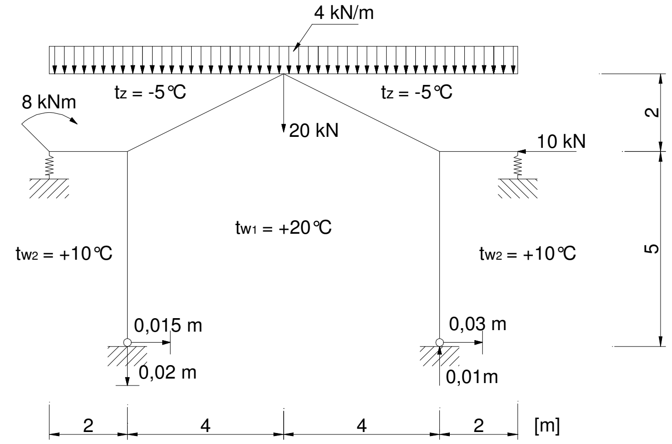
METODA SIŁ - RAMA

Schemat układu:



Przyjęto następujące przekroje prętów:

- słupek ⇒ I200 ⇒ I_1 = 2140 cm^4, h = 20 cm = 0,20 m
- rygiel ⇒ I220 ⇒ I_2 = 3060 cm^4, h = 22 cm = 0,22 m

E = 205 GPa = 20500 kN/cm^2

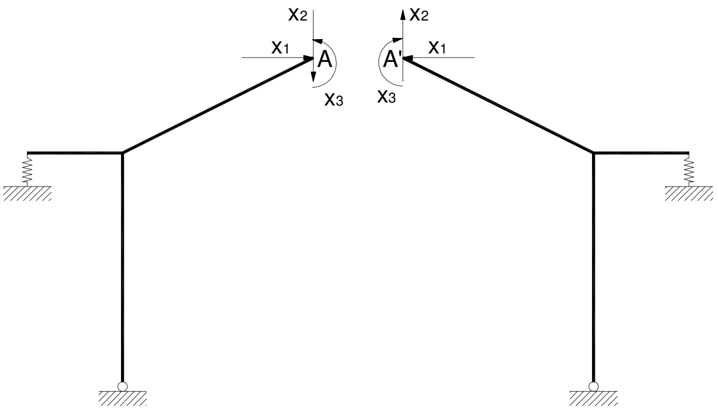
$$EI_1 = 20500 \cdot 2140 = 4387\,kNm^2$$

$$EI_2 = 20500 \cdot 3060 = 6273\,kNm^2$$

$k = aEI_1 \quad a = \frac{1}{5}\left[\frac{1}{m^3}\right]$ $\qquad k = \frac{1}{5} \cdot 4387 = 877{,}4\,{}^{kN}\!/_{m}$

SIŁY WEWNĘTRZNE OD OBCIĄŻENIA ZEWNĘTRZNEGO

Układ podstawowy:



Układ równań kanonicznych:

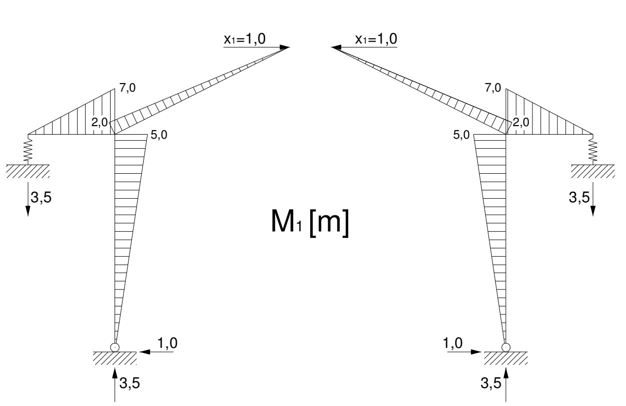
$\begin{cases} \Delta H = 0 \\ \Delta V = 0 \\ \Delta M = 0 \end{cases}$ wzajemne przem.poziome przekrojów A i A'; wzajemne przem. pionowe przekrojów A i A'; wzajemny obrót przekrojów A i A'

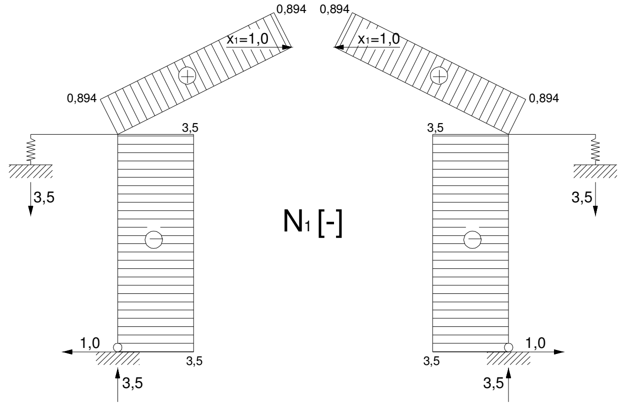
$$\begin{cases} \delta_{11} x_1 + \delta_{12} x_2 + \delta_{13} x_3 + \delta_{1P} = 0 \\ \delta_{21} x_1 + \delta_{22} x_2 + \delta_{23} x_3 + \delta_{2P} = 0 \\ \delta_{31} x_1 + \delta_{32} x_2 + \delta_{33} x_3 + \delta_{3P} = 0 \end{cases}$$

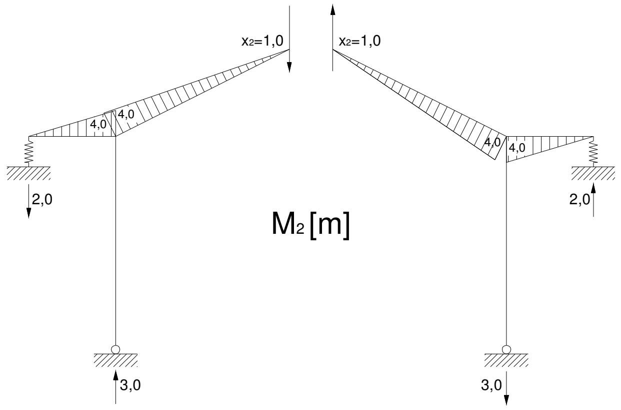
Wyznaczanie współczynników równań kanonicznych:

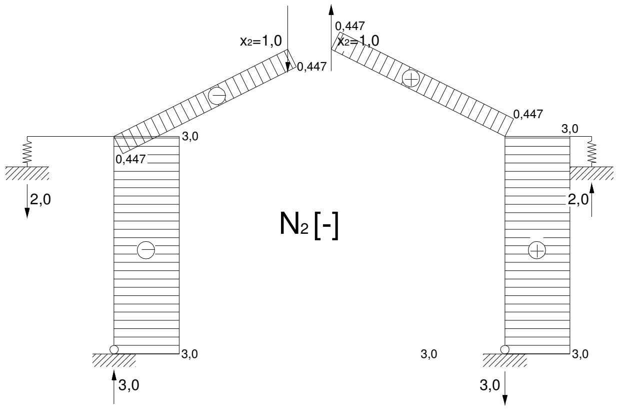
$$\delta_{ik} = \sum \int_s \frac{M_i M_k}{EI} ds + \sum R_i R_k \frac{1}{k}$$

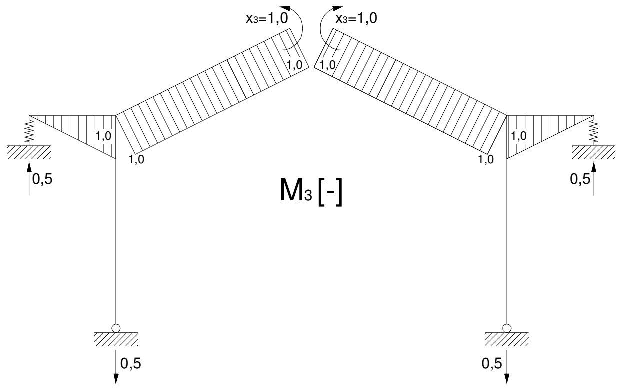
$$\delta_{iP} = \sum \int_s \frac{M_i M_P}{EI} ds + \sum R_i R_k \frac{1}{k}$$

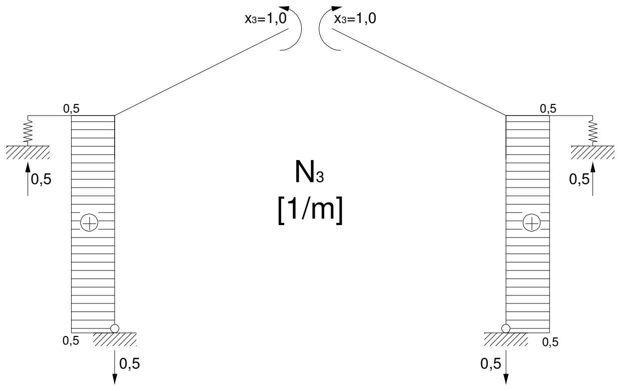

x1=1,0
x1=1,0
7,0
2,0
5,0
7,0
2,0
5,0
3,5
3,5
M1 [m]
1,0
1,0
3,5
3,5

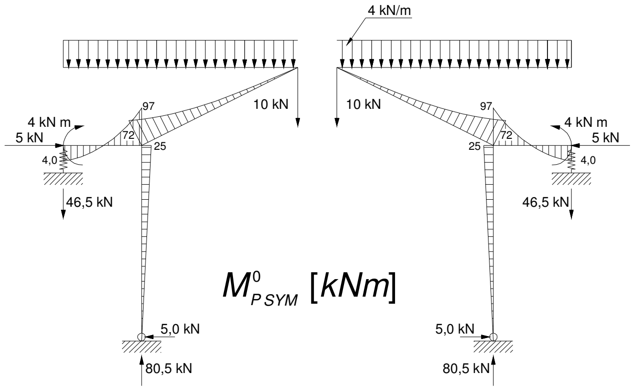

0,894
0,894
x1=1,0
x1=1,0
0,894
0,894
3,5
3,5
3,5
3,5
N1 [-]
1,0
1,0
3,5
3,5
3,5
3,5

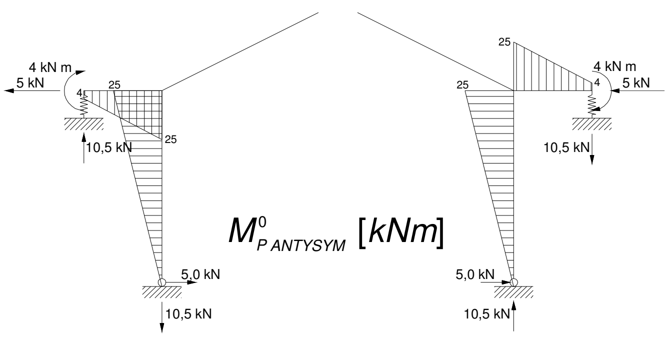

x2=1,0
x2=1,0
4,0
4,0
4,0
4,0
2,0
2,0
M2 [m]
3,0
3,0


x2=1,0
0,447
x2=1,0
0,447
0,447
0,447
3,0
3,0
2,0
2,0
N2 [-]
3,0
3,0
3,0
3,0
3,0


x3=1,0
x3=1,0
1,0
1,0
1,0
1,0
1,0
1,0
0,5
0,5
M3 [-]
0,5
0,5


x3=1,0
x3=1,0
0,5
0,5
0,5
0,5
N3
[1/m]
0,5
0,5
0,5
0,5


4 kN/m
10 kN
10 kN
97
97
4 kN m
5 kN
72
25
4,0
4 kN m
5 kN
72
25
4,0
46,5 kN
46,5 kN
$M^0_{P\,SYM}$ [kNm]
5,0 kN
5,0 kN
80,5 kN
80,5 kN


25
4 kN m
5 kN
25
4
25
4 kN m
5 kN
4
25
10,5 kN
10,5 kN
$M^0_{P\,ANTYSYM}$ [kNm]
5,0 kN
5,0 kN
10,5 kN
10,5 kN

$$\delta_{11}=\frac{2}{EI_1}\left[\frac{1}{2}\cdot5\cdot5\cdot\frac{2}{3}\cdot5\right]+\frac{2}{EI_2}\left[\left(\frac{1}{2}\cdot2\cdot7\cdot\frac{2}{3}\cdot7\right)+\left(\frac{1}{2}\cdot2\sqrt{5}\cdot2\cdot\frac{2}{3}\cdot2\right)\right]+\frac{2}{k}[3{,}5\cdot3{,}5]=\underline{0{,}0592350}\quad\left[\frac{m}{kN}\right]$$

$$\delta_{22}=\frac{2}{EI_2}\left[\left(\frac{1}{2}\cdot2\cdot4\cdot\frac{2}{3}\cdot4\right)+\left(\frac{1}{2}\cdot2\sqrt{5}\cdot4\cdot\frac{2}{3}\cdot4\right)\right]+\frac{2}{k}[2\cdot2]=\underline{0{,}0201231}\quad\left[\frac{m}{kN}\right]$$

$$\delta_{33}=\frac{2}{EI_2}\left[\left(\frac{1}{2}\cdot2\cdot1\cdot\frac{2}{3}\cdot1\right)+\left(2\sqrt{5}\cdot1\cdot1\right)\right]+\frac{2}{k}[0{,}5\cdot0{,}5]=\underline{0{,}0022083}\quad\left[\frac{1}{kNm}\right]$$

$$\delta_{12}=\delta_{21}=0\quad\left[\frac{m}{kN}\right]$$

$$\delta_{23}=\delta_{32}=0\quad\left[\frac{1}{kN}\right]$$

$$\delta_{13}=\delta_{31}=\frac{2}{EI_2}\left[\left(-\frac{1}{2}\cdot2\cdot7\cdot\frac{2}{3}\cdot1\right)+\left(-\frac{1}{2}\cdot2\sqrt{5}\cdot2\cdot1\right)\right]+\frac{2}{k}[(-3{,}5\cdot0{,}5)]=\underline{-0{,}0069028}\quad\left[\frac{1}{kN}\right]$$

$$\delta_{1P}=\frac{2}{EI_1}\left(\frac{1}{2}\cdot5\cdot25\cdot\frac{2}{3}\cdot5\right)+\frac{2}{EI_2}\left[\left(\frac{2\sqrt{5}}{6}(2\cdot97\cdot7-4\cdot7)-\frac{2}{3}\cdot2\cdot\frac{4\cdot2^2}{8}\cdot\frac{1}{2}\cdot7\right)+\left(\frac{1}{2}\cdot2\sqrt{5}\cdot72\cdot\frac{2}{3}\cdot2-\frac{2}{3}\cdot2\sqrt{5}\cdot\frac{4\cdot4^2}{8}\cdot\frac{1}{2}\cdot2\right)\right]+\frac{2}{k}\cdot[3{,}5\cdot46{,}5]=\underline{0{,}6651665}\quad[m]$$

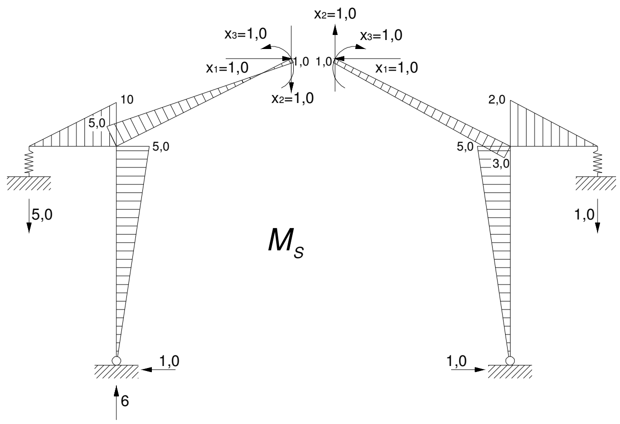
$$\delta_{2P}=\frac{2}{EI_2}\left[\frac{2}{6}(-2\cdot4\cdot25-4\cdot4)\right]+\frac{2}{k}[(-10{,}5\cdot2)]=\underline{-0{,}0708242}\quad[m]$$

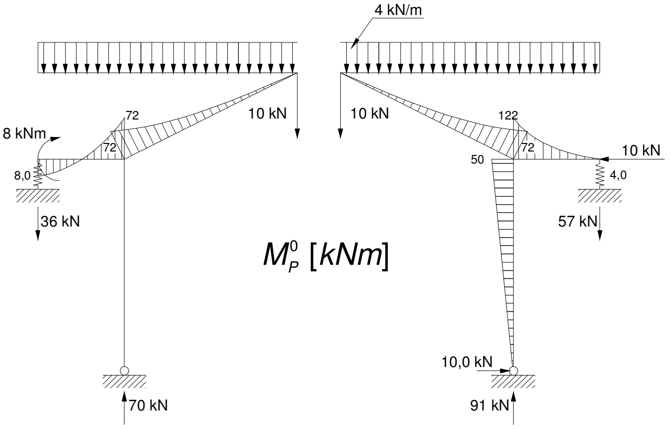
$$\delta_{3P}=\frac{2}{EI_2}\left[\left(\frac{2}{6}(-2\cdot97\cdot1+4\cdot1)+\frac{2}{3}\cdot2\cdot\frac{4\cdot2^2}{8}\cdot\frac{1}{2}\cdot1\right)+\left(-\frac{1}{2}\cdot2\sqrt{5}\cdot72\cdot1+\frac{2}{3}\cdot2\sqrt{5}\cdot\frac{4\cdot(4)^2}{8}\cdot1\right)\right]+\frac{2}{k}[(-46{,}5\cdot0{,}5)]=\underline{-0{,}1164904}\quad[-]$$

Sprawdzenie globalne współczynników δ_{ik}, δ_{iP} :

$$\delta_{SS}=\sum\int_S\frac{M_{SS}^2}{EI}ds\qquad\qquad\delta_{SP}=\sum\int_S\frac{M_S M_P^0}{EI}ds$$

$$\delta_{SS}=\sum_{i=1}^{n}\sum_{k=1}^{n}\delta_{ik}\qquad\qquad\delta_{SP}=\sum_{i=1}^{n}\delta_{iP}\qquad\qquad M_S=M_1+M_2+M_3$$





$$\delta_{SS}=\frac{2}{EI_1}\left[\frac{1}{2}\cdot5\cdot5\cdot\frac{2}{3}\cdot5\right]+\frac{1}{EI_2}\left[\left(\frac{1}{2}\cdot2\cdot10\cdot\frac{2}{3}\cdot10\right)+\frac{2\sqrt{5}}{6}(2\cdot5\cdot5+2\cdot1\cdot1-2\cdot5\cdot1)+\frac{2\sqrt{5}}{6}(2\cdot1\cdot1+2\cdot3\cdot3+2\cdot1\cdot3)+\right.$$

$$\left.+\left(\frac{1}{2}\cdot2\cdot2\cdot\frac{2}{3}\cdot2\right)\right]+\frac{1}{k}[5\cdot5+1\cdot1]=\underline{0{,}067760}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{n}\delta_{ik}=\delta_{11}+\delta_{22}+\delta_{33}+2\cdot\delta_{12}+2\cdot\delta_{13}+2\cdot\delta_{23}=0{,}059235+0{,}020123+0{,}002208+2\cdot(-0{,}006903)=\underline{0{,}067760}$$

$$\delta_{SP}=\frac{1}{EI_1}\left(\frac{1}{2}\cdot5\cdot5\cdot\frac{2}{3}\cdot50\right)+\frac{1}{EI_2}\left[\frac{1}{2}\cdot2\cdot10\cdot\left(\frac{2}{3}\cdot72-\frac{1}{3}\cdot8\right)-\frac{2}{3}\cdot2\cdot\frac{4\cdot2^2}{8}\cdot\frac{1}{2}\cdot10+\frac{1}{2}\cdot2\sqrt{5}\cdot72\cdot\left(\frac{2}{3}\cdot5-\frac{1}{3}\right)-\frac{2}{3}\cdot2\sqrt{5}\cdot\frac{4\cdot4^2}{8}\cdot\frac{1}{2}\cdot(5-1)+\right]$$

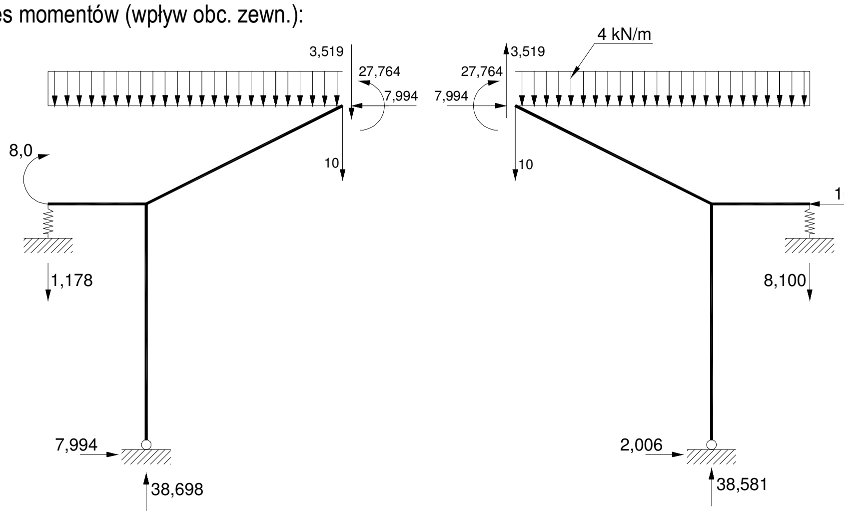
$$+\frac{1}{EI_2}\left[-\frac{1}{2}\cdot2\sqrt{5}\cdot72\cdot\left(\frac{2}{3}\cdot3+\frac{1}{3}\right)+\frac{2}{3}\cdot2\sqrt{5}\cdot\frac{4\cdot4^2}{8}\cdot\frac{1}{2}\cdot(3+1)+\frac{1}{2}\cdot2\cdot2\cdot\frac{2}{3}\cdot122-\frac{2}{3}\cdot2\cdot\frac{4\cdot2^2}{8}\cdot1\right]+\frac{1}{k}\cdot[5\cdot36+1\cdot57]=\underline{0.477851}$$

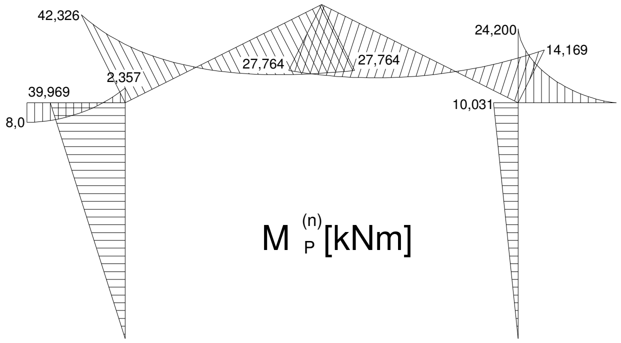
$$\sum_{i=1}^{n}\delta_{iP}=\delta_{1P}+\delta_{2P}+\delta_{3P}=0{,}665167-0{,}070824-0{,}116490=\underline{0{,}477851}$$

$$\begin{cases} 0{,}059235X_1 + 0X_2 - 0{,}006903X_3 = -0{,}665167 \\ 0X_1 + 0{,}020123X_2 + 0X_3 = 0{,}070824 \\ -0{,}006903X_1 + 0X_2 + 0{,}002208X_3 = 0{,}116490 \end{cases}$$

$$\begin{cases} X_1 = -7{,}994\ kN \\ X_2 = 3{,}520\ kN \\ X_3 = 27{,}764\ kNm \end{cases}$$

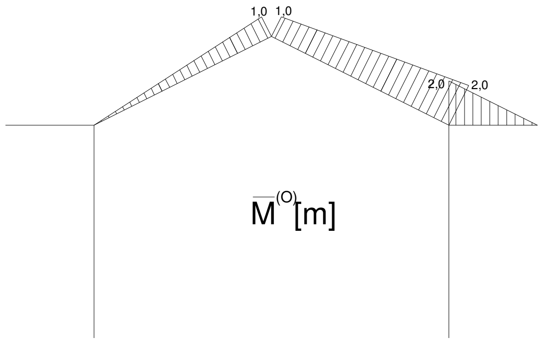
Ostateczny wykres momentów (wpływ obc. zewn.):





Sprawdzenie kinematyczne:

$$1{,}0 \cdot V_k = \sum \int_S \frac{\overline{M}^{(0)} M_P^{(n)}}{EI} ds$$

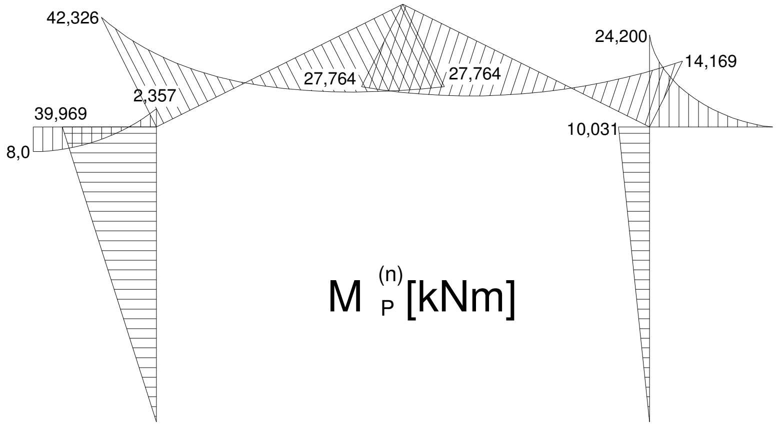


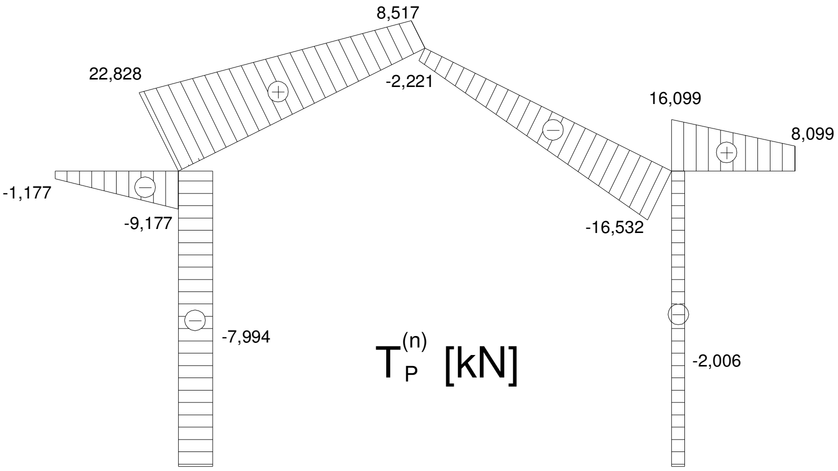
$$1{,}0 \cdot V_k = \frac{1}{EI_2}\left[\frac{2\sqrt{5}}{6}\left(-2\cdot 27{,}764\cdot 1 + 1\cdot 42{,}326\right) - \frac{2}{3}\cdot 2\sqrt{5}\cdot\frac{4\cdot 4^2}{8}\cdot\frac{1}{2}\cdot 1 + \frac{2\sqrt{5}}{6}\left(-2\cdot 27{,}764\cdot 1 + 2\cdot 14{,}169\cdot 2 - 27{,}764\cdot 2 + 14{,}169\cdot 1\right) - \frac{2}{3}\cdot 2\sqrt{5}\cdot\frac{4\cdot 4^2}{8}\cdot 1{,}5\right] +$$

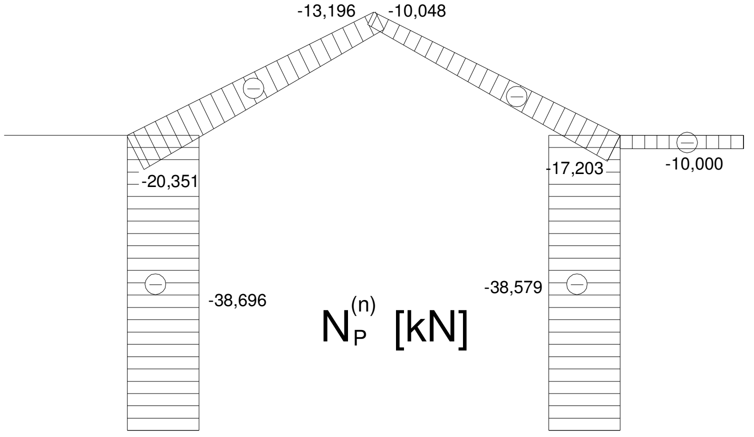
$$+\frac{1}{2}\cdot 2\cdot 24{,}200\cdot\frac{2}{3}\cdot 2 - \frac{2}{3}\cdot 2\cdot\frac{4\cdot 2^2}{8}\cdot\frac{1}{2}\cdot 2\Bigg] = -0{,}0092321\,\text{m}$$ (przemieszczenie przeciwnie do kierunku siły jednostkowej -> w górę)

$$V_k = R_P^{(n)}\cdot\frac{1}{k} = 8{,}100\cdot\frac{1}{877{,}4} = 0{,}0092321\,\text{m}$$ (ugięcie sprężyny; siła rozciągająca –> przemieszczenie w górę)

Ostateczne wykresy *N*, *M*, *T* (wpływ obc. zewn.):

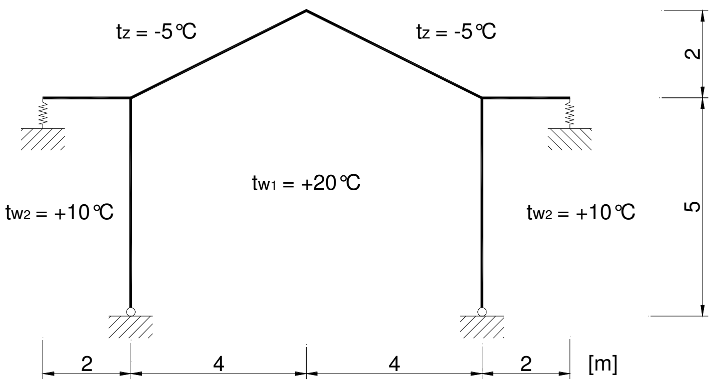






Siły wewnętrzne od wpływu temperatur:

Schemat obciążenia układu:



$t_{W1} = +20^\circ C$
$t_{W2} = +10^\circ C$
$t_Z = -5^\circ C$
$t_m = +8^\circ C$

$|\Delta t_1| = |t_{w1} - t_{w2}| = 10^\circ C$
$|\Delta t_2| = |t_{w1} - t_z| = 25^\circ C$
$|\Delta t_3| = |t_{w2} - t_z| = 15^\circ C$

$t_{01} = 15 - 8 = +7^\circ C$
$t_{02} = 7{,}5 - 8 = -0{,}5^\circ C$
$t_{03} = 2{,}5 - 8 = -5{,}5^\circ C$

$\alpha_t = 1{,}2 \cdot 10^{-5} \left[\frac{1}{^\circ C}\right]$; $h_1 = 0{,}20m$; $h_2 = 0{,}22m$

Przyjęto układ podstawowy jak w poprzednim zadaniu.

Układ równań kanonicznych:

$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{13}x_3 + \delta_{1t} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{23}x_3 + \delta_{2t} = 0 \\ \delta_{31}x_1 + \delta_{32}x_2 + \delta_{33}x_3 + \delta_{3t} = 0 \end{cases}$$

$$\delta_{it} = \sum_S \int M_i \cdot \alpha_t \cdot \frac{|\Delta t|}{h} ds + \sum_S \int N_i \cdot \alpha_t \cdot t_0 ds$$

$$\delta_{1t} = 2 \cdot \left[\left(\frac{1}{2} \cdot 5 \cdot 5 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{10}{0{,}20} \right) - \left(\frac{1}{2} \cdot 2 \cdot 7 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{15}{0{,}22} \right) - \left(\frac{1}{2} \cdot 2\sqrt{5} \cdot 2 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{25}{0{,}22} \right) \right] +$$
$$+ 2 \cdot \left[\left(5 \cdot (-3{,}5) \cdot 1{,}2 \cdot 10^{-5} \cdot 7\right) + \left(2\sqrt{5} \cdot \frac{2}{\sqrt{5}} \cdot 1{,}2 \cdot 10^{-5} \cdot (-0{,}5) \right) \right] = \underline{-0{,}0116393} \quad [m]$$

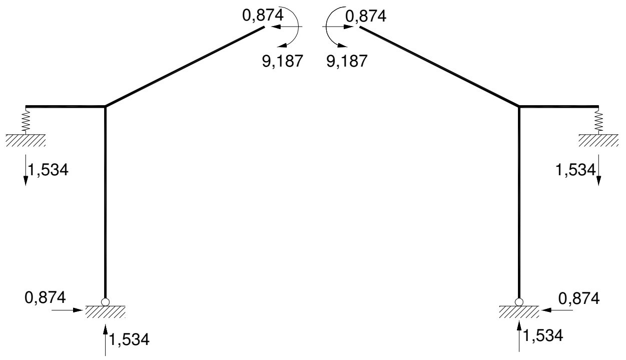
$\delta_{2t} = 0 \quad [m]$

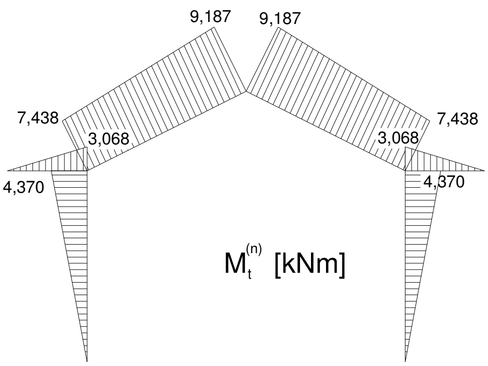
$$\delta_{3t} = 2 \cdot \left[\left(\frac{1}{2} \cdot 2 \cdot 1 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{15}{0{,}22} \right) + \left(2\sqrt{5} \cdot 1 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{25}{0{,}22} \right) \right] + 2 \cdot \left[\left(5 \cdot 0{,}5 \cdot 1{,}2 \cdot 10^{-5} \cdot 7\right) \right] = \underline{0{,}0142531} \quad [-]$$

$$\begin{cases} 0{,}0592350 X_1 + 0 X_2 - 0{,}0069028 X_3 = 0{,}0116393 \\ 0 X_1 + 0{,}020123 X_2 + 0 X_3 = 0 \\ -0{,}0069028 X_1 + 0 X_2 + 0{,}0022083 X_3 = -0{,}0142531 \end{cases}$$

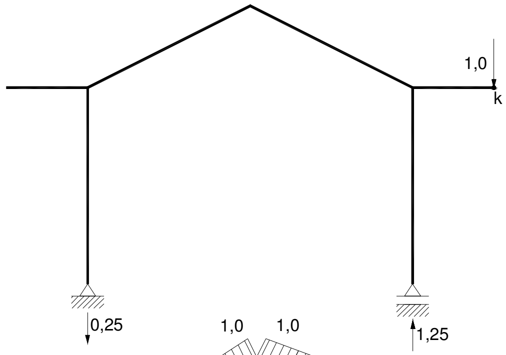
$$\begin{cases} X_1 = -0{,}874 kN \\ X_2 = 0{,}000 kN \\ X_3 = -9{,}187 kNm \end{cases}$$

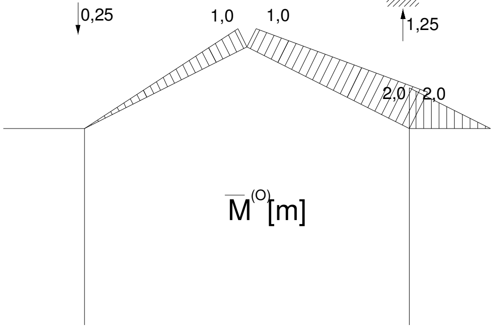
Ostateczny wykres momentów (wpływ temperatury):

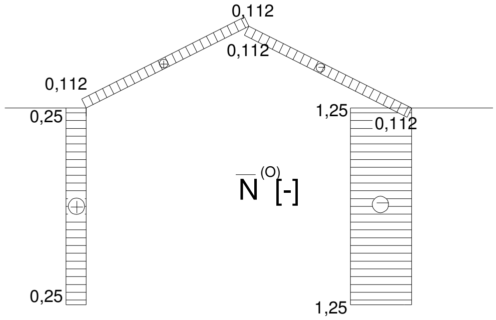




Sprawdzenie kinematyczne:







$$\overline{1{,}0}\cdot V_k = \sum\int_S \frac{\overline{M}^{(o)}M_t^{(n)}}{EI}ds + \sum\int_S \overline{M}^{(0)}\cdot\alpha_t\cdot\frac{\Delta t}{h}ds + \sum\int_S \overline{N}^{(0)}\cdot\alpha_t\cdot t_0 ds$$

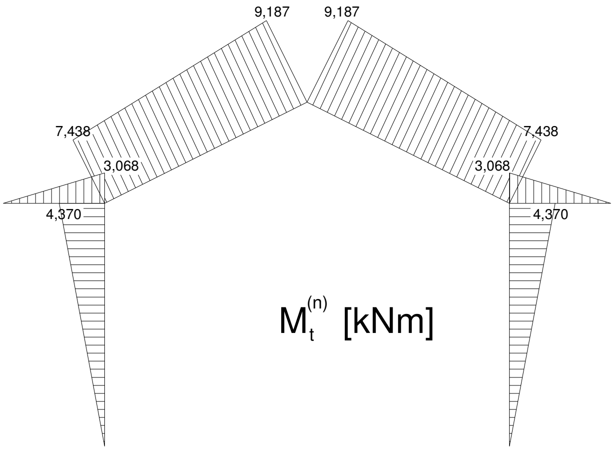
$$\overline{1{,}0}\cdot V_k = \frac{1}{EI_2}\left[\frac{2\sqrt5}{6}(2\cdot 9{,}187\cdot 1+1\cdot 7{,}439)+\frac{2\sqrt5}{6}(2\cdot 9{,}187\cdot 1+2\cdot 7{,}438\cdot 2+9{,}187\cdot 2+7{,}438\cdot 1)+\frac12\cdot 2\cdot 3{,}068\cdot\frac23\cdot 2\right]+$$

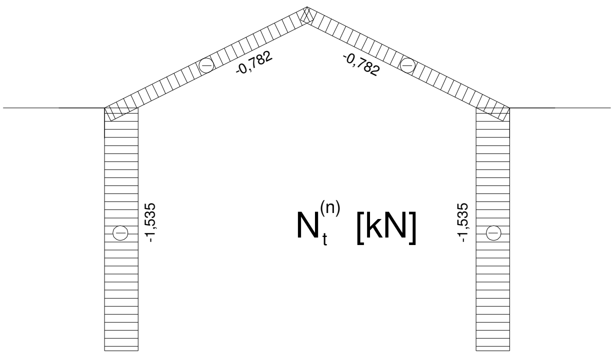
$$+1{,}2\cdot 10^{-5}\cdot\left(-\frac12\cdot 2\sqrt5\cdot 1\cdot\frac{25}{0{,}22}-1{,}5\cdot 2\sqrt5\cdot\frac{25}{0{,}22}-\frac12\cdot 2\cdot 2\cdot\frac{15}{0{,}22}\right)+1{,}2\cdot 10^{-5}\cdot 7\cdot(0{,}25\cdot 5-1{,}25\cdot 5)=\frac{78{,}4410}{EI_2}-0{,}014253=$$

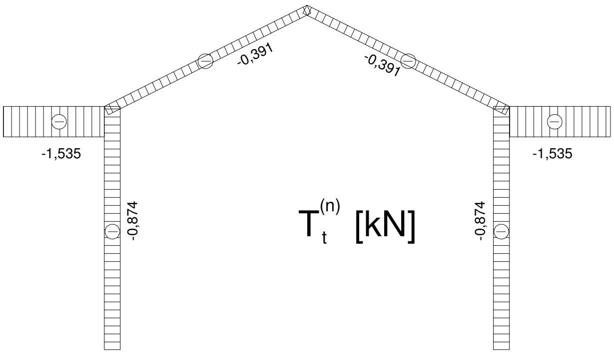
$=0{,}012505-0{,}014253=-0{,}001748m$ (przemieszczenie przeciwnie do kierunku siły jednostkowej -> w górę)

$V_k = R_t^{(n)}\frac{1}{k} = 1{,}534\cdot\frac{1}{877{,}4} = 0{,}001748m$ (ugięcie sprężyny; siła rozciągająca –> przemieszczenie w górę)

Ostateczne wykresy *N*, *M*, *T* (wpływ temperatury):

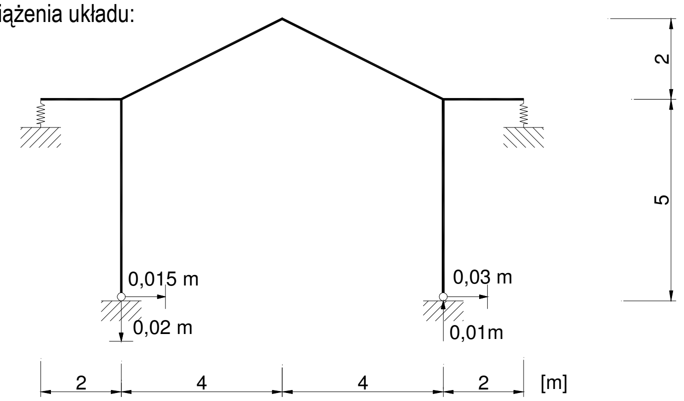






Siły wewnętrzne - wpływ osiadania podpór:

Schemat obciążenia układu:



Przyjęto układ podstawowy jak w poprzednim zadaniu.
Układ równań kanonicznych:

$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{13}x_3 + \delta_{1\Delta} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{23}x_3 + \delta_{2\Delta} = 0 \\ \delta_{31}x_1 + \delta_{32}x_2 + \delta_{33}x_3 + \delta_{3\Delta} = 0 \end{cases} \qquad \delta_{i\Delta} = -\sum R_i \Delta$$

$$\delta_{1\Delta} = -[(-0{,}02 \cdot 3{,}5) + (-0{,}015 \cdot 1{,}0) + (0{,}01 \cdot 3{,}5) + (0{,}03 \cdot 1{,}0)] = \underline{0{,}020} \quad [m]$$

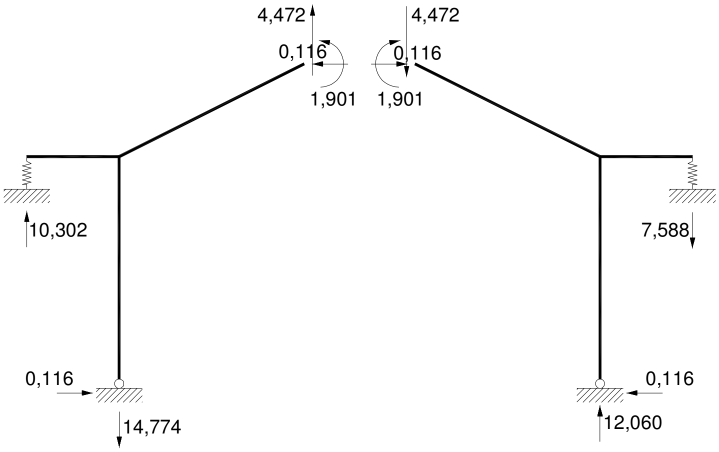
$$\delta_{2\Delta} = -[(-0{,}02 \cdot 3{,}0) + (-0{,}01 \cdot 3{,}0)] = \underline{0{,}090} \quad [m]$$

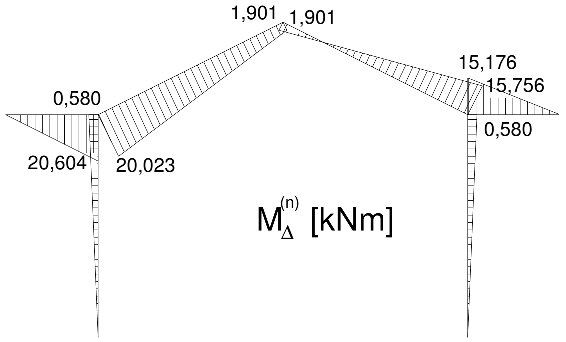
$$\delta_{3\Delta} = -[(0{,}02 \cdot 0{,}5) + (-0{,}01 \cdot 0{,}5)] = \underline{-0{,}005} \quad [-]$$

$$\begin{cases} 0{,}0592350x_1 + 0x_2 - 0{,}0069028x_3 = -0{,}0200 \\ 0x_1 + 0{,}0201231x_2 + 0x_3 = -0{,}0900 \\ -0{,}0069028x_1 + 0x_2 + 0{,}0022083x_3 = 0{,}0050 \end{cases}$$

$$\begin{cases} X_1 = -0{,}116kN \\ X_2 = -4{,}472kN \\ X_3 = 1{,}901kNm \end{cases}$$

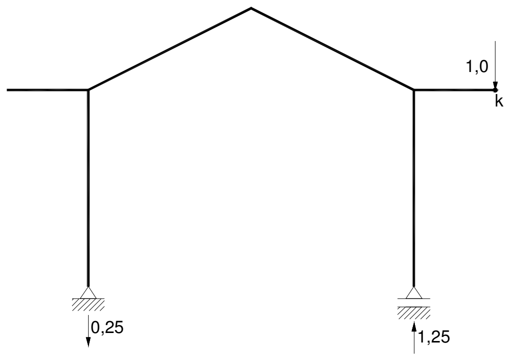
Ostateczny wykres momentów (wpływ osiadania podpór):

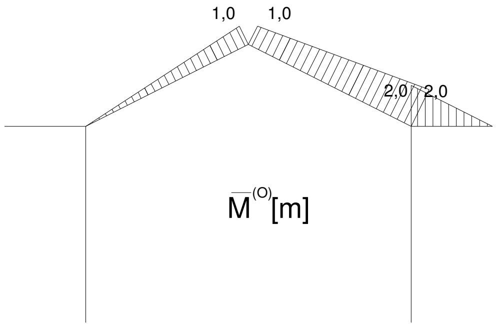




Sprawdzenie kinematyczne:

$$1{,}0 \cdot V_k = \sum_S \int \frac{\overline{M}^{(o)} M_{\Delta}^{(n)}}{EI} ds - \sum_j \overline{R}_i^{(0)} \Delta_j$$





$$1{,}0 \cdot V_k = \frac{1}{EI_2}\left[\frac{2\sqrt{5}}{6}\left(-2\cdot 1{,}901\cdot 1 - 20{,}023\cdot 1\right) + \frac{2\sqrt{5}}{6}\left(-2\cdot 1{,}901\cdot 1 + 2\cdot 15{,}756\cdot 2 - 1{,}901\cdot 2 + 15{,}756\cdot 1\right) + \frac{1}{2}\cdot 2\cdot 15{,}176\cdot \frac{2}{3}\cdot 2\right]$$

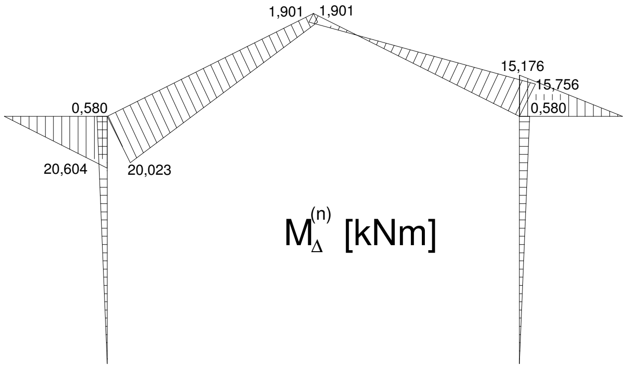
$$-\left[\left(0{,}02\cdot 0{,}25\right) + \left(0{,}01\cdot 1{,}25\right)\right] = \frac{55{,}52684}{EI_2} - 0{,}0175 = \frac{55{,}52684}{6273{,}0} - 0{,}0175 = 0{,}008851 - 0{,}0175 = -0{,}00865\,m$$

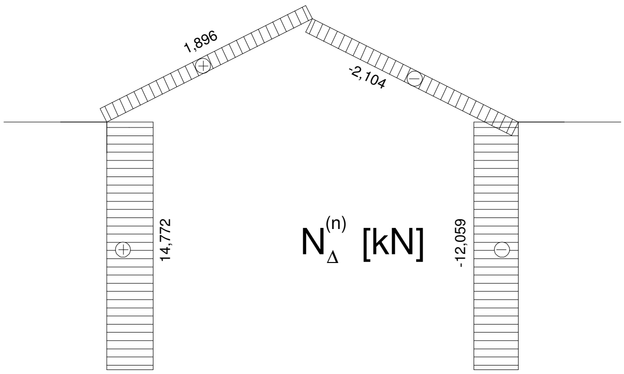
(przemieszczenie przeciwnie do kierunku siły jednostkowej -> w górę)

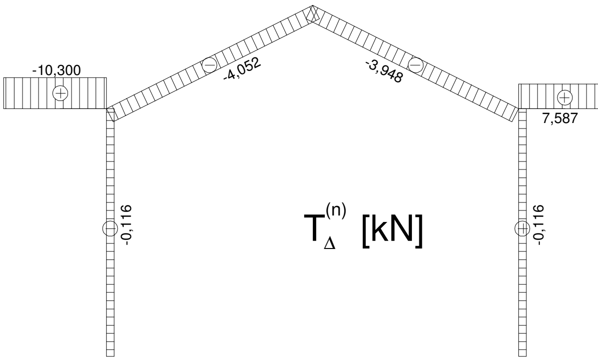
$$V_k = R_{\Delta}^{(n)} \cdot \frac{1}{k} = 7{,}588 \cdot \frac{1}{877{,}4} = 0{,}00865\,m$$

(ugięcie sprężyny; siła rozciągająca –> przemieszczenie w górę)

Ostateczne wykresy *N*, *M*, *T* (wpływ osiadania podpór):

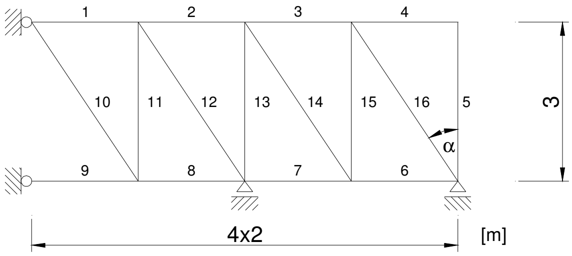






METODA SIŁ - KRATOWNICA

Zadana kratownica:



Pas górny kratownicy doznał ogrzania o t_0 [°C]

Dane:

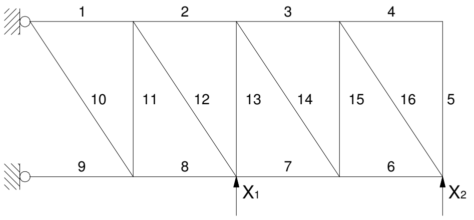
$$t_0 = 30\left[C^\circ\right]$$

$$\alpha_t = 1{,}2 \cdot 10^{-5}\left[\frac{1}{C^\circ}\right]$$

Przyjęto profil prętów I120

A=14,20 cm²
E = 205GPa
EA = 291100 kN

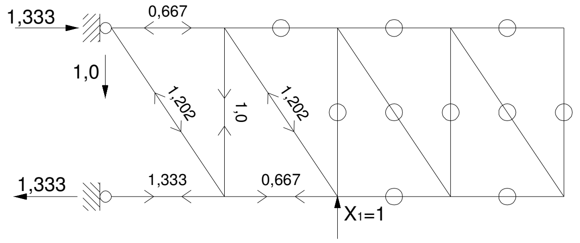
Stopień statycznej niewyznaczalności układu: SSN = 2



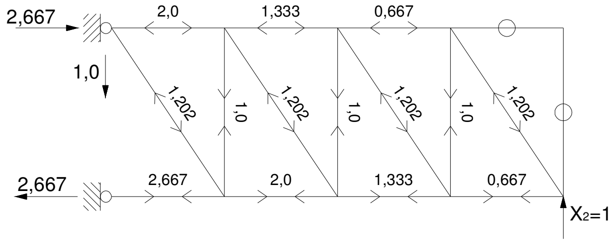
Układ podstawowy:

Układ równań kanonicznych: $\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{1t} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{2t} = 0 \end{cases}$ $\delta_{it} = \sum N_i \cdot t_0 \cdot \alpha_t \cdot l$ $\delta_{ik} = \sum \frac{N_i N_k}{EA} \cdot l$

Stan X_1=1



Stan X_2=1



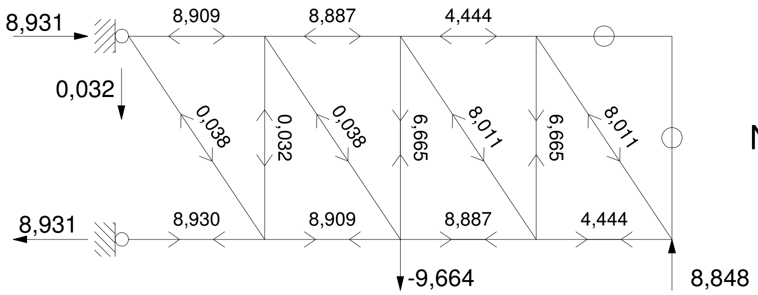
Lp.	l	N_1	N_2	$N_1^2 \cdot l/EA$	$N_2^2 \cdot l/EA$	$N_1 N_2 \cdot l/EA$	δ_{1t}	δ_{2t}	$N^{(n)}$
1	2,000	-0,667	-2,000	0,889	8,000	2,667	-0,00048	-0,00144	-25,934
2	2,000	0,000	-1,333	0,000	3,556	0,000	0	-0,00096	-25,871
3	2,000	0,000	-0,667	0,000	0,889	0,000	0	-0,00048	-12,936
4	2,000	0,000	0,000	0,000	0,000	0,000	0	0	0,000
5	3,000	0,000	0,000	0,000	0,000	0,000	-	-	0,000
6	2,000	0,000	0,667	0,000	0,889	0,000	-	-	12,936
7	2,000	0,000	1,333	0,000	3,556	0,000	-	-	25,871
8	2,000	0,667	2,000	0,889	8,000	2,667	-	-	25,934
9	2,000	1,333	2,667	3,556	14,222	7,111	-	-	25,996
10	3,606	-1,202	-1,202	5,208	5,208	5,208	-	-	-0,112
11	3,000	1,000	1,000	3,000	3,000	3,000	-	-	0,093
12	3,606	-1,202	-1,202	5,208	5,208	5,208	-	-	-0,112
13	3,000	0,000	1,000	0,000	3,000	0,000	-	-	19,404
14	3,606	0,000	-1,202	0,000	5,208	0,000	-	-	-23,320
15	3,000	0,000	1,000	0,000	3,000	0,000	-	-	19,404
16	3,606	0,000	-1,202	0,000	5,208	0,000	-	-	-23,320
				18,749	68,943	25,860	-0,00048	-0,00288	
				$EA \cdot \delta_{11}$	$EA \cdot \delta_{22}$	$EA \cdot \delta_{12}$	δ_{1t}	δ_{2t}	
				6,441E-05	0,000237	8,884E-05			

$$EA \cdot \delta_{11} = 18{,}749 \left[\frac{m}{kN}\right] \qquad \delta_{1t} = -0{,}00048\ [m]$$

$$EA \cdot \delta_{22} = 68{,}943 \left[\frac{m}{kN}\right] \qquad \delta_{2t} = -0{,}00288\ [m]$$

$$EA \cdot \delta_{12} = 25{,}860 \left[\frac{m}{kN}\right]$$

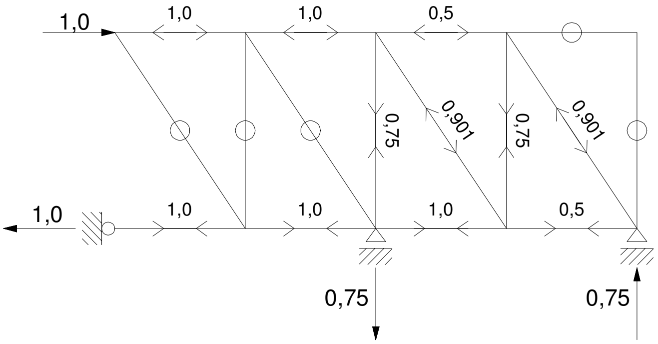
$$\begin{cases}18{,}749x_1 + 25{,}860x_2 = 0{,}00048 \cdot EA \\ 25{,}860x_1 + 68{,}943x_2 = 0{,}00288 \cdot EA\end{cases} \quad \begin{cases}18{,}749x_1 + 25{,}860x_2 = 139{,}728 \\ 25{,}860x_1 + 68{,}943x_2 = 838{,}368\end{cases} \quad \begin{cases}x_1 = -19{,}3104kN \\ x_2 = 19{,}40357kN\end{cases}$$



$N_t^{(n)}$ [kN]

Sprawdzenie kinematyczne:

$$\overline{1{,}0} \cdot V_K = \sum_i \frac{N_t^{(n)} \overline{N}^o}{EA} \cdot l + \sum_i \overline{N}^o \cdot t_0 \cdot \alpha_t \cdot l$$



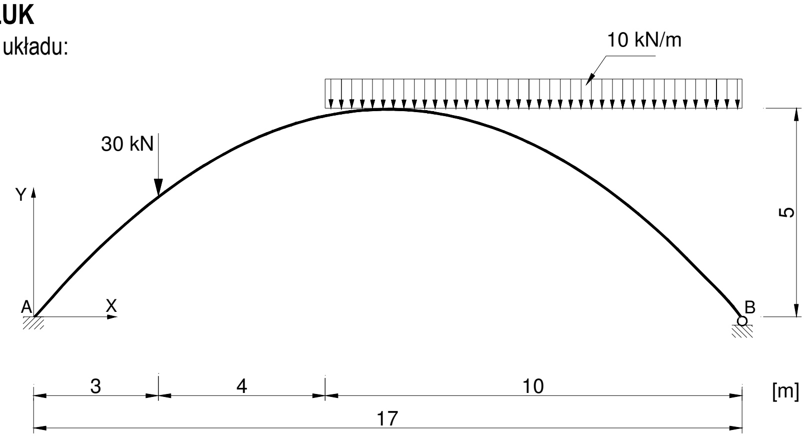
$\overline{N}^{(o)}$ [-]

Lp.	l	$N^{(n)}$	$N^{(o)}$	$N^{(n)} \cdot N^{(o)} \cdot l/EA$	No·α·t·l
1	2,000	-25,934	-1.000	51.867098	-0.00072
2	2,000	-25,871	-1.000	51.742859	-0.00072
3	2,000	-12,936	-0.500	12.935715	-0.00036
4	2,000	0,000	0.000	0	0
5	3,000	0,000	0.000	0	-
6	2,000	12,936	0.500	12.935715	-
7	2,000	25,871	1.000	51.742859	-
8	2,000	25,934	1.000	51.867098	-
9	2,000	25,996	1.000	51.991336	-
10	3,606	-0,112	0.000	0	-
11	3,000	0,093	0.000	0	-
12	3,606	-0,112	0.000	0	-
13	3,000	19,404	0.750	43.658038	-
14	3,606	-23,320	-0.901	75.790623	-
15	3,000	19,404	0.750	43.658038	-
16	3,606	-23,320	-0.901	75.790623	-
				523.98	-0.0018

$$\overline{1{,}0} \cdot V_K = \frac{523{,}98}{EA} - 0{,}0018 = 0{,}00$$

METODA SIŁ - ŁUK

Schemat układu:



Stopień statycznej niewyznaczalności SSN = 2

Łuk ma kształt paraboli o równaniu:

$$y = \frac{4f}{l^2}x(l - x)$$

$f = 5\ \text{m}$

$l = 17\ \text{m}$

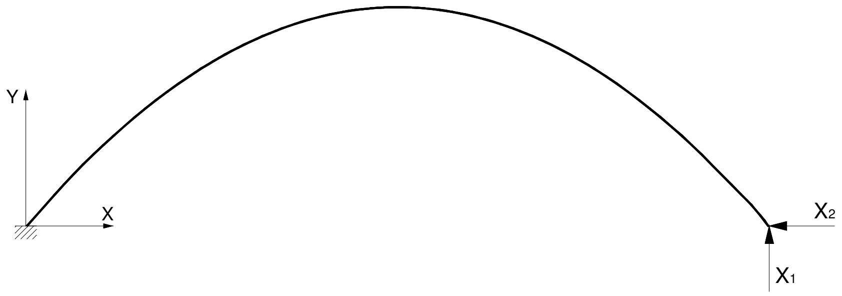
$$y = \frac{4 \cdot 5}{17^2}x(17 - x)$$

$$y = \frac{340}{289}x - \frac{20}{289}x^2$$

Funkcja stycznej w dowolnym punkcie: $\text{tg}\alpha = \frac{dy}{dx} = \frac{340}{289} - \frac{40}{289}x$

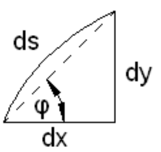
Pomijam wpływ T i N zgodnie z warunkiem $\frac{f}{l} = \frac{5}{17} \geq \frac{1}{5}$

Przyjęty układ podstawowy:



Układ równań kanonicznych:

$$\begin{cases} H = 0 \\ V = 0 \end{cases} \qquad \begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{1P} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{2P} = 0 \end{cases} \qquad \delta_{ik} = \sum \int_s \frac{M_i M_k}{EI} ds \qquad \delta_{iP} = \sum \int_s \frac{M_i M_P}{EI} ds$$



$$\frac{dx}{ds} = \cos\varphi \quad \rightarrow \quad ds = \frac{dx}{\cos\varphi}$$

$$\delta_{ik} = \sum \int_s \frac{M_i M_k}{EI\cos\varphi} dx$$

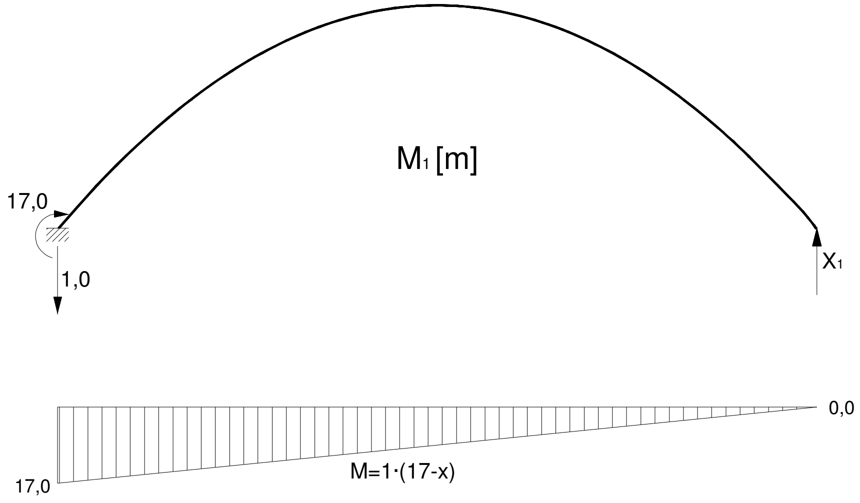
$$\delta_{iP} = \sum \int_s \frac{M_i M_P}{EI\cos\varphi} dx$$

Całki w łuku obliczone zostaną metodą Simsona:

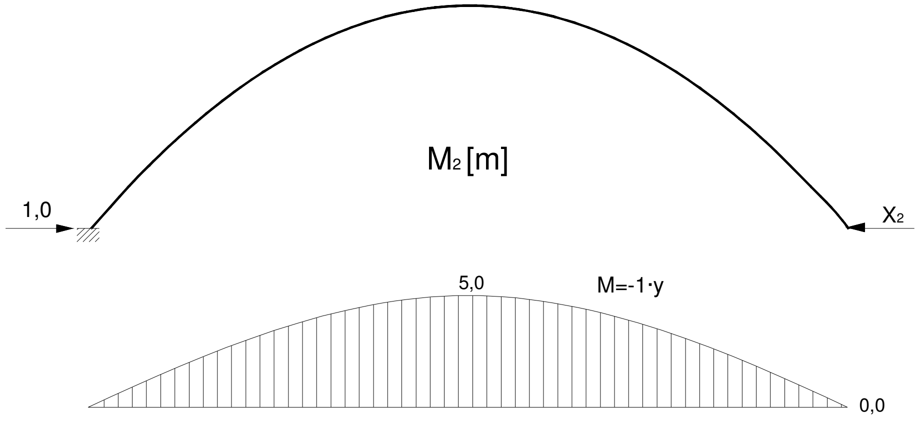
$$\int_a^b f(x) = \frac{\Delta x}{3}\left(f_0 + 4f_1 + 2f_2 + 4f_3 + \ldots + 2f_{n-2} + 4f_{n-1} + f_n\right)$$

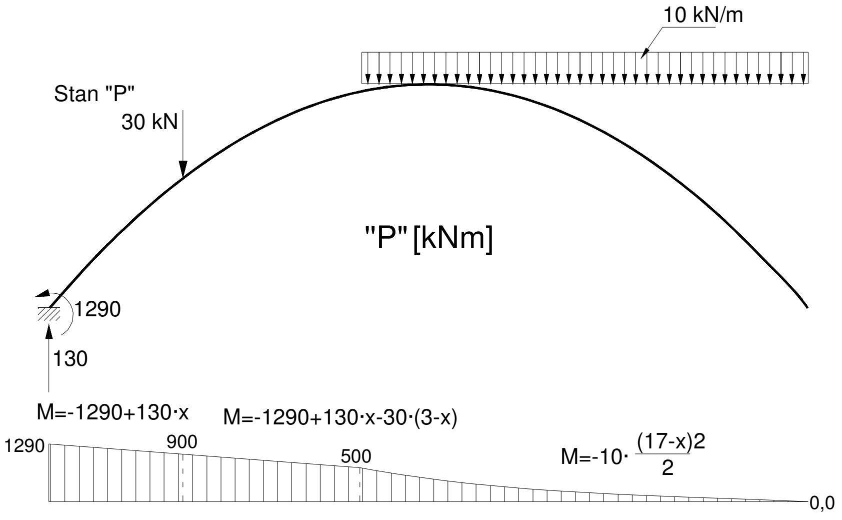
$$f(x) = \frac{M_i M_k}{\cos\varphi}; \qquad \Delta X = 0{,}85\text{m}$$

Stan X_1=1



Stan X_2=1





Lp.	X	Y	$\frac{dy}{dx} = tg\varphi$	φ	$\frac{1}{\cos\varphi}$	Stan M1	Stan M2	Stan MP	$\frac{M_1^2}{\cos\varphi}$	$\frac{M_2^2}{\cos\varphi}$	$\frac{M_1M_2}{\cos\varphi}$	$\frac{M_1M_p}{\cos\varphi}$	$\frac{M_2M_p}{\cos\varphi}$	$M^{(n)}$
0	0	0	1,17647	0,866302	1,544048	17,000	0,00	-1290,000	446,2298	0	0	-33860,96	0	20.6591
1	0,85	0,95	1,05882	0,813962	1,456402	16,150	-0,95	-1179,500	379,8625	1,314402954	-22,34485	-27742,9	1631,935	9.3265
2	1,7	1,8	0,94118	0,755104	1,373249	15,300	-1,80	-1069,000	321,4639	4,449327153	-37,81928	-22460,45	2642,406	3.9203
3	2,55	2,55	0,82353	0,688924	1,295454	14,450	-2,55	-958,500	270,494	8,423688696	-47,73424	-17942,46	3166,3159	4.4403
4	3,4	3,2	0,70588	0,614663	1,224038	13,600	-3,20	-860,000	226,3981	12,53415276	-53,27015	-14316,35	3368,5536	-1.1134
5	4,25	3,75	0,58824	0,531724	1,160181	12,750	-3,75	-775,000	188,602	16,31505021	-55,47117	-11464,04	3371,777	-14.2408
6	5,1	4,2	0,47059	0,439843	1,105194	11,900	-4,20	-690,000	156,5065	19,49561825	-55,23759	-9074,746	3202,8516	-21.4420
7	5,95	4,55	0,35294	0,339293	1,060456	11,050	-4,55	-605,000	129,4844	21,95409567	-53,31709	-7089,415	2919,171	-22.7169
8	6,8	4,8	0,23529	0,231091	1,027309	10,200	-4,80	-520,000	106,8812	23,66919421	-50,29704	-5448,846	2564,1627	-18.0655
9	7,65	4,95	0,11765	0,117109	1,006897	9,350	-4,95	-437,113	88,02542	24,67148476	-46,60169	-4115,188	2178,6292	-9.6004
10	8,5	5	0	0	1	8,500	-5,00	-361,250	72,25	25	-42,5	-3070,625	1806,25	-2.2340
11	9,35	4,95	-0,11765	-0,11711	1,006897	7,650	-4,95	-292,613	58,92611	24,67148476	-38,12866	-2253,924	1458,4212	3.8337
12	10,2	4,8	-0,23529	-0,23109	1,027309	6,800	-4,80	-231,200	47,50276	23,66919421	-33,53136	-1615,094	1140,0662	8.6027
13	11,05	4,55	-0,35294	-0,33929	1,060456	5,950	-4,55	-177,013	37,5428	21,95409567	-28,7092	-1116,898	854,09876	12.0729
14	11,9	4,2	-0,47059	-0,43984	1,105194	5,100	-4,20	-130,050	28,74609	19,49561825	-23,67325	-733,0253	603,66789	14.2444
15	12,75	3,75	-0,58824	-0,53172	1,160181	4,250	-3,75	-90,313	20,95578	16,31505021	-18,49039	-445,3102	392,92079	15.1171
16	13,6	3,2	-0,70588	-0,61466	1,224038	3,400	-3,20	-57,800	14,14988	12,53415276	-13,31754	-240,548	226,39813	14.6912
17	14,45	2,55	-0,82353	-0,68892	1,295454	2,550	-2,55	-32,513	8,423689	8,423688696	-8,423689	-107,402	107,40203	12.9665
18	15,3	1,8	-0,94118	-0,7551	1,373249	1,700	-1,80	-14,450	3,96869	4,449327153	-4,202142	-33,73386	35,71821	9.9430
19	16,15	0,95	-1,05882	-0,81396	1,456402	0,850	-0,95	-3,612	1,052251	1,314402954	-1,176045	-4,472065	4,9981902	5.6209
20	17	0	-1,17647	-0,8663	1,544048	0,000	0,00	0,000	0	0	0	0	0	0.0000
									2021,708	247,0731	-540,964	-123809,8	27064,8	
									$EI\delta_{11}$	$EI\delta_{22}$	$EI\delta_{12}=EI\delta_{21}$	$EI\delta_{1P}$	$EI\delta_{2P}$	

$$\delta_{11} = \sum\int \frac{M_1 M_1}{EI\cos\varphi}dx = \frac{2021{,}708}{EI}$$

$$\delta_{1P} = \sum\int \frac{M_1 M_P}{EI\cos\varphi}dx = \frac{-123809{,}8}{EI}$$

$$\delta_{22} = \sum\int \frac{M_2 M_2}{EI\cos\varphi}dx = \frac{247{,}0731685}{EI}$$

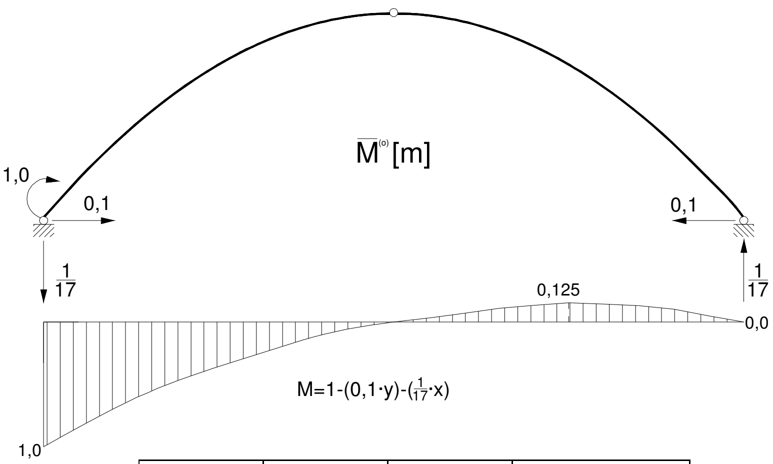
$$\delta_{2P} = \sum\int \frac{M_2 M_P}{EI\cos\varphi}dx = \frac{27064{,}8}{EI}$$

$$\delta_{12} = \sum\int \frac{M_1 M_2}{EI\cos\varphi}dx = \frac{-540{,}964}{EI}$$

$$\begin{cases} 2021{,}708\,x_1 - 540{,}9640205\,x_2 = 123809{,}82 \\ -540{,}9640205\,x_1 + 247{,}0731685\,x_2 = -27064{,}80 \end{cases} \Rightarrow \begin{cases} x_1 = 77{,}09751kN \\ x_2 = 59{,}2626kN \end{cases}$$

$$M^{(n)} = M_1 \cdot X_1 + M_2 \cdot X_2 + M_P$$

Sprawdzenie kinematyczne: $\overline{1}\cdot\varphi = \int \frac{\overline{M}^{(o)} M^{(o)}}{EI\cos\varphi}dx$



$\overline{M}^{(O)}$	$M^{(n)}$	$\frac{1}{\cos\varphi}$	$\frac{\overline{M}^{(o)}M^{(n)}}{EI\cos\varphi}$
1,0000	20,6591	1,5440	31,8986
0,8550	9,3265	1,4564	11,6136
0,7200	3,9203	1,3732	3,8761
0,5950	4,4403	1,2955	3,4226
0,4800	-1,1134	1,2240	-0,6542
0,3750	-14,2408	1,1602	-6,1957
0,2800	-21,4420	1,1052	-6,6353
0,1950	-22,7169	1,0605	-4,6976
0,1200	-18,0655	1,0273	-2,2271
0,0550	-9,6004	1,0069	-0,5317
0,0000	-2,2340	1,0000	0,0000
-0,0450	3,8337	1,0069	-0,1737
-0,0800	8,6027	1,0273	-0,7070
-0,1050	12,0729	1,0605	-1,3443
-0,1200	14,2444	1,1052	-1,8891
-0,1250	15,1171	1,1602	-2,1923
-0,1200	14,6912	1,2240	-2,1579
-0,1050	12,9665	1,2955	-1,7637
-0,0800	9,9430	1,3732	-1,0923
-0,0450	5,6209	1,4564	-0,3684
0,0000	0,0000	1,5440	0,0000
		Σ	**-1,3E-12**

$$EI\cdot\varphi_A = -1{,}3\cdot 10^{-12} \approx 0$$

Wyznaczenie funkcji siły normalnej N i tnącej T od zmiennej φ:

- $x \in \langle 0,3 \rangle$

$T = 52{,}9024 \cdot \cos\varphi - 59{,}2627 \cdot \sin\varphi$

$N = -52{,}9024 \cdot \sin\varphi - 59{,}2627 \cdot \cos\varphi$

- $x \in \langle 3,7 \rangle$

$T = 52{,}9024 \cdot \cos\varphi - 59{,}2627 \cdot \sin\varphi - 30 \cdot \cos\varphi$

$N = -52{,}9024 \cdot \sin\varphi - 59{,}2627 \cdot \cos\varphi + 30 \cdot \sin\varphi$

- $x \in \langle 7,12 \rangle$

$T = 52{,}9024 \cdot \cos\varphi - 59{,}2627 \cdot \sin\varphi - 30 \cdot \cos\varphi - 10 \cdot (x-7) \cdot \cos\varphi$

$N = -529024 \cdot \sin\varphi - 59{,}2627 \cdot \cos\varphi + 30 \cdot \cos\varphi + 10 \cdot (x-7) \cdot \sin\varphi$