

Mechanika Budowli
Ćwiczenie nr 2

UKŁADY STATYCZNIE NIEWYZNACZALNE – METODA SIŁ

Prowadzący:

mgr inż. Anita Kaczor

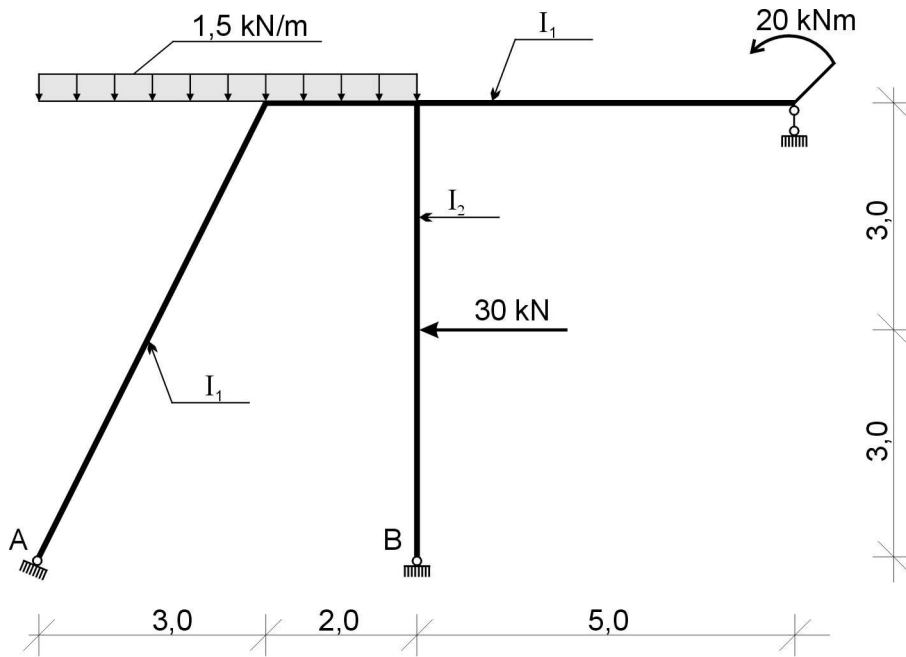
Wykonał:

Wiesław Ludzkowski

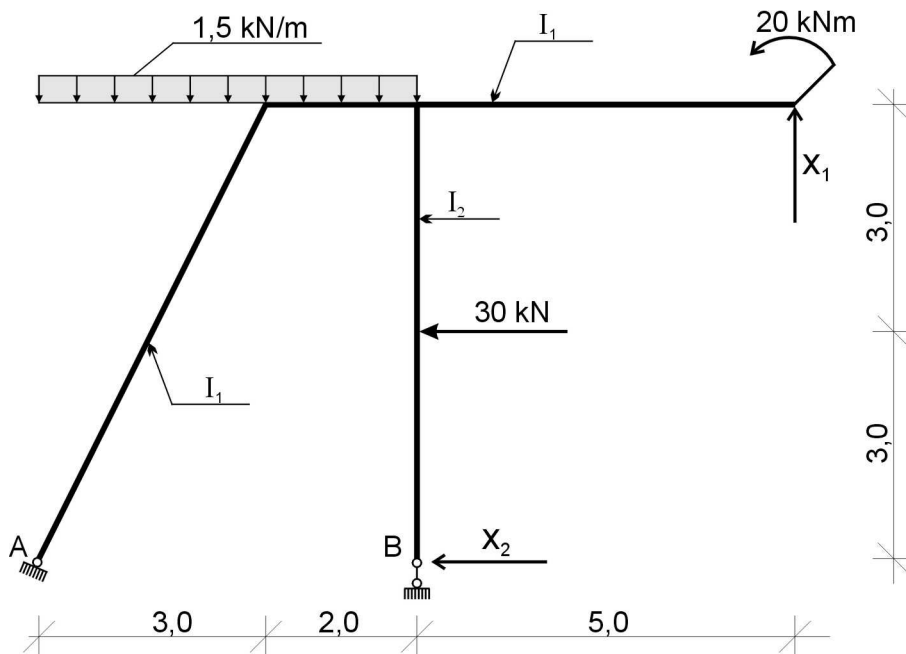
II rok III grupa

Schemat układu:

SSN=2



Układ podstawowy

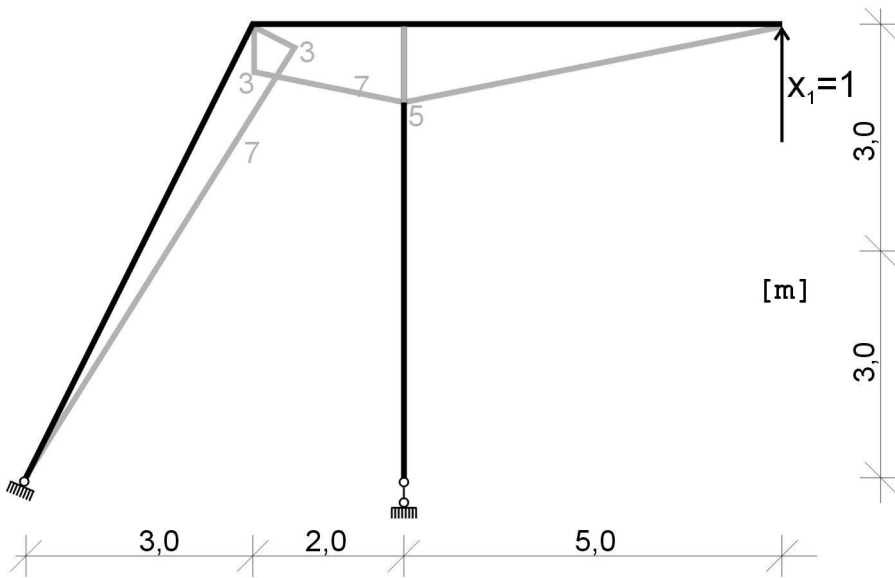


I140 $I_1 = 573 \text{ cm}^4$ $E = 205 \text{ GPa}$
 I200 $I_2 = 2140 \text{ cm}^4$

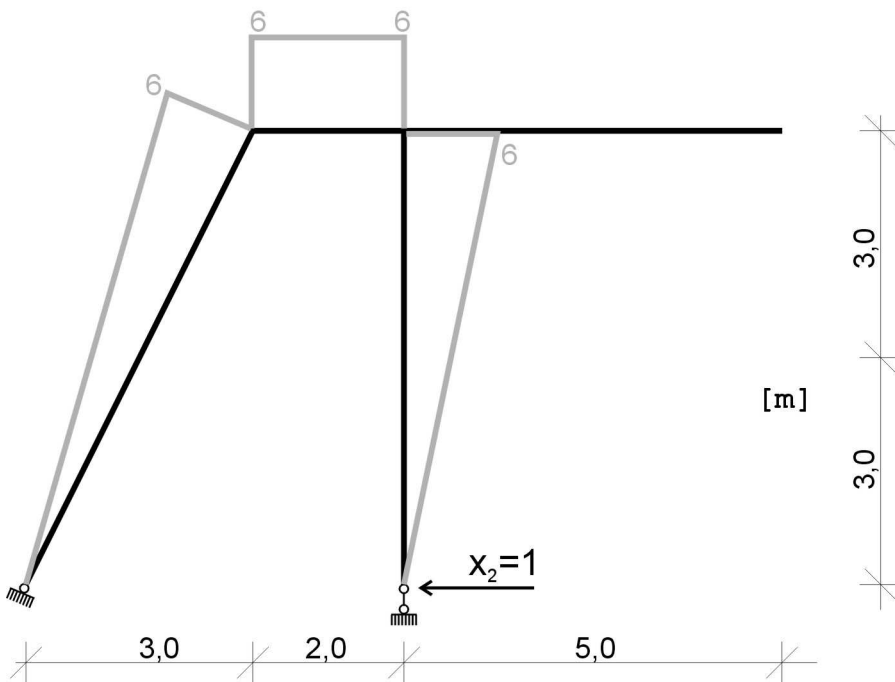
$I_1 = EI$
 $I_2 = 2,85 EI$

Wykres momentów zginających od sił jednostkowych

Stan od $x_1=1$

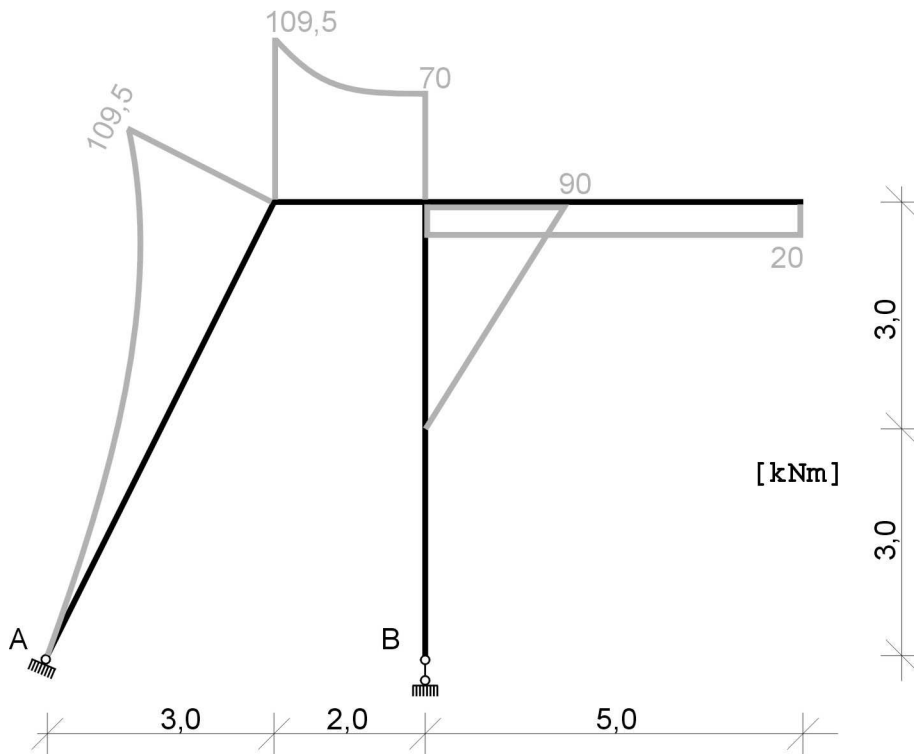


Stan od $x_2=1$



Wykres momentów od obciążenia zewnętrznego

Stan P



Równanie kanoniczne

$$\delta_{11}x_1 + \delta_{12}x_2 + \delta_{1P} = 0$$

$$\delta_{21}x_1 + \delta_{22}x_2 + \delta_{2P} = 0$$

$$\delta_{12} = \delta_{21}$$

$$\delta_{11} = \int_s \frac{M_1^0 \cdot M_1^0}{EI} ds$$

$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \cdot 3 \cdot 6,708 \cdot \frac{2}{3} \cdot 3 + \frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{2}{3} \cdot 5 \right) + \frac{1}{EI} \left[\frac{2}{6} (2 \cdot 3 \cdot 3 + 2 \cdot 5 \cdot 5 + 3 \cdot 5 + 3 \cdot 5) \right] = \mathbf{94,4574/EI}$$

$$\delta_{12} = \int_s \frac{M_1^0 \cdot M_2^0}{EI} ds$$

$$\delta_{12} = -\frac{1}{EI} \left(\frac{1}{2} \cdot 3 \cdot 6,708 \cdot \frac{2}{3} \cdot 6 \right) - \frac{1}{EI} \left[\frac{2}{6} (2 \cdot 3 \cdot 6 + 2 \cdot 5 \cdot 6 + 3 \cdot 6 + 5 \cdot 6) \right] = \mathbf{-88,248/EI}$$

$$\delta_{22} = \int_s \frac{M_2^0 \cdot M_2^0}{EI} ds$$

$$\delta_{22} = 1/EI [(1/2 \cdot 6 \cdot 6,708 \cdot 2/3 \cdot 6) + 1/EI (6 \cdot 2 \cdot 6) + 1/2,85EI (1/2 \cdot 6 \cdot 6 \cdot 2/3 \cdot 6)] = \mathbf{177,7592/EI}$$

$$\delta_{1P} = \int_s \frac{M_1^0 \cdot M_P^0}{EI} ds$$

$$\delta_{1P} = -1/EI (1/2 \cdot 3 \cdot 6,708 \cdot 2/3 \cdot 109,5) + 1/EI [2/3 \cdot (1,5 \cdot 3^2)/8 \cdot 6,708 \cdot 1/2 \cdot 3] - \\ -1/EI [2/6(2 \cdot 3 \cdot 109,5 + 2 \cdot 70 \cdot 5 + 109,5 \cdot 5 + 70 \cdot 3)] + \\ + 1/EI [2/3 \cdot (1,5 \cdot 2^2)/8 \cdot 2 \cdot (3+5)/2] + 1/EI (20 \cdot 5 \cdot 1/2 \cdot 5) = \mathbf{-235,6273/EI}$$

$$\delta_{2P} = \int_s \frac{M_2^0 \cdot M_P^0}{EI} ds$$

$$\delta_{2P} = 1/EI (1/2 \cdot 106,5 \cdot 6,708 \cdot 2/3 \cdot 6) - 1/EI [2/3 \cdot (1,5 \cdot 3^2)/8 \cdot 6,708 \cdot 1/2 \cdot 6] + \\ + 1/EI [(1/2 \cdot 106,5 \cdot 2 \cdot 6 + 1/2 \cdot 70 \cdot 6) - 1/EI [(2/3 \cdot (1,5 \cdot 2^2)/8 \cdot 2 \cdot 6) + \\ + 1/2,85 EI [1/2 \cdot 90 \cdot 3 \cdot (2/3 \cdot 6 + 1/3 \cdot 3)]] = \mathbf{2696,006605/EI}$$

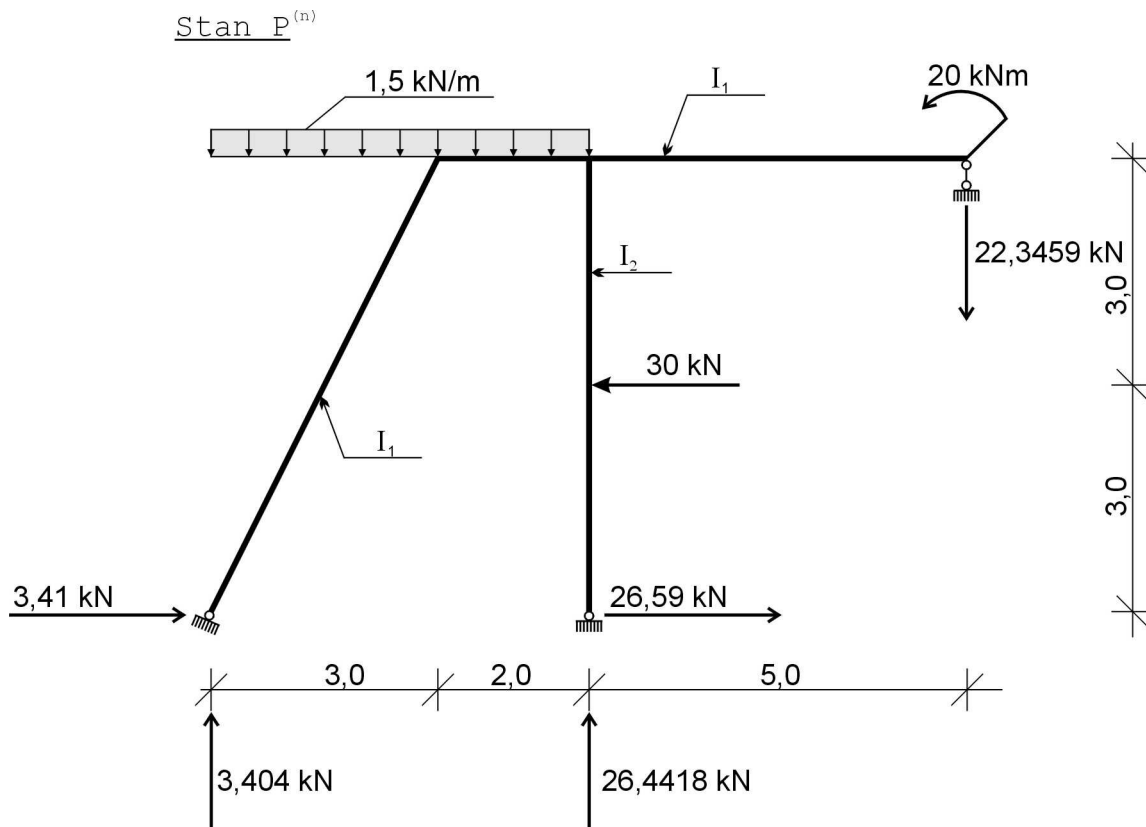
$$\frac{94,4574}{EI} x_1 + \frac{-88,248}{EI} x_2 + \frac{-235,6273}{EI} = 0$$

$$\frac{-88,248}{EI} x_1 + \frac{177,7592}{EI} x_2 + \frac{2754,25605}{EI} = 0$$

Po rozwiązaniu układu równań otrzymujemy wartości sił nadliczbowych x_1 i x_2

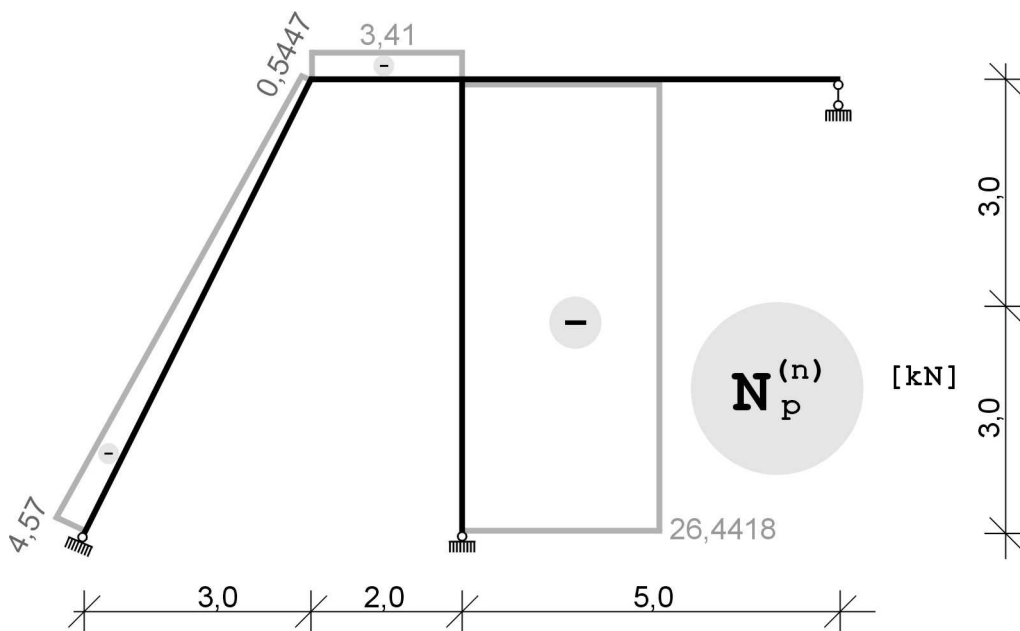
$$\mathbf{x_1 = -22,3459 \text{ kN}}$$

$$\mathbf{x_2 = -26,59 \text{ kN}}$$

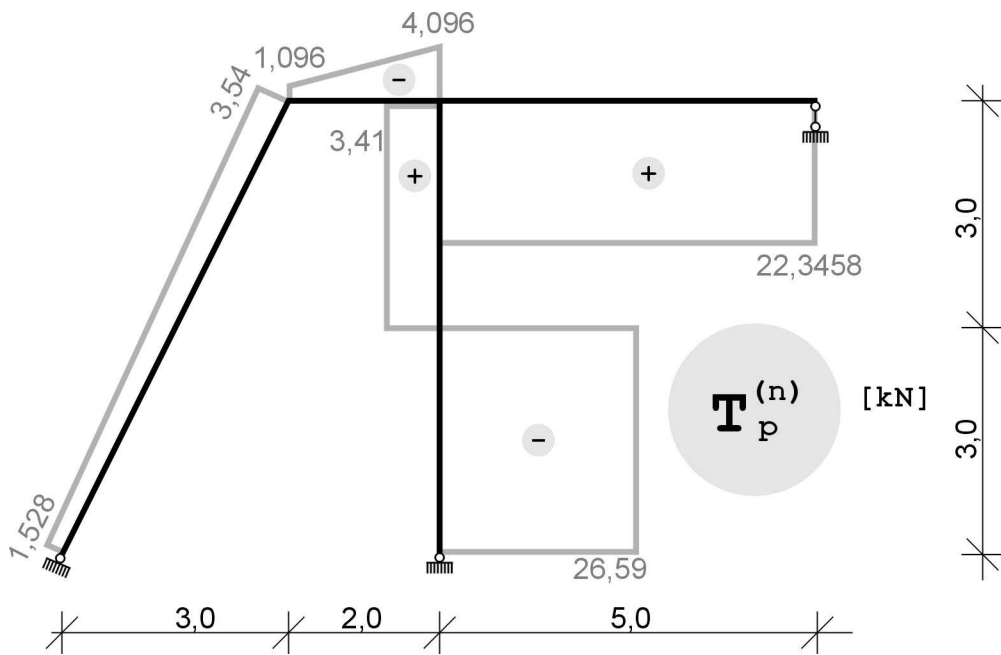


Wykresy sił wewnętrznych

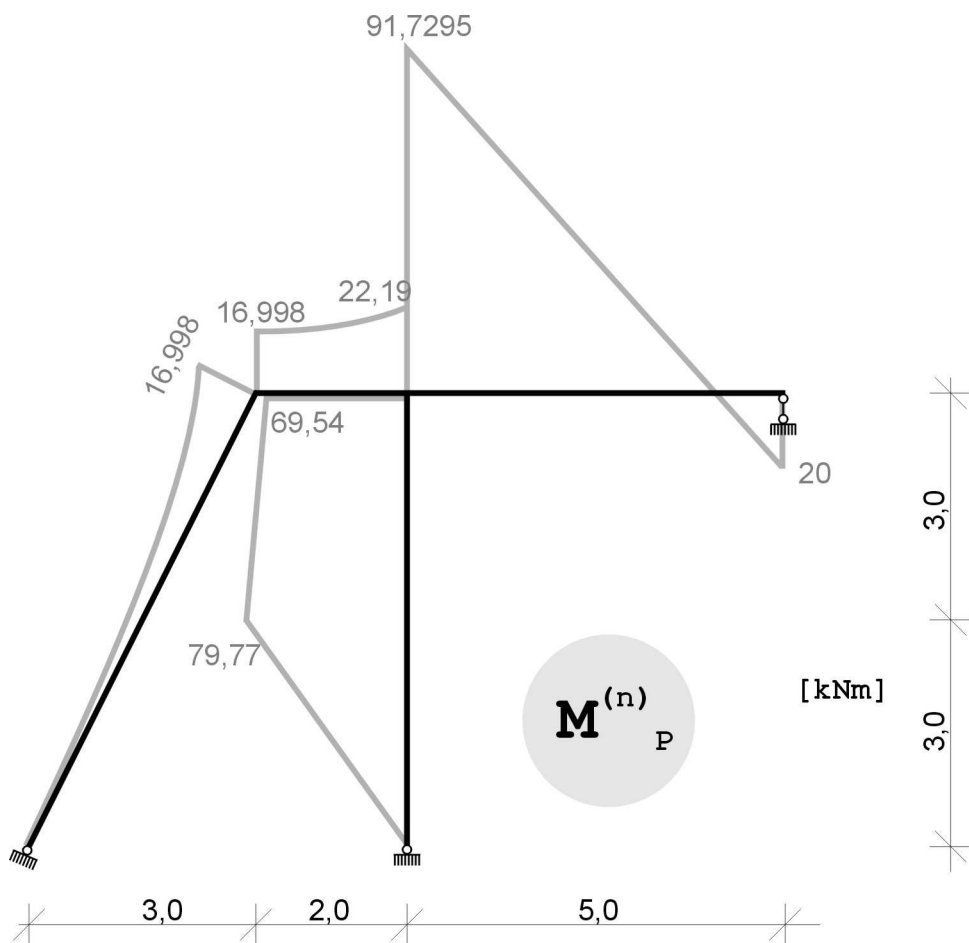
Siły normalne:



Sily tnące:



Momenty zginające:

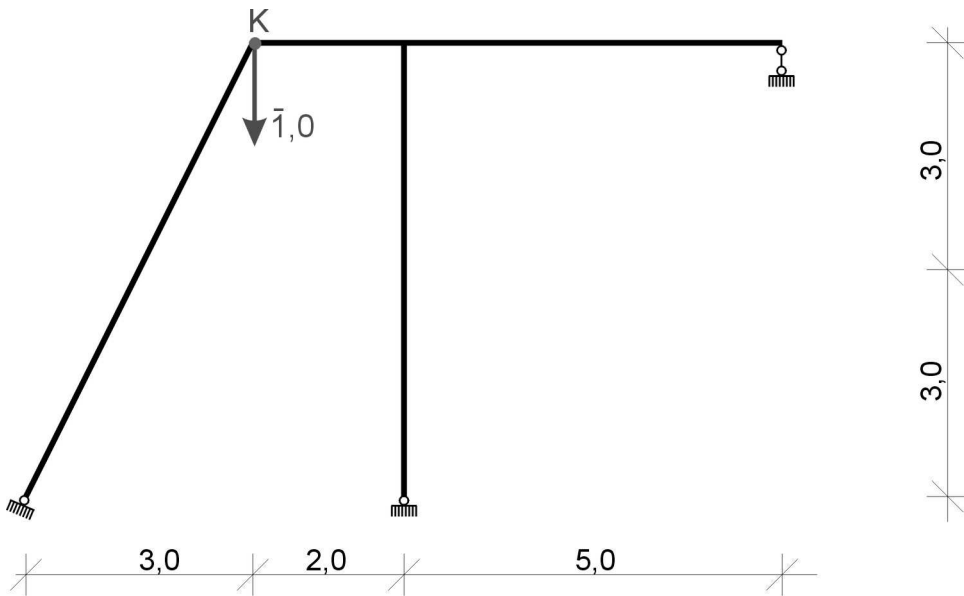


Sprawdzenie kinematyczne

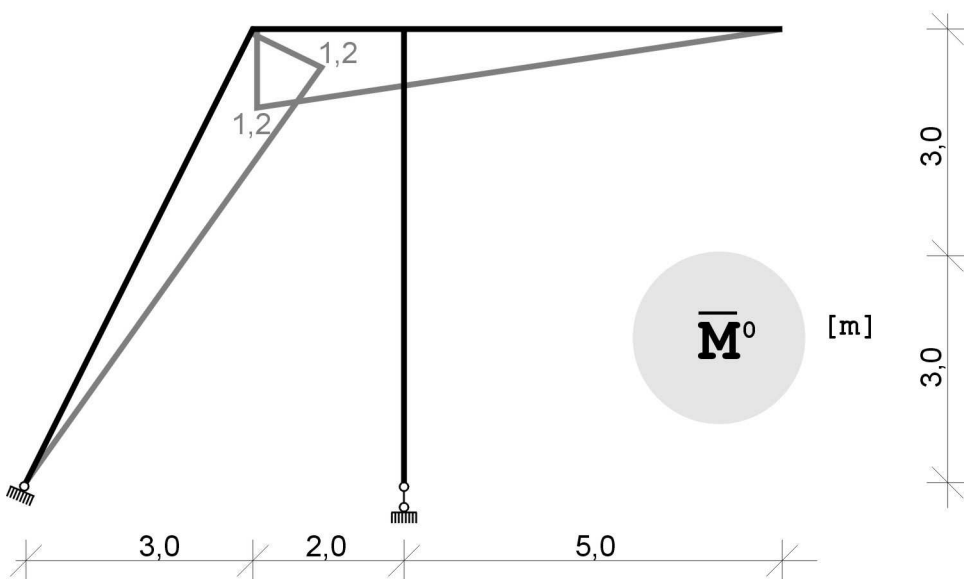
$$\bar{\delta}^T H_B = \int_s \frac{M_2^0 M_p^n}{EI} d_s = 0$$

$$\begin{aligned} & 1/EI (1/2 * 6 * 6,708 * 2/3 * 16,998) - 1/EI [2/3 * (1,5 * 3^2) / 8 * 6,708 * 1/2 * 6] + \\ & + 1/EI [2/6 (2 * 16,998 * 6 + 2 * 22,19 * 6 + 16,998 * 6 + 22,19 * 6)] - 1/EI (2/3 * (1,5 * 2^2) / 8 * 2 * 6) - \\ & - 1/2,85 EI [3 * 6 (2 * 3 * 79,77 + 2 * 69,54 * 6 + 69,54 * 3 + 79,77 * 6)] - \\ & - 1/2,85 EI (1/2 * 3 * 3 * 2/3 * 79,77) = -0,37/EI < 1/EI \sim 0 \end{aligned}$$

Przemieszczenie pionowe punktu K



Wykres momentów od obciążenia wirtualnego



$$\bar{1} \cdot V_K = \int_s \frac{\bar{M}^0 M_p^n}{EI} ds = -1/EI (1/2 \cdot 16,998 \cdot 6,708 \cdot 2/3 \cdot 1,2) + 1/EI [2/3 \cdot (1,5 \cdot 3^2)/8 \cdot 6,708 \cdot 1/2 \cdot 1,2] - 1/EI [2/6 (2 \cdot 16,998 \cdot 1,2 + 2 \cdot 22,19 \cdot 0,86 + 16,998 \cdot 0,86 + 22,19 \cdot 1,2)] + 1/EI [2/3 \cdot (1,5 \cdot 2^2)/8 \cdot 2 \cdot 1/2 \cdot (1,2 + 0,86)] + 1/EI [5/6 (2 \cdot 22,19 \cdot (-0,86) + 2 \cdot (-20) \cdot 0 + 22,19 \cdot 0 + (-20) \cdot (-0,86))] = -98,1093/EI$$

$$EI = 1174,53 \text{ kNm}^2$$

$$V_K = -98,1093/1174,53 = -0,0835 \text{ m} = \underline{\underline{-8,35 \text{ cm}}}$$