphi_1 + r_{12}u_2 + r_{1\Delta} = 0 \\ r_{21}\varphi_1 + r_{22}u_2 + r_{2\Delta} = 0 \end{cases}$$

stan $\varphi_1 = 1$



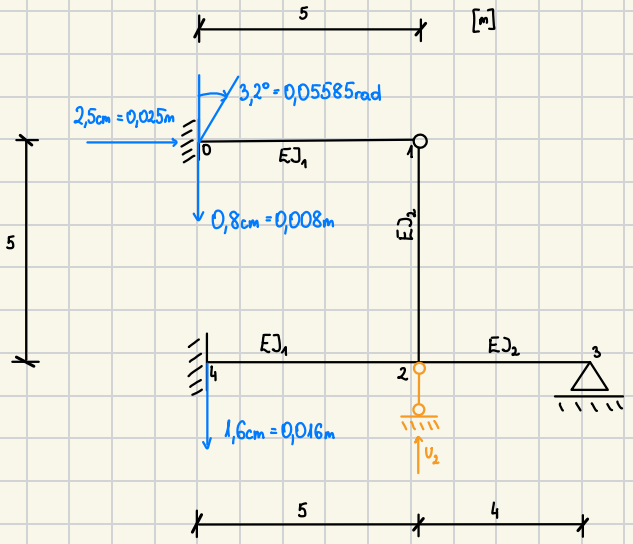
stan $u_2 = 1$

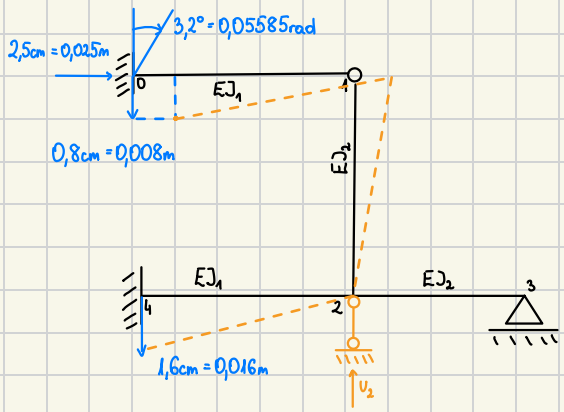


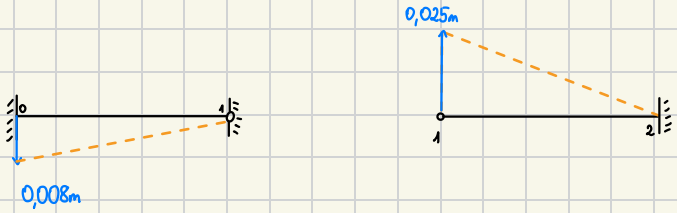
macierz sztywności

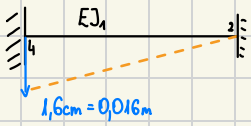
$$K = \begin{bmatrix} 22,24EJ_2 & 6,079EJ_2 \\ 6,079EJ_2 & 3,180EJ_2 \end{bmatrix}$$

stan „Δ"

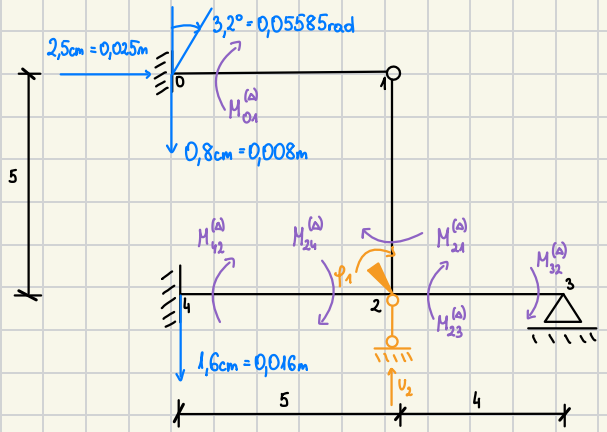








$\psi_{01} = \frac{0-0,008}{5} = -0,0016$ $\psi_{21} = \frac{0+0,025}{5} = 0,005$ $\psi_{23} = 0$ $\psi_{42} = \frac{0-0,016}{5} = -0,0032$



$M_{01}^{(a)} = \frac{3EJ_1}{l}\left(\varphi_0^{(a)} - \psi_{01}^{(a)}\right) = \frac{3 \cdot 26,11EJ_2}{5}(0,05585 + 0,0016) = 0,3000EJ_2$

$M_{10}^{(a)} = 0$

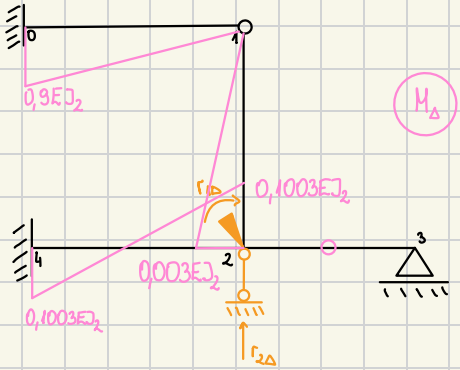
$M_{12}^{(a)} = 0$

$M_{21}^{(a)} = \frac{3EJ_2}{l}\left(\varphi_2^{(a)} - \psi_{21}^{(a)}\right) = \frac{3EJ_2}{5}(0 - 0,005) = -0,003EJ_2$

$M_{42}^{(a)} = \frac{2EJ_1}{l}\left(2\varphi_4^{(a)} + \varphi_2^{(a)} - 3\psi_{42}^{(a)}\right) = \frac{2 \cdot 26,11EJ_2}{5}(0 + 0 + 3 \cdot 0,0032) = 0,1003EJ_2$ ✓

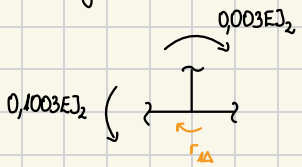
$M_{24}^{(a)} = \frac{2EJ_1}{l}\left(2\varphi_2^{(a)} + \varphi_4^{(a)} - 3\psi_{24}^{(a)}\right) = \frac{2 \cdot 26,11EJ_2}{5}(0 + 0 + 3 \cdot 0,0032) = 0,1003EJ_2$

$M_{23}^{(a)} = \frac{3EJ_2}{l}\left(\varphi_2^{(a)} - \psi_{24}^{(a)}\right) = \frac{3EJ_2}{4}(0 - 0) = 0$



wyznaczanie r_{ip}

równowaga węzła 2



$r_{1\Delta} = 0{,}1003EJ_2 - 0{,}003EJ_2$

$r_{1\Delta} = 0{,}0973EJ_2$

$r_{2\Delta}$ - RPW

$\overline{\psi}_{10} = -\frac{1}{5}$

$\overline{\psi}_{24} = -\frac{1}{5}$

$\overline{\psi}_{23} = \frac{1}{4}$

$$r_{2\Delta} \cdot \overline{1} + \sum M_{ik}^{(a)} \cdot \overline{\psi}_{ik} = 0$$

$$r_{2\Delta} \cdot \overline{1} + 0{,}9EJ_2 \cdot \left(-\frac{1}{5}\right) + \left(0{,}1003EJ_2 + 0{,}1003EJ_2\right) \cdot \left(-\frac{1}{5}\right) = 0$$

$$r_{2\Delta} = 0{,}2201EJ_2$$

wyznaczanie niewiadomych ρ_1 i u_2

$J_2 = 3060cm^4$

$E = 210GPa$

$EJ_2 = 2{,}1 \cdot 3060 = 6426\ kNm^2$

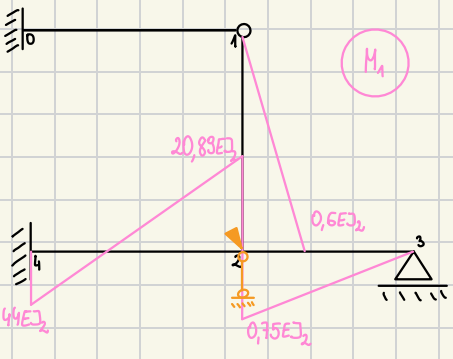
$$\begin{cases} 22{,}24EJ_2\rho_1 + 6{,}079EJ_2u_2 + 0{,}0973 \cdot 6426 = 0 \\ 6{,}079EJ_2\rho_1 + 3{,}180EJ_2u_2 + 0{,}2201 \cdot 6426 = 0 \end{cases}$$

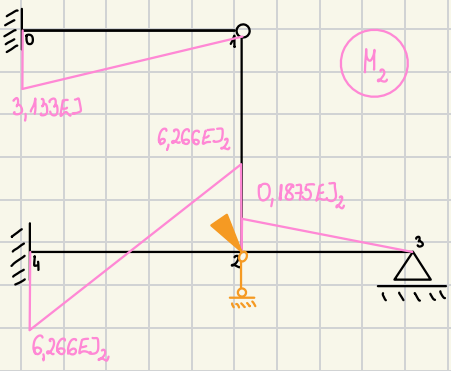
$$\begin{cases} \rho_1 = \frac{195{,}6}{EJ_2} \\ u_2 = \frac{-818{,}3}{EJ_2} \end{cases} \quad \left(\frac{195{,}7}{EI_2}, \frac{-818{,}9}{EI_2}\right)$$

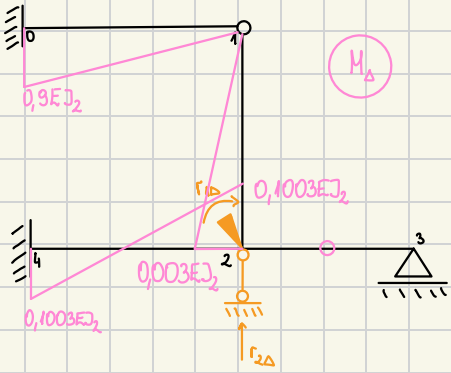
rozwiązałam powyższy układ równań i otrzymałam trochę inne wyniki

wyznaczanie rozkładu momentów zginających

$$M_{ik}^{\wedge} = M_{ik}^{(1)} \cdot \rho_1 + M_{ik}^{(2)} \cdot u_2 + M_{ik}^{(a)}$$







dla $\varphi_1 = \frac{195,7}{EI_2}$ $u_2 = \frac{-818,9}{EI_2}$

$M^n_{01} = 3,133EJ_2 \cdot \left(-\frac{818,3}{EJ_2}\right) + 0,9 \cdot 6426 = 3220\,kNm$ — 3217 kNm

$M^n_{10} = M^n_{12} = 0kNm$

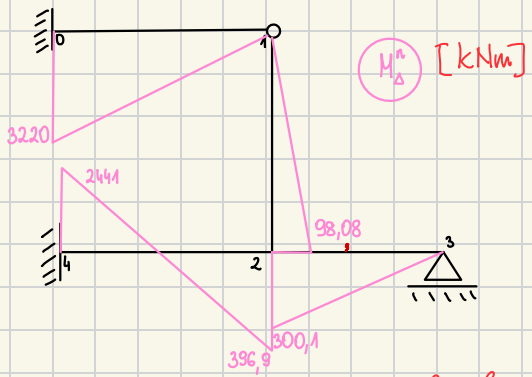
$M^n_{21} = 0,6EJ_2 \cdot \frac{195,6}{EJ_2} - 0,003 \cdot 6426 = 98,08\,kNm$ — 98,14 kNm

$M^n_{23} = 0,75EJ_2 \cdot \frac{195,6}{EJ_2} - 0,1875EJ_2 \cdot \left(\frac{-818,3}{EJ_2}\right) = 300,1\,kNm$ — 300,3 kNm

$M^n_{32} = 0kNm$

$M^n_{24} = 20,89 \cdot \frac{195,6}{EJ_2} + 6,266EJ_2 \cdot \left(\frac{-818,3}{EJ_2}\right) + 0,1003 \cdot 6426 = -396,9\,kNm$ — −398,5 kNm

$M^n_{42} = 10,44EJ_2 \cdot \frac{195,6}{EJ_2} + 6,266EJ_2 \cdot \left(\frac{-818,3}{EJ_2}\right) + 0,1003 \cdot 6426 = -244,1\,kNm$ — −244 kNm



M^n_Δ [kNm]

równowaga węzła 2

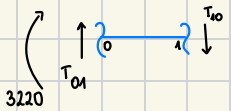
98,08 ↺ 396,9 ↑ $\frac{1}{2}$ ↑ 300,1

$-396,9 + 300,1 + 98,08 = 1,18$

$-398,5 + 300,3 + 98,14 = -0,06$ ✓

wyznaczanie rozkładu sił poprzecznych

dalsze obliczenia tej byłyby chyba inne



$\Sigma M_1 = 0$

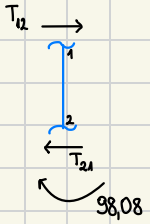
$5 \cdot T_{01} + 3220 = 0$

$T_{01} = -644\,kN$

$\Sigma M_0 = 0$

$5 \cdot T_{10} + 3220 = 0$

$T_{10} = -644kN$



$\Sigma M_1 = 0$

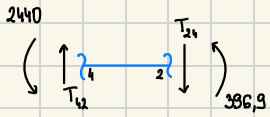
$5 \cdot T_{21} + 98,08 = 0$

$T_{21} = -19,62kN$

$\Sigma M_2 = 0$

$5 \cdot T_{12} + 98,08 = 0$

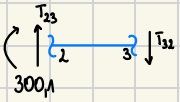
$T_{12} = -19,62\,kN$



$\Sigma M_4 = 0$

$5 \cdot T_{24} - 396,9 - 2440 = 0$

$T_{24} = 567,4kN$



$\Sigma M_2 = 0$

$4 \cdot T_{32} + 300,1 = 0$

$T_{32} = -75,03kN$

$\Sigma M_3 = 0$

$4 \cdot T_{23} + 300,1 = 0$

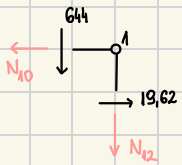
$T_{23} = -75,03kN$

$\Sigma M_2 = 0$

$5 \cdot T_{24} - 396,9 - 2440 = 0$

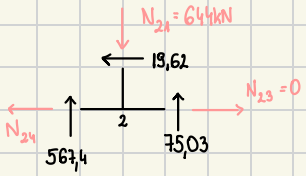
$T_{24} = 567,4kN$

wyznaczanie sił normalnych

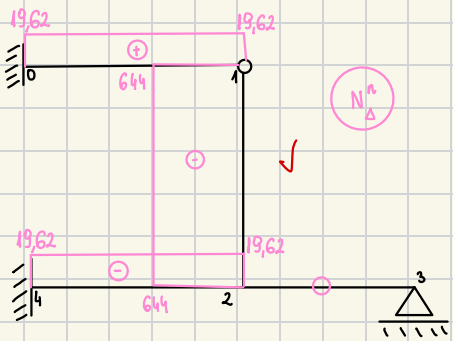


$N_{10} = 19,62kN$

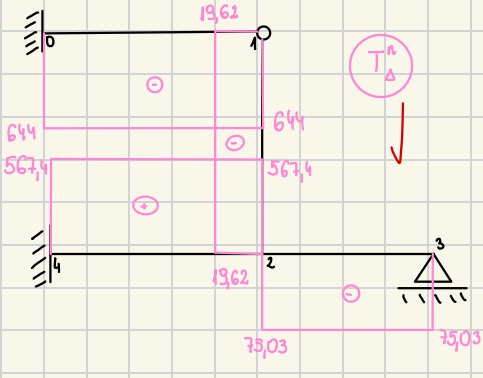
$N_{12} = -644kN$



$N_{24} = -19,62kN$

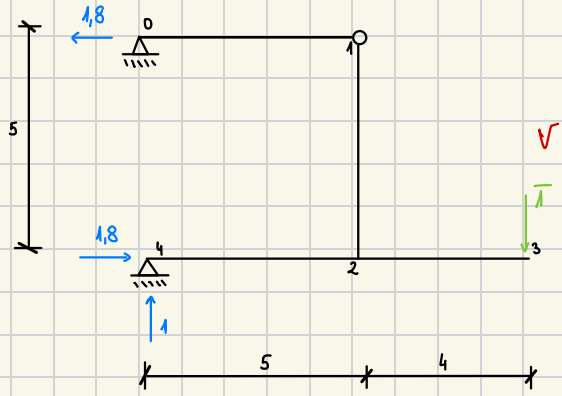


N^n_Δ ✓

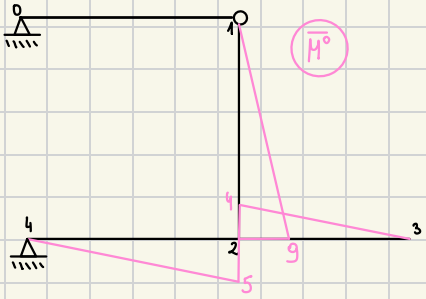


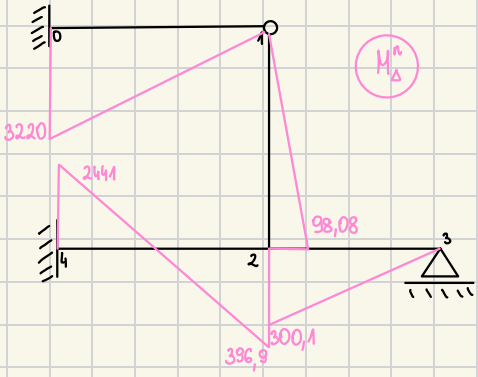
T^n_Δ ✓

sprawdzenie kinematyczne (można było zrobić tak samo jak przy obciążeniu siłami)



✓





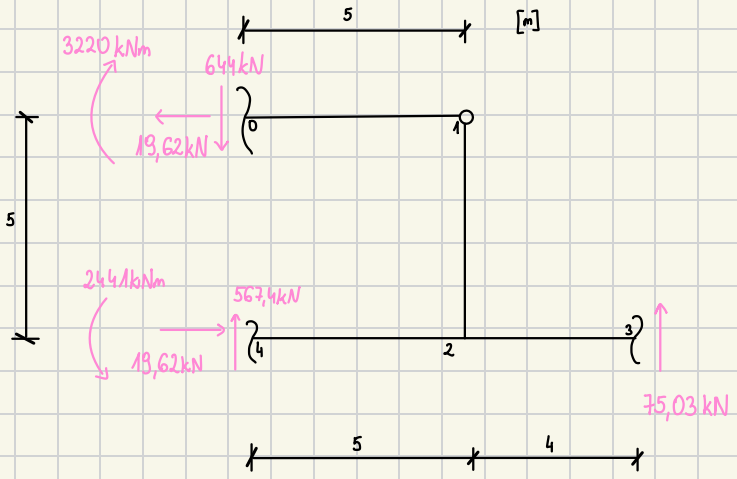
$$V_0 = \int \frac{M_p^n \cdot \overline{M}^0}{EJ}\,dx - \sum \overline{R}^0_\Delta \cdot \Delta = \frac{1}{EJ_1}\left[-\frac{1}{2}\cdot 2441\cdot 5\cdot\frac{1}{3}\cdot 5 + \frac{1}{2}\cdot 3969\cdot 5\cdot\frac{2}{3}\cdot 5\right] + \frac{1}{EJ_2}\left[-\frac{1}{2}\cdot 300{,}1\cdot 4\cdot\frac{2}{3}\cdot 4 + \frac{1}{2}\cdot 98{,}08\cdot 5\cdot\frac{2}{3}\cdot 9\right] + \left(0{,}025\cdot 1{,}8 - 0{,}016\right) = \frac{-392{,}2}{EJ_2} - (-0{,}061) \approx 0$$

✓

$$= -\frac{392{,}2}{6426} + 0{,}061 = -3\cdot 10^{-5} \cong 0$$

✓

kontrola statyczna

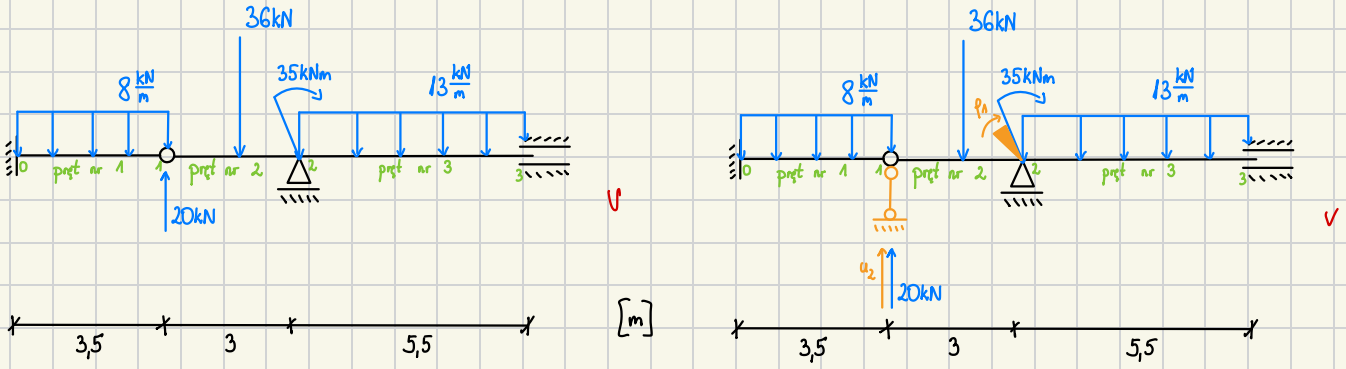


$\Sigma X = 0$ $\quad 19{,}62 - 19{,}62 = 0$

$\Sigma Y = 0$ $\quad 567{,}4 + 75{,}03 - 644 = -1{,}57 \cong 0$ ✓

$\Sigma M_3 = 0$ $\quad 3220 - 2441 - 644\cdot 3 - 19{,}62\cdot 5 + 567{,}4\cdot 9 \approx 0$

BELKA



$\Sigma \varphi = 1$

$\Sigma u = 1$

$SGN = 1+1 = 2$

wstępne przyjęcie przekroju

pręty 1,3 - przekrój I 240

$J_1 = 4250 cm^4$

pręt 2 - przekrój I 360 HEA

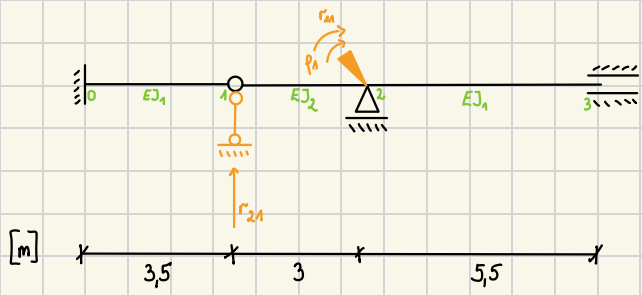
$J_2 = 33090 cm^4$

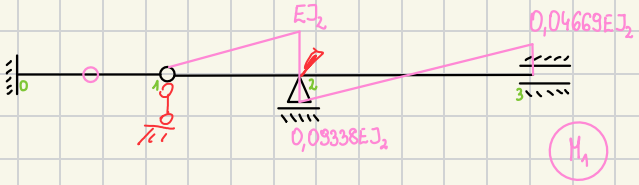
$$\frac{J_1}{J_2} = \frac{4250}{33090} = 0,1284$$

$$J_1 = 0,1284 J_2$$

$$\begin{cases} r_{11}\varphi_1 + r_{12}u_2 + r_{1P} = 0 \\ r_{21}\varphi_1 + r_{22}u_2 + r_{2P} = 0 \end{cases}$$

stan $\varphi_1 = 1$





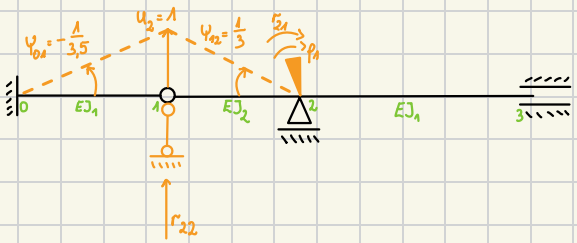
$$M_{01}^{(1)} = 0 = M_{10}^{(1)}$$

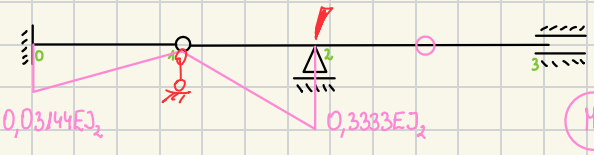
$$M_{12}^{(1)} = \frac{3EJ_2}{3} = EJ_2$$

$$M_{23}^{(1)} = \frac{4 \cdot EJ_1}{5,5} = \frac{4 \cdot E \cdot 0,1284 J_2}{5,5} = 0,09338 EJ_2$$

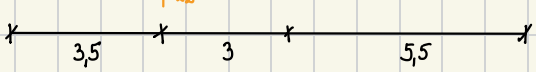
$$M_{32}^{(1)} = \frac{2 \cdot EJ_1}{5,5} = \frac{2 \cdot E \cdot 0,1284 J_2}{5,5} = 0,04669 EJ_2$$

stan $u_2 = 1$





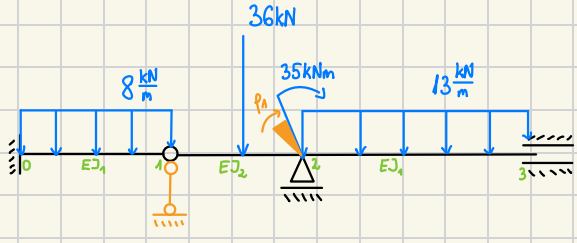
[m]

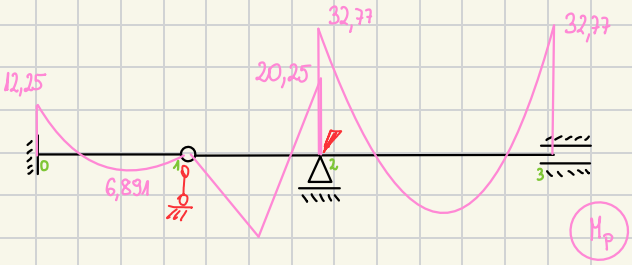


$$M_{01}^{(2)} = -\frac{3EJ_1}{l_{01}} \cdot \psi_{01} = -\frac{3 \cdot E \cdot 0,1284 J_2}{3,5} \cdot \left(-\frac{1}{3,5}\right) = 0,03144 EJ_2$$

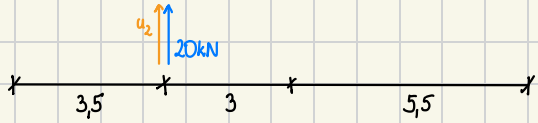
$$M_{12}^{(2)} = -\frac{3EJ_2}{l_{12}} \cdot \psi_{12} = -\frac{3EJ_2}{3} \cdot \left(\frac{1}{3}\right) = -0,3333 EJ_2$$

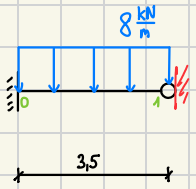
stan P





[m]

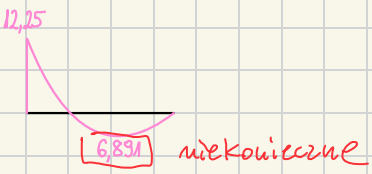




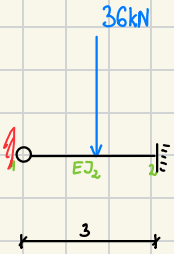
$$M_{01} = \frac{ql^2}{8} = \frac{8 \cdot 3,5^2}{8} = 12,25 \text{ kNm}$$

$M_{01} = -12,25$ kNm

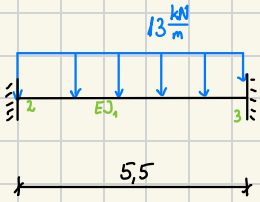
$$\frac{9}{128} \cdot ql^2 = \frac{9}{128}\left(8 \cdot 3,5^2\right) = 6,891 \text{ kNm}$$



niekorzystne



$$M = \frac{3Pl}{16} = \frac{3 \cdot 36 \cdot 3}{16} = 20,25 \text{ kNm}$$

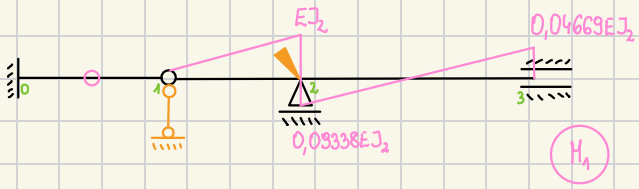


$$M = -\frac{ql^2}{12} = -\frac{13 \cdot 5,5^2}{12} = -32,77 \text{ kNm}$$

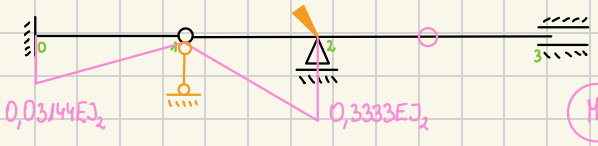
$$M_{23} = -\frac{ql^2}{12}$$

$$M_{32} = \frac{ql^2}{12}$$

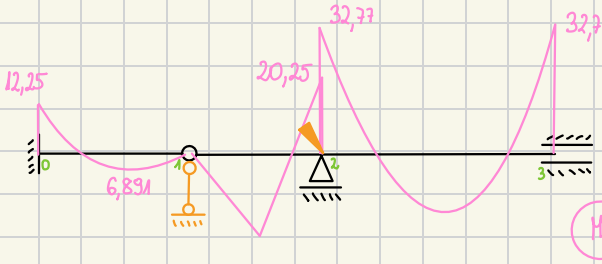
wyznaczanie r_{ik}



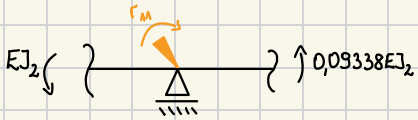
H_1



H_2

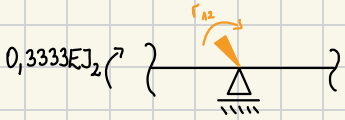


H_P



$r_{11} = 0,09338EJ_2 + EJ_2$

$r_{11} = 1,0934EJ_2$



$r_{12} = -0,3333EJ_2$

$$\sum_j R_j^{(1)} \cdot \overline{\delta}_j + \sum_n M_{ik}^{(1)} \cdot \Psi_{ik} = 0$$

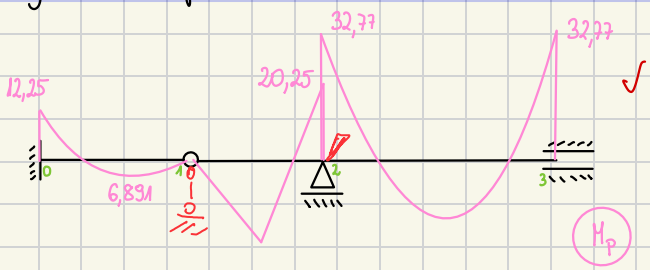
$$r_{21} \cdot \overline{1} + EJ_2 \cdot \frac{\overline{1}}{3} = 0$$

$r_{21} = -0,3333EJ_2$

$$r_{22} \cdot \overline{1} + 0,03144EJ_2 \cdot \left(-\frac{\overline{1}}{3,5}\right) - 0,3333 \cdot \frac{\overline{1}}{3} = 0$$

$r_{22} = 0,1201EJ_2$

wyznaczanie reakcji r_{iP}

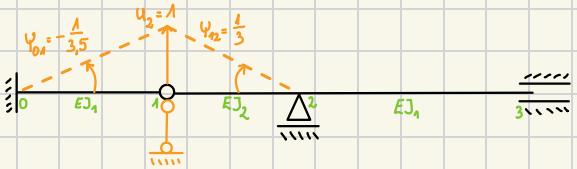


H_P ✓

równowaga w węźle 2

$$\sum H_2^{(P)} = 0$$

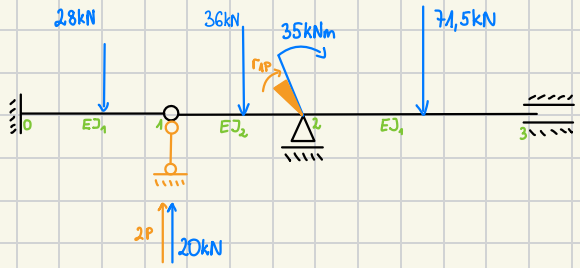
$r_{1P} = -47,52$ kNm ✓



$$\sum_j R_j^{(P)} \cdot \overline{\delta}_j + \sum_k P_k^{(P)} \cdot \overline{\delta}_k + \sum_n M_{ik}^{(P)} \cdot \overline{\Psi}_{ik} = 0$$

$$r_{2P} \cdot \overline{1} - 28 \cdot \frac{\overline{1}}{2} + 20 \cdot \overline{1} - 36 \cdot \frac{\overline{1}}{2} - 12,25 \cdot \left(-\frac{\overline{1}}{3,5}\right) + 20,25 \cdot \frac{\overline{1}}{3} = 0$$

$r_{2P} = 1,75$ kN



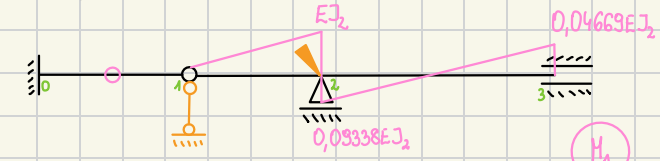
wyznaczanie niewiadomych φ_1 oraz u_2

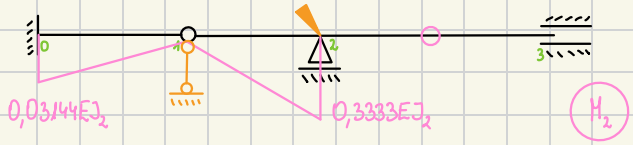
$$\begin{cases} 1{,}0934\,EJ_2\,\varphi_1 - 0{,}3333EJ_2\,u_2 - 47{,}52 = 0 \\ -0{,}3333\,EJ_2\,\varphi_1 + 0{,}1201EJ_2\,u_2 + 1{,}75 = 0 \end{cases}$$

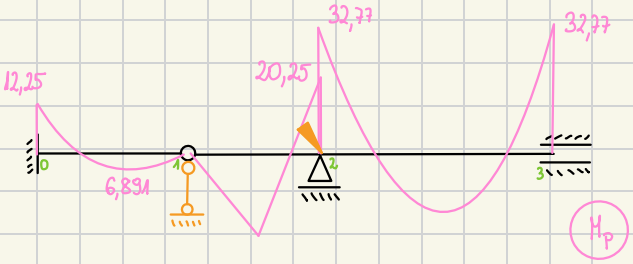
$$\begin{cases} \varphi_1 = \dfrac{253{,}3}{EJ_2} \\ u_2 = \dfrac{688{,}4}{EJ_2} \end{cases}$$

wyznaczanie rozkładu momentów zginających

$$M_{ik}^{n} = M_{ik}^{(1)} \cdot \varphi_1 + M_{ik}^{(2)} \cdot u_2 + M_{ik}^{(P)}$$







$$M_{01}^{n} = 0{,}03144EJ_2 \cdot \frac{688{,}4}{EJ_2} - 12{,}25 = 9{,}393\ \text{kNm}$$

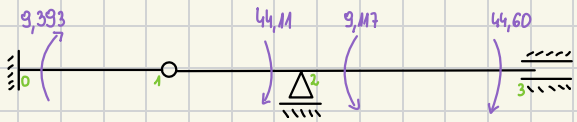
$$M_{10}^{n} = 0\ \text{kNm}$$

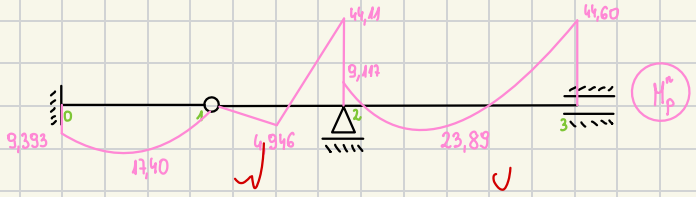
$$M_{12}^{n} = 0\ \text{kNm}$$

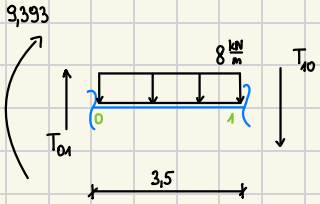
$$M_{21}^{n} = EJ_2 \cdot \frac{253{,}3}{EJ_2} - 0{,}3333EJ_2 \cdot \frac{688{,}4}{EJ_2} + 20{,}25 = 44{,}11\ \text{kNm}$$

$$M_{23}^{n} = 0{,}09338EJ_2 \cdot \frac{253{,}3}{EJ_2} - 32{,}77 = -9{,}117\ \text{kNm}$$

$$M_{32}^{n} = 0{,}04669 \cdot \frac{253{,}3}{EJ_2} + 32{,}77 = 44{,}60\ \text{kNm}$$







$\Sigma M_0 = 0$

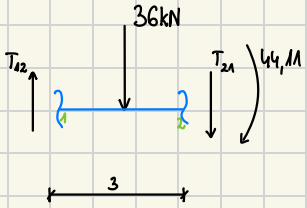
$3,5T_{10} + 9,393 + 8 \cdot 3,5 \cdot \frac{3,5}{2} = 0$

$T_{10} = -16,68\,kN$

$\Sigma M_1 = 0$

$3,5T_{01} + 9,393 - 8 \cdot 3,5 \cdot \frac{3,5}{2} = 0$

$T_{01} = 11,32\,kN$



$\Sigma M_1 = 0$

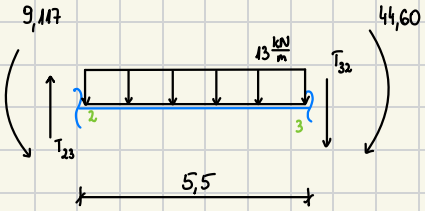
$36 \cdot 1,5 + 3T_{21} + 44,11 = 0$

$T_{21} = -32,70\,kN$

$\Sigma M_2 = 0$

$3 \cdot T_{12} - 36 \cdot 1,5 + 44,11 = 0$

$T_{12} = 3,297\,kN$



$\Sigma M_2 = 0$

$-9,117 + 13 \cdot 5,5 \cdot \frac{5,5}{2} + T_{32} \cdot 5,5 + 44,60 = 0$

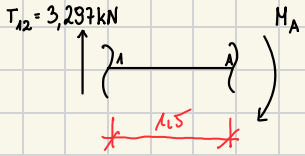
$T_{32} = -42,20\,kN$

$\Sigma M_3 = 0$

$-9,117 + 5,5 \cdot T_{23} - 13 \cdot 5,5 \cdot \frac{5,5}{2} + 44,60 = 0$

$T_{23} = 29,30\,kN$

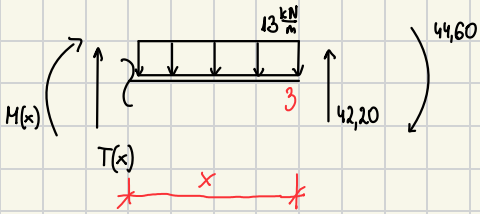
brakujące momenty



$\Sigma H_A = 0$

$H_A + 3,297 \cdot 1,5 = 0$

$H_A = -4,946\,kNm$



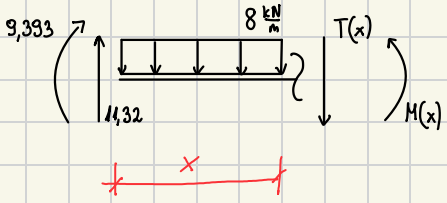
$M(x) = 42,2x - 13 \cdot x \cdot \frac{x}{2} - 44,60$

$T(x) = -42,2 + 13x$

$-42,2 = -13x$

$x = 3,246m$

$M(3,246) = 23,89\,kNm$



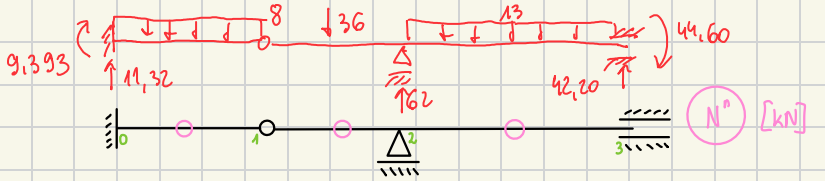
$M(x) = 9,393 + 11,32x - 8 \cdot x \cdot \frac{x}{2}$

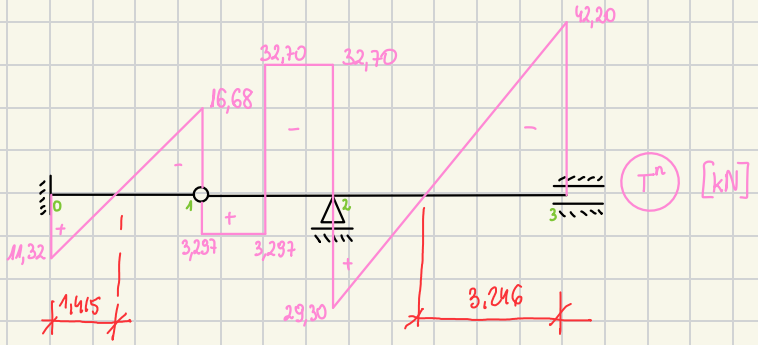
$T(x) = 11,32 - 8x$

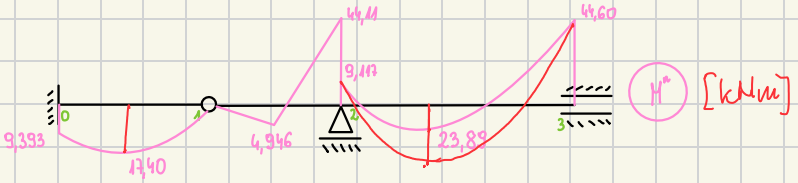
$11,32 - 8x = 0$

$x = 1,415m$

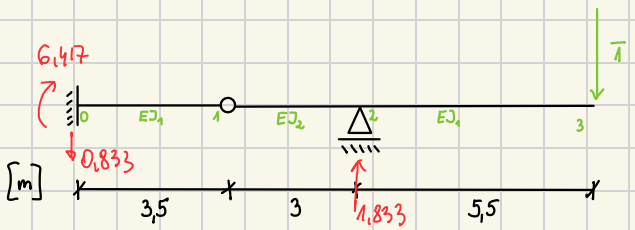
$M(1,415) = 17,40\,kNm$

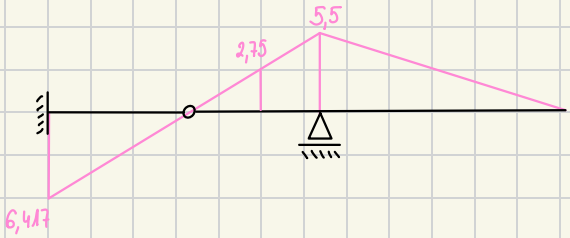






sprawdzenie kinematyczne

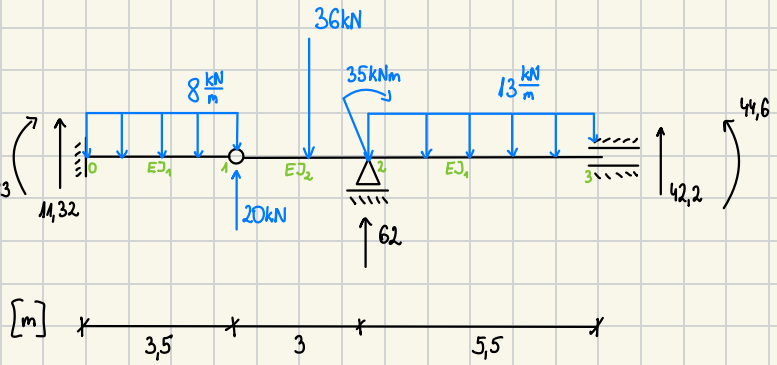




$$V_0 = \frac{1}{EJ_1}\left[\frac{1}{2}\cdot 9,393\cdot 3,5\cdot\frac{2}{3}\cdot 6,417 + \frac{2}{3}\cdot\frac{8\cdot 3,5^3}{8}\cdot\frac{1}{2}\cdot 6,417 + \frac{1}{2}\cdot 9,117\cdot 5,5\cdot\frac{2}{3}\cdot 5,5 - \frac{2}{3}\cdot\frac{13\cdot 5,5^3}{8}\cdot\frac{1}{2}\cdot 5,5 + \frac{1}{2}\cdot 44,6\cdot 5,5\cdot\frac{1}{3}\cdot 5,5\right] + \frac{1}{EJ_2}\left[-\frac{1}{2}\cdot 2,27\cdot\frac{2}{3}\cdot 4,946\cdot 1,5 + \frac{1}{2}\cdot 44,11\cdot 1,5\left(\frac{2}{3}\cdot 5,5 + \frac{1}{3}\cdot 2,75\right)\right.$$

$$\left. - \frac{1}{2}\cdot 4,946\cdot 1,5\left(\frac{2}{3}\cdot 2,75 + \frac{1}{3}\cdot 5,5\right)\right] = \frac{-16,84}{0,1284EJ_2} + \frac{131,2}{EJ_2} = \frac{0,04735}{EJ_2}$$

kontrola statyczna

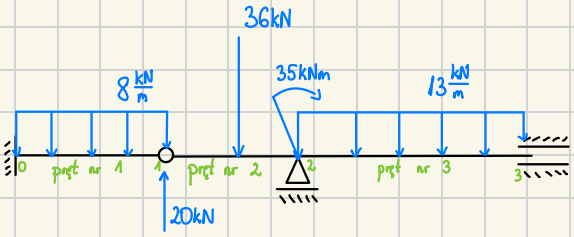


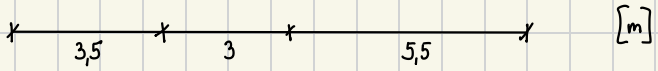
$$\Sigma M = 9,393 - 44,6 + 35 = -0,21 \approx 0$$

$$\Sigma X = 0$$

$$\Sigma Y: \; -8\cdot 3,5 + 20 - 36 - 13\cdot 5,5 + 62 + 42,2 + 11,32 = 0,02 \approx 0$$

BELKA - M. PRZEMIESZCZEŃ W UJĘCIU MACIERZOWYM





[m]

wstępne przyjęcie przekroju

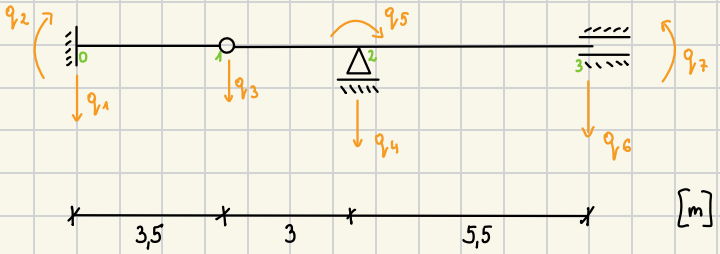
pręty 1, 3 - przekrój I 240
$J_1 = 4250 cm^4$

pręt 2 - przekrój I 360 HEA
$J_2 = 33090 cm^4$

$$\frac{J_1}{J_2} = \frac{4250}{33090} = 0,1284$$

$$J_1 = 0,1284 J_2$$

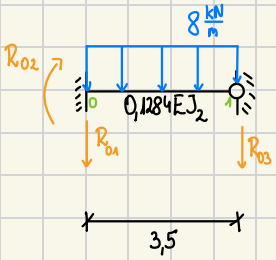
niewiadome przemieszczenia węzłowe



[m]

pręty - elementy

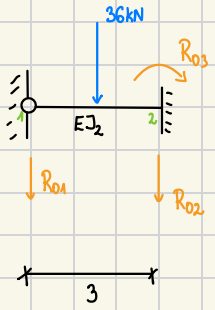
①



$$K_1 = \begin{bmatrix} \frac{3\cdot 0,1284EJ_2}{L^3} & \frac{3\cdot 0,1284EJ_2}{L^2} & -\frac{3\cdot 0,1284EJ_2}{L^3} \\ \frac{3\cdot 0,1284EJ_2}{L^2} & \frac{3\cdot 0,1284EJ_2}{L} & -\frac{3\cdot 0,1284EJ_2}{L^2} \\ -\frac{3\cdot 0,1284EJ_2}{L^3} & -\frac{3\cdot 0,1284EJ_2}{L^2} & \frac{3\cdot 0,1284EJ_2}{L^3} \end{bmatrix} = EJ_2 \begin{bmatrix} 0,008943 & 0,03144 & -0,008943 \\ 0,03144 & 0,1101 & -0,03144 \\ -0,008943 & -0,03144 & 0,008943 \end{bmatrix}$$ ✓

$$R_{01} = \begin{bmatrix} -\frac{5qL}{8} \\ -\frac{qL^2}{8} \\ -\frac{3qL^2}{8} \end{bmatrix} = \begin{bmatrix} -17,5 \\ -12,25 \\ -10,5 \end{bmatrix}$$ ✓

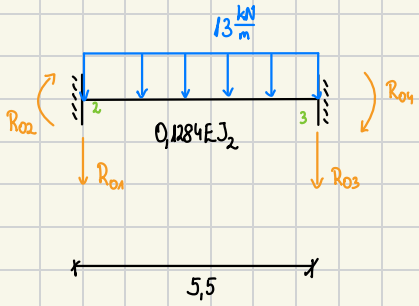
(2)



$$K_2 = \begin{bmatrix} \frac{3EJ_2}{l^3} & -\frac{3EJ_2}{l^3} & \frac{3EJ_2}{l^2} \\ -\frac{3EJ_2}{l^3} & \frac{3EJ_2}{l^3} & -\frac{3EJ_2}{l^2} \\ \frac{3EJ_2}{l^2} & -\frac{3EJ_2}{l^2} & \frac{3EJ_2}{l} \end{bmatrix} = EJ_2 \begin{bmatrix} 0,1111 & -0,1111 & 0,3333 \\ -0,1111 & 0,1111 & -0,3333 \\ 0,3333 & -0,3333 & 1 \end{bmatrix} \quad \surd$$

$$R_{12} = \begin{bmatrix} -\frac{5P}{16} \\ -\frac{11P}{16} \\ \frac{3PL}{16} \end{bmatrix} = \begin{bmatrix} -11,25 \\ -24,75 \\ 20,25 \end{bmatrix}$$

(3)



$$K_3 = \begin{bmatrix} \frac{12\cdot 0,1284EJ_2}{l^3} & \frac{6\cdot 0,1284EJ_2}{l^2} & -\frac{12\cdot 0,1284EJ_2}{l^3} & \frac{6\cdot 0,1284EJ_2}{l^2} \\ \frac{6\cdot 0,1284EJ_2}{l^2} & \frac{4\cdot 0,1284EJ_2}{l} & -\frac{6\cdot 0,1284EJ_2}{l^2} & \frac{2\cdot 0,1284EJ_2}{l} \\ -\frac{12\cdot 0,1284EJ_2}{l^3} & -\frac{6\cdot 0,1284EJ_2}{l^2} & \frac{12\cdot 0,1284EJ_2}{l^3} & -\frac{6\cdot 0,1284EJ_2}{l^2} \\ \frac{6\cdot 0,1284EJ_2}{l^2} & \frac{2\cdot 0,1284EJ_2}{l} & -\frac{6\cdot 0,1284EJ_2}{l^2} & \frac{4\cdot 0,1284EJ_2}{l} \end{bmatrix} = EJ_2 \begin{bmatrix} 0,009261 & 0,02547 & -0,009261 & 0,02547 \\ 0,02547 & 0,09338 & -0,02547 & 0,04669 \\ -0,009261 & -0,02547 & 0,009261 & -0,02547 \\ 0,02547 & 0,04669 & -0,02547 & 0,09338 \end{bmatrix} \quad \downarrow$$

$$R_{23} = \begin{bmatrix} -\frac{qL}{2} \\ -\frac{qL^2}{12} \\ -\frac{qL}{2} \\ \frac{qL^2}{12} \end{bmatrix} = \begin{bmatrix} -35,75 \\ -32,77 \\ -35,75 \\ 32,77 \end{bmatrix} \quad \surd$$

tabela powiązań

numer elementu	numer przemieszczeń			
	1	2	3	4
1	1	2	3	—
2	3	4	5	—
3	4	5	6	7

$$K_1 = EJ_2 \begin{bmatrix} 1 & 2 & 3 & \\ 0{,}008943 & 0{,}03144 & -0{,}008943 & 1 \\ 0{,}03144 & 0{,}1101 & -0{,}03144 & 2 \\ -0{,}008943 & -0{,}03144 & 0{,}008943 & 3 \end{bmatrix}$$

$$K_2 = EJ_2 \begin{bmatrix} 3 & 4 & 5 & \\ 0{,}1111 & -0{,}1111 & 0{,}3333 & 3 \\ -0{,}1111 & 0{,}1111 & -0{,}3333 & 4 \\ 0{,}3333 & -0{,}3333 & 1 & 5 \end{bmatrix}$$

$$K_3 = EJ_2 \begin{bmatrix} 4 & 5 & 6 & 7 & \\ 0{,}009261 & 0{,}02547 & -0{,}009261 & 0{,}02547 & 4 \\ 0{,}02547 & 0{,}09338 & -0{,}02547 & 0{,}04669 & 5 \\ -0{,}009261 & -0{,}02547 & 0{,}009261 & -0{,}02547 & 6 \\ 0{,}02547 & 0{,}04669 & -0{,}02547 & 0{,}09338 & 7 \end{bmatrix}$$

$$K = \begin{bmatrix} 0{,}008943 & 0{,}03144 & -0{,}008943 & 0 & 0 & 0 & 0 \\ 0{,}03144 & 0{,}1101 & -0{,}03144 & 0 & 0 & 0 & 0 \\ -0{,}008943 & -0{,}03144 & 0{,}008943 + 0{,}1111 & -0{,}1111 & 0{,}3333 & 0 & 0 \\ 0 & 0 & -0{,}1111 & 0{,}1111 + 0{,}009261 & -0{,}3333 + 0{,}02547 & -0{,}009261 & 0{,}02547 \\ 0 & 0 & 0{,}3333 & -0{,}3333 + 0{,}2547 & 1 + 0{,}09338 & -0{,}02547 & 0{,}04669 \\ 0 & 0 & 0 & -0{,}009261 & -0{,}02547 & 0{,}009261 & -0{,}02547 \\ 0 & 0 & 0 & 0{,}02547 & 0{,}04669 & -0{,}02547 & 0{,}09338 \end{bmatrix}$$

$$K = \begin{bmatrix} 0{,}008943 & 0{,}03144 & -0{,}008943 & 0 & 0 & 0 & 0 \\ 0{,}03144 & 0{,}1101 & -0{,}03144 & 0 & 0 & 0 & 0 \\ -0{,}008943 & -0{,}03144 & 0{,}1200 & -0{,}1111 & 0{,}3333 & 0 & 0 \\ 0 & 0 & -0{,}1111 & 0{,}1204 & -0{,}3078 & -0{,}009261 & 0{,}02547 \\ 0 & 0 & 0{,}3333 & -0{,}0786 & 1{,}09338 & -0{,}02547 & 0{,}04663 \\ 0 & 0 & 0 & -0{,}009261 & -0{,}02547 & 0{,}009261 & -0{,}02547 \\ 0 & 0 & 0 & 0{,}02547 & 0{,}04663 & -0{,}02547 & 0{,}09338 \end{bmatrix}$$

agregacja wektora reakcji przywęzłowych od obciążenia przęsłowego

$$R_{01} = \begin{bmatrix} -17{,}5 \\ -12{,}25 \\ -10{,}5 \end{bmatrix} \qquad R_{12} = \begin{bmatrix} -11{,}25 \\ -24{,}75 \\ 20{,}25 \end{bmatrix} \qquad R_{23} = \begin{bmatrix} -35{,}75 \\ -32{,}77 \\ -35{,}75 \\ 32{,}77 \end{bmatrix}$$

$$R_0 = \begin{bmatrix} -17{,}5 \\ -12{,}25 \\ -10{,}5 - 11{,}25 \\ -24{,}75 - 35{,}75 \\ 20{,}25 - 32{,}77 \\ -35{,}75 \\ 32{,}77 \end{bmatrix} = \begin{bmatrix} -17{,}5 \\ -12{,}25 \\ -21{,}75 \\ -60{,}5 \\ -12{,}52 \\ -35{,}75 \\ 32{,}77 \end{bmatrix} \surd$$

wektor sił węzłowych

$$P_w = \begin{bmatrix} 0 \\ 0 \\ -20 \\ 0 \\ 35 \\ 0 \\ 0 \end{bmatrix}$$

✓

uwzględnienie warunków podparcia belki

$$K = \begin{bmatrix} 0,008943 & 0,003144 & 0,008943 & 0 & 0 & 0 & 0 \\ 0,003144 & 0,001104 & -0,003144 & 0 & 0 & 0 & 0 \\ -0,008943 & -0,003144 & 0,1200 & -0,1111 & 0,3333 & 0 & 0 \\ 0 & 0 & -0,1111 & 0,1204 & -0,3078 & -0,009261 & 0,02547 \\ 0 & 0 & 0,3333 & -0,01786 & 1,09338 & -0,02547 & 0,04663 \\ 0 & 0 & 0 & -0,009261 & -0,02547 & 0,009261 & 0,02547 \\ 0 & 0 & 0 & 0,02547 & 0,04663 & -0,02547 & 0,04338 \end{bmatrix}$$

✓

$$R_0 = \begin{bmatrix} 17,5 \\ 12,25 \\ -21,75 \\ 69,5 \\ -12,52 \\ 33,75 \\ 32,71 \end{bmatrix} \qquad P_w = \begin{bmatrix} 0 \\ 0 \\ -20 \\ 0 \\ 35 \\ 0 \\ 0 \end{bmatrix}$$

równanie równowagi

$$K_{red}\, q_{red} = P_{w,red} - R_{red}$$

$$\begin{bmatrix} 0{,}1200 & 0{,}3333 \\ 0{,}3333 & 1{,}09338 \end{bmatrix} \begin{bmatrix} q_3 \\ q_5 \end{bmatrix} = \begin{bmatrix} -20 \\ 35 \end{bmatrix} - \begin{bmatrix} -21{,}75 \\ -12{,}52 \end{bmatrix}$$

$$\begin{bmatrix} 0{,}1200 & 0{,}3333 \\ 0{,}3333 & 1{,}0934 \end{bmatrix} \begin{bmatrix} q_3 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1{,}75 \\ 47{,}52 \end{bmatrix}$$

$$\begin{bmatrix} q_3 \\ q_5 \end{bmatrix} = \frac{1}{EJ_2} \begin{bmatrix} -692{,}1 \\ 254{,}4 \end{bmatrix}$$

wektory przemieszczeń dla elementów

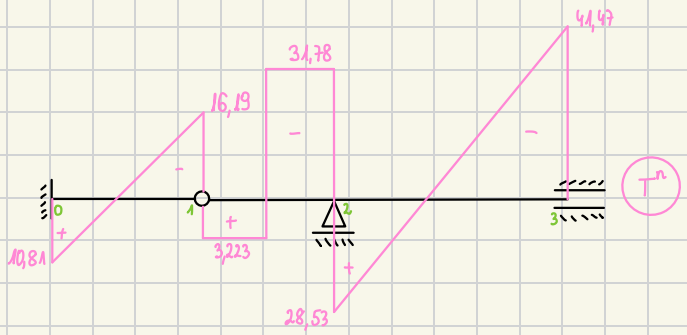
$$q_1 = \frac{1}{EJ_2} \begin{bmatrix} 0 \\ 0 \\ -692{,}1 \end{bmatrix} \qquad q_2 = \frac{1}{EJ_2} \begin{bmatrix} -692{,}1 \\ 0 \\ 254{,}4 \end{bmatrix} \qquad q_3 = \frac{1}{EJ_2} \begin{bmatrix} 0 \\ 254{,}4 \\ 0 \\ 0 \end{bmatrix}$$

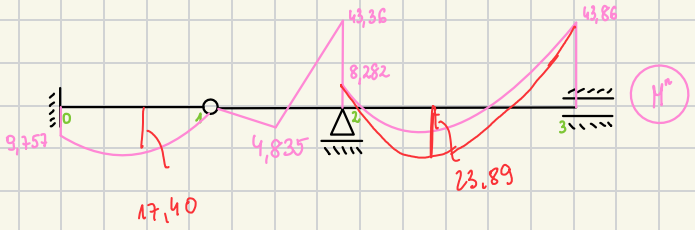
wektory końcowych reakcji węzłowych

$$R_1 = K_1 q_1 + R_{01} = EJ_2 \begin{bmatrix} 0{,}008943 & 0{,}03144 & -0{,}008943 \\ 0{,}03144 & 0{,}1101 & -0{,}03144 \\ -0{,}008943 & -0{,}03144 & 0{,}008943 \end{bmatrix} \frac{1}{EJ_2} \begin{bmatrix} 0 \\ 0 \\ -692{,}1 \end{bmatrix} + \begin{bmatrix} -17{,}5 \\ -12{,}25 \\ -10{,}5 \end{bmatrix} = \begin{bmatrix} -10{,}81 \\ 9{,}757 \\ -16{,}19 \end{bmatrix}$$

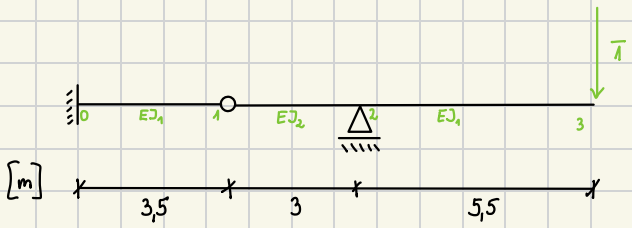
$$R_2 = K_2 q_2 + R_{02} = EJ_2 \begin{bmatrix} 0{,}1111 & -0{,}1111 & 0{,}3333 \\ -0{,}1111 & 0{,}1111 & -0{,}3333 \\ 0{,}3333 & -0{,}3333 & 1 \end{bmatrix} \frac{1}{EJ_2} \begin{bmatrix} -692{,}1 \\ 0 \\ 254{,}4 \end{bmatrix} + \begin{bmatrix} -11{,}25 \\ -24{,}75 \\ 20{,}25 \end{bmatrix} = \begin{bmatrix} -3{,}223 \\ -31{,}78 \\ 43{,}36 \end{bmatrix}$$

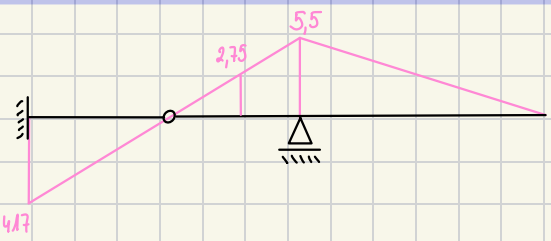
$$R_3 = K_3 q_3 + R_{03} = \begin{bmatrix} 0{,}009261 & 0{,}02547 & -0{,}009261 & 0{,}02547 \\ 0{,}02547 & 0{,}09338 & -0{,}02547 & 0{,}04669 \\ -0{,}009261 & -0{,}02547 & 0{,}009261 & -0{,}02547 \\ 0{,}02547 & 0{,}04669 & -0{,}02547 & 0{,}09338 \end{bmatrix} \frac{1}{EJ_2} \begin{bmatrix} 0 \\ 254{,}4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -35{,}75 \\ -32{,}77 \\ -35{,}75 \\ 32{,}77 \end{bmatrix} = \begin{bmatrix} -28{,}53 \\ -8{,}282 \\ -41{,}47 \\ 43{,}86 \end{bmatrix}$$





sprawdzenie kinematyczne

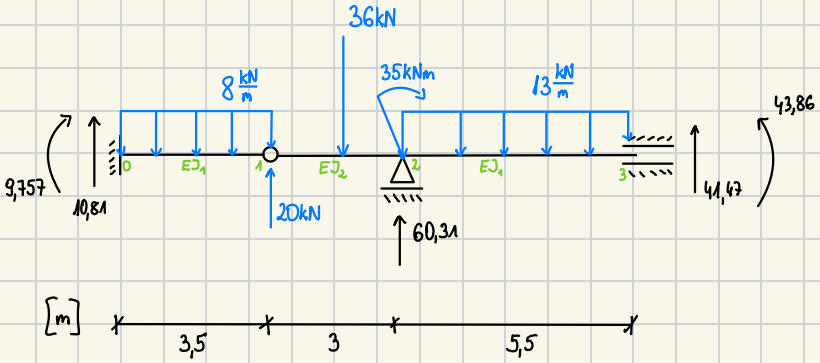




$$V_0 = \frac{1}{EJ_1}\left[\frac{1}{2}\cdot 9,757\cdot 3,5\cdot\frac{2}{3}\cdot 6,417 + \frac{2}{3}\cdot\frac{8\cdot 3,5^3}{8}\cdot\frac{1}{2}\cdot 6,417 + \frac{1}{2}\cdot 8,282\cdot 5,5\cdot\frac{2}{3}\cdot 5,5 - \frac{2}{3}\cdot\frac{13\cdot 5,5^3}{8}\cdot\frac{1}{2}\cdot 5,5 + \frac{1}{2}\cdot 43,86\cdot 5,5\cdot\frac{1}{3}\cdot 5,5\right] + \frac{1}{EJ_2}\left[-\frac{1}{2}\cdot 2,27\cdot\frac{2}{3}\cdot 4,835\cdot 1,5 + \frac{1}{2}\cdot 43,36\cdot 1,5\left(\frac{2}{3}\cdot 5,5 + \frac{1}{3}\cdot 2,75\right)\right.$$

$$\left. - \frac{1}{2}\cdot 4,835\cdot 1,5\left(\frac{2}{3}\cdot 2,75 + \frac{1}{3}\cdot 5,5\right)\right] = \frac{-26,27}{0,128EJ_2} + \frac{130,3}{EJ_2} = \frac{-0,001078}{EJ_2}$$

kontrola statyczna



$$\Sigma M = 9,757 - 43,86 + 35 = 0,897 \approx 0$$

$$\Sigma X = 0$$

$$\Sigma Y: \; -8\cdot 3,5 + 20 - 36 - 13\cdot 5,5 + 60,31 + 41,47 + 10,81 \approx 0$$