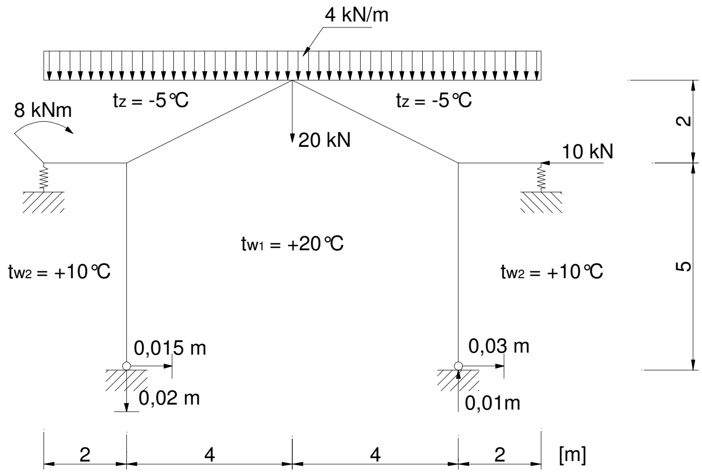
RAMA

Schemat układu:



Przyjęto następujące przekroje prętów:

- słupek ⇒ I200 ⇒ $I_1 = 2140\ cm^4$, h = 20 cm = 0,20 m
- rygiel ⇒ I220 ⇒ $I_2 = 3060\ cm^4$, h = 22 cm = 0,22 m

E = 205 GPa = 20500 kN/cm²

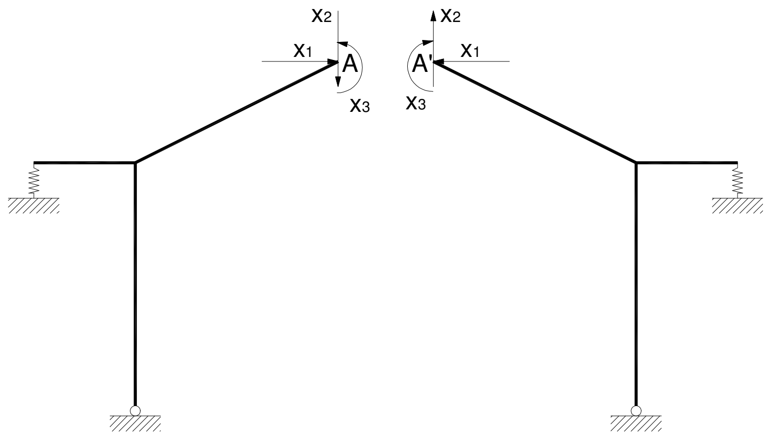
$$EI_1 = 20500 \cdot 2140 = 4387\,kNm^2$$

$$EI_2 = 20500 \cdot 3060 = 6273\,kNm^2$$

$$k = aEI_1 \quad a = \frac{1}{5}\left[\frac{1}{m^3}\right] \qquad k = \frac{1}{5}\cdot 4387 = 877{,}4\,{}^{kN}\!/_{m}$$

SIŁY WEWNĘTRZNE OD OBCIĄŻENIA ZEWNĘTRZNEGO

Układ podstawowy:



Układ równań kanonicznych:

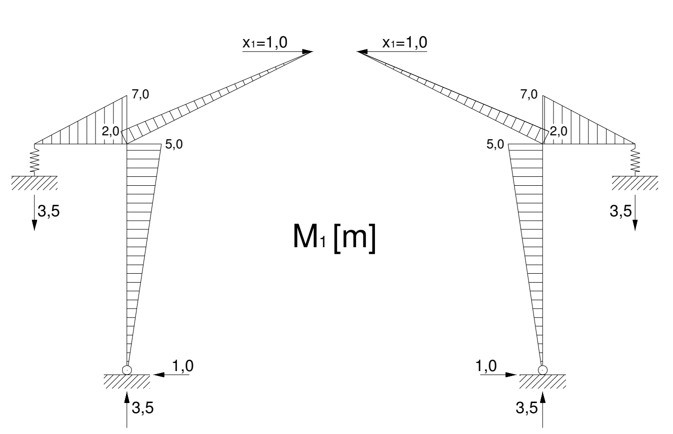
$$\begin{cases} \Delta H = 0 & \text{wzajemne przem.poziome przekrojów A i A'} \\ \Delta V = 0 & \text{wzajemne przem. pionowe przekrojów A i A'} \\ \Delta M = 0 & \text{wzajemny obrót przekrojów A i A'} \end{cases}$$

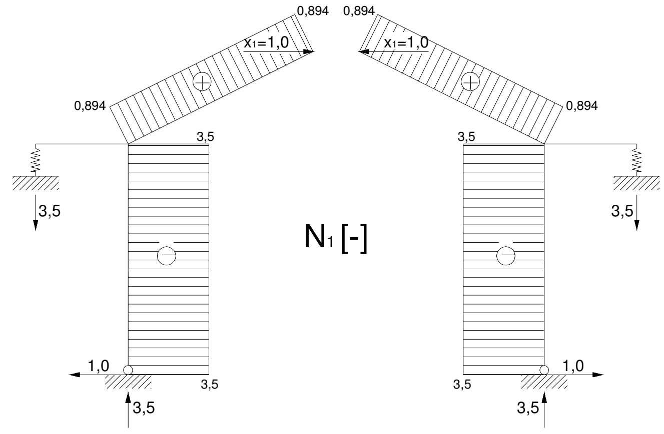
$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{13}x_3 + \delta_{1P} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{23}x_3 + \delta_{2P} = 0 \\ \delta_{31}x_1 + \delta_{32}x_2 + \delta_{33}x_3 + \delta_{3P} = 0 \end{cases}$$

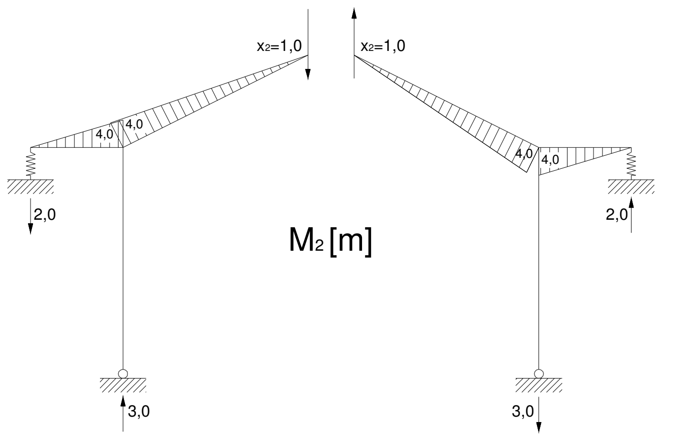
Wyznaczanie współczynników równań kanonicznych:

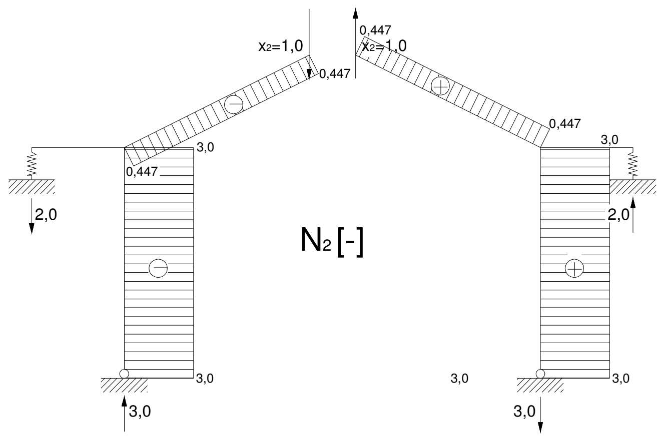
$$\delta_{ik} = \sum \int_s \frac{M_i M_k}{EI} ds + \sum R_i R_k \frac{1}{k}$$

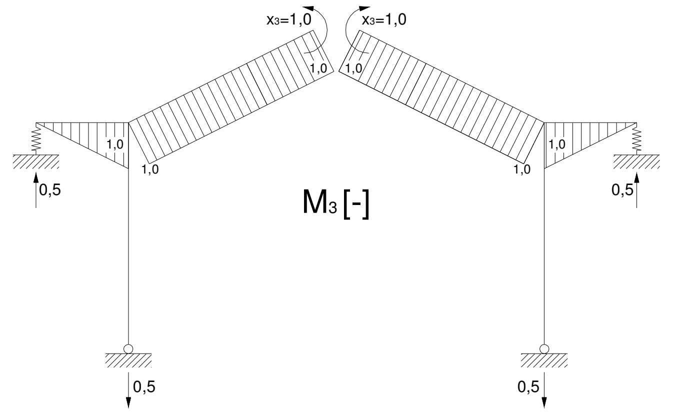
$$\delta_{iP} = \sum \int_s \frac{M_i M_P}{EI} ds + \sum R_i R_k \frac{1}{k}$$

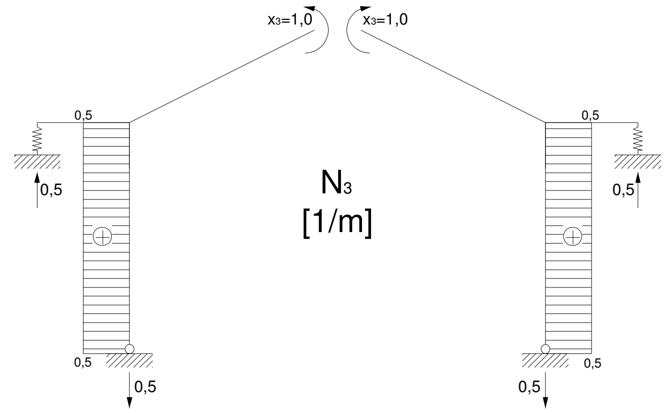

x1=1,0
x1=1,0
7,0
7,0
2,0
2,0
5,0
5,0
3,5
3,5
M1 [m]
1,0
1,0
3,5
3,5

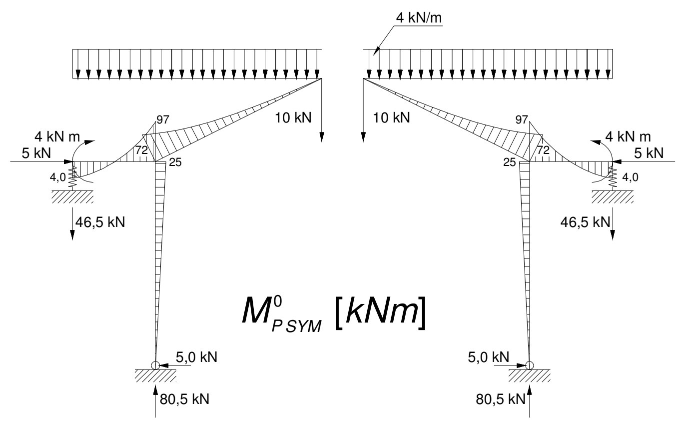

0,894
0,894
x1=1,0
x1=1,0
0,894
0,894
3,5
3,5
3,5
3,5
N1 [-]
1,0
1,0
3,5
3,5
3,5
3,5

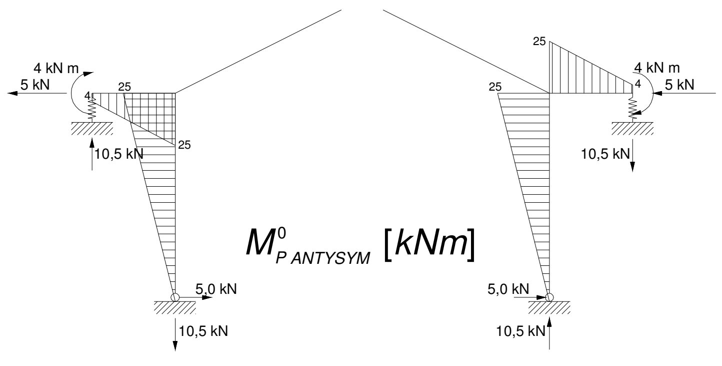

x2=1,0
x2=1,0
4,0
4,0
4,0
4,0
2,0
2,0
M2 [m]
3,0
3,0


x2=1,0
0,447
x2=1,0
0,447
0,447
3,0
0,447
3,0
2,0
2,0
N2 [-]
3,0
3,0
3,0
3,0
3,0


x3=1,0
x3=1,0
1,0
1,0
1,0
1,0
1,0
1,0
0,5
0,5
M3 [-]
0,5
0,5


x3=1,0
x3=1,0
0,5
0,5
0,5
0,5
N3
[1/m]
0,5
0,5
0,5
0,5


4 kN/m
10 kN
10 kN
97
97
72
72
25
25
4 kN m
5 kN
4 kN m
5 kN
4,0
4,0
46,5 kN
46,5 kN
$M^0_{P\,SYM}$ [kNm]
5,0 kN
5,0 kN
80,5 kN
80,5 kN


4 kN m
5 kN
4
25
25
25
25
4 kN m
5 kN
4
10,5 kN
10,5 kN
$M^0_{P\,ANTYSYM}$ [kNm]
5,0 kN
5,0 kN
10,5 kN
10,5 kN

$$\delta_{11} = \frac{2}{EI_1}\left[\frac{1}{2}\cdot 5\cdot 5\cdot \frac{2}{3}\cdot 5\right] + \frac{2}{EI_2}\left[\left(\frac{1}{2}\cdot 2\cdot 7\cdot \frac{2}{3}\cdot 7\right) + \left(\frac{1}{2}\cdot 2\sqrt{5}\cdot 2\cdot \frac{2}{3}\cdot 2\right)\right] + \frac{2}{k}\left[3{,}5\cdot 3{,}5\right] = \underline{0{,}0592350} \quad \left[\frac{m}{kN}\right]$$

$$\delta_{22} = \frac{2}{EI_2}\left[\left(\frac{1}{2}\cdot 2\cdot 4\cdot \frac{2}{3}\cdot 4\right) + \left(\frac{1}{2}\cdot 2\sqrt{5}\cdot 4\cdot \frac{2}{3}\cdot 4\right)\right] + \frac{2}{k}\left[2\cdot 2\right] = \underline{0{,}0201231} \quad \left[\frac{m}{kN}\right]$$

$$\delta_{33} = \frac{2}{EI_2}\left[\left(\frac{1}{2}\cdot 2\cdot 1\cdot \frac{2}{3}\cdot 1\right) + \left(2\sqrt{5}\cdot 1\cdot 1\right)\right] + \frac{2}{k}\left[0{,}5\cdot 0{,}5\right] = \underline{0{,}0022083} \quad \left[\frac{1}{kNm}\right]$$

$$\delta_{12} = \delta_{21} = 0 \quad \left[\frac{m}{kN}\right]$$

$$\delta_{23} = \delta_{32} = 0 \quad \left[\frac{1}{kN}\right]$$

$$\delta_{13} = \delta_{31} = \frac{2}{EI_2}\left[\left(-\frac{1}{2}\cdot 2\cdot 7\cdot \frac{2}{3}\cdot 1\right) + \left(-\frac{1}{2}\cdot 2\sqrt{5}\cdot 2\cdot 1\right)\right] + \frac{2}{k}\left[\left(-3{,}5\cdot 0{,}5\right)\right] = \underline{-0{,}0069028} \quad \left[\frac{1}{kN}\right]$$

$$\delta_{1P} = \frac{2}{EI_1}\left(\frac{1}{2}\cdot 5\cdot 25\cdot \frac{2}{3}\cdot 5\right) + \frac{2}{EI_2}\left[\left(\frac{2\sqrt{5}}{6}\left(2\cdot 97\cdot 7 - 4\cdot 7\right) - \frac{2}{3}\cdot 2\cdot \frac{4\cdot 2^2}{8}\cdot \frac{1}{2}\cdot 7\right) + \left(\frac{1}{2}\cdot 2\sqrt{5}\cdot 72\cdot \frac{2}{3}\cdot 2 - \frac{2}{3}\cdot 2\sqrt{5}\cdot \frac{4\cdot 4^2}{8}\cdot \frac{1}{2}\cdot 2\right)\right] + \frac{2}{k}\cdot\left[3{,}5\cdot 46{,}5\right] = \underline{0{,}6651665} \quad [m]$$

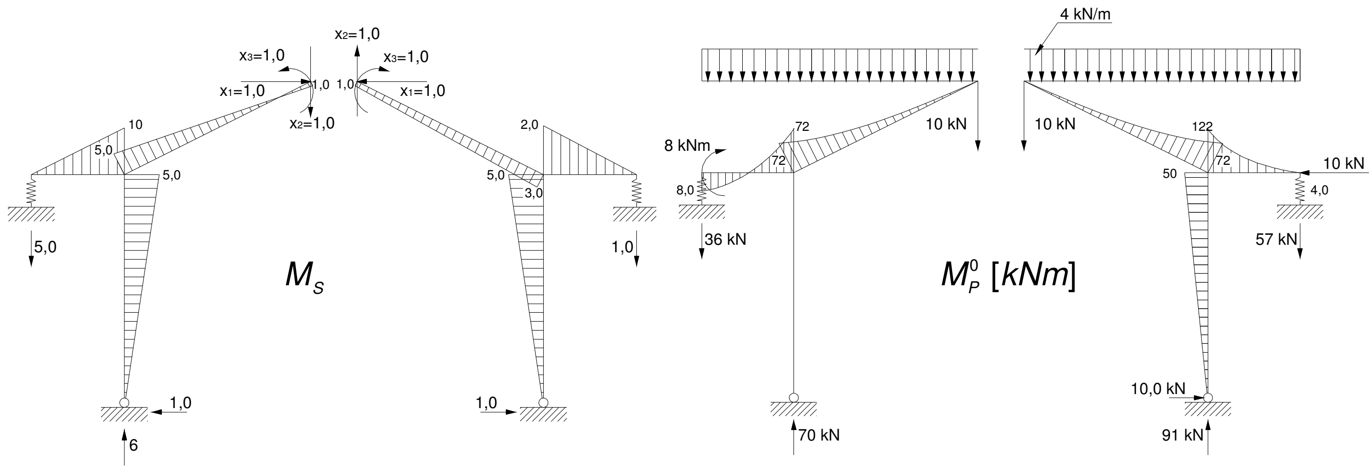
$$\delta_{2P} = \frac{2}{EI_2}\left[\frac{2}{6}\left(-2\cdot 4\cdot 25 - 4\cdot 4\right)\right] + \frac{2}{k}\left[\left(-10{,}5\cdot 2\right)\right] = \underline{-0{,}0708242} \quad [m]$$

$$\delta_{3P} = \frac{2}{EI_2}\left[\left(\frac{2}{6}\left(-2\cdot 97\cdot 1 + 4\cdot 1\right) + \frac{2}{3}\cdot 2\cdot \frac{4\cdot 2^2}{8}\cdot \frac{1}{2}\cdot 1\right) + \left(-\frac{1}{2}\cdot 2\sqrt{5}\cdot 72\cdot 1 + \frac{2}{3}\cdot 2\sqrt{5}\cdot \frac{4\cdot (4)^2}{8}\cdot 1\right)\right] + \frac{2}{k}\left[\left(-46{,}5\cdot 0{,}5\right)\right] = \underline{-0{,}1164904} \quad [-]$$

Sprawdzenie globalne współczynników δ_{ik}, δ_{iP} :

$$\delta_{SS} = \sum\int_S \frac{M_{SS}^2}{EI}ds \qquad \delta_{SP} = \sum\int_S \frac{M_S M_P^0}{EI}ds$$

$$\delta_{SS} = \sum_{i=1}^{n}\sum_{k=1}^{n}\delta_{ik} \qquad \delta_{SP} = \sum_{i=1}^{n}\delta_{iP} \qquad M_S = M_1 + M_2 + M_3$$



$$\delta_{SS} = \frac{2}{EI_1}\left[\frac{1}{2}\cdot 5\cdot 5\cdot \frac{2}{3}\cdot 5\right] + \frac{1}{EI_2}\left[\left(\frac{1}{2}\cdot 2\cdot 10\cdot \frac{2}{3}\cdot 10\right) + \frac{2\sqrt{5}}{6}\left(2\cdot 5\cdot 5 + 2\cdot 1\cdot 1 - 2\cdot 5\cdot 1\right) + \frac{2\sqrt{5}}{6}\left(2\cdot 1\cdot 1 + 2\cdot 3\cdot 3 + 2\cdot 1\cdot 3\right) + \right.$$

$$\left. + \left(\frac{1}{2}\cdot 2\cdot 2\cdot \frac{2}{3}\cdot 2\right)\right] + \frac{1}{k}\left[5\cdot 5 + 1\cdot 1\right] = \underline{0{,}067760}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{n}\delta_{ik} = \delta_{11} + \delta_{22} + \delta_{33} + 2\cdot\delta_{12} + 2\cdot\delta_{13} + 2\cdot\delta_{23} = 0{,}059235 + 0{,}020123 + 0{,}002208 + 2\cdot(-0{,}006903) = \underline{0{,}067760}$$

$$\delta_{SP} = \frac{1}{EI_1}\left(\frac{1}{2}\cdot 5\cdot 5\cdot \frac{2}{3}\cdot 50\right) + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 2\cdot 10\cdot\left(\frac{2}{3}\cdot 72 - \frac{1}{3}\cdot 8\right) - \frac{2}{3}\cdot 2\cdot \frac{4\cdot 2^2}{8}\cdot \frac{1}{2}\cdot 10 + \frac{1}{2}\cdot 2\sqrt{5}\cdot 72\cdot\left(\frac{2}{3}\cdot 5 - \frac{1}{3}\right) - \frac{2}{3}\cdot 2\sqrt{5}\cdot \frac{4\cdot 4^2}{8}\cdot \frac{1}{2}\cdot (5-1) + \right]$$

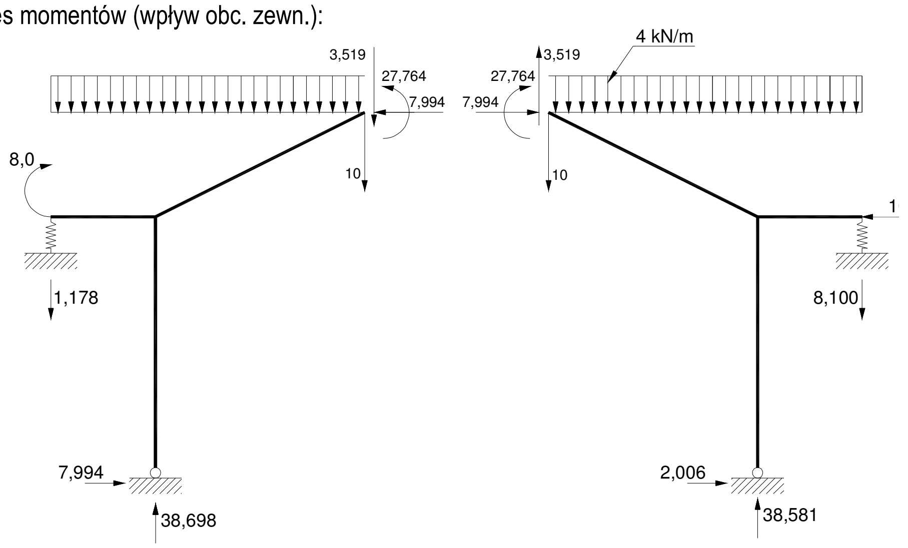
$$+ \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 2\sqrt{5}\cdot 72\cdot\left(\frac{2}{3}\cdot 3 + \frac{1}{3}\right) + \frac{2}{3}\cdot 2\sqrt{5}\cdot \frac{4\cdot 4^2}{8}\cdot \frac{1}{2}\cdot (3+1) + \frac{1}{2}\cdot 2\cdot 2\cdot \frac{2}{3}\cdot 122 - \frac{2}{3}\cdot 2\cdot \frac{4\cdot 2^2}{8}\cdot 1\right] + \frac{1}{k}\cdot\left[5\cdot 36 + 1\cdot 57\right] = \underline{0.477851}$$

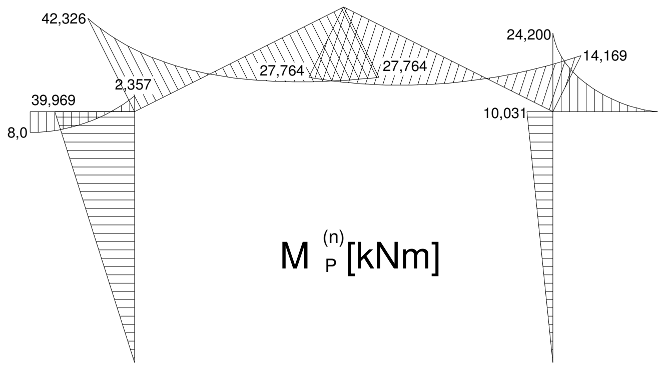
$$\sum_{i=1}^{n}\delta_{iP} = \delta_{1P} + \delta_{2P} + \delta_{3P} = 0{,}665167 - 0{,}070824 - 0{,}116490 = \underline{0{,}477851}$$

$$\begin{cases} 0{,}059235X_1 + 0X_2 - 0{,}006903X_3 = -0{,}665167 \\ 0X_1 + 0{,}020123X_2 + 0X_3 = 0{,}070824 \\ -0{,}006903X_1 + 0X_2 + 0{,}002208X_3 = 0{,}116490 \end{cases}$$

$$\begin{cases} X_1 = -7{,}994\ kN \\ X_2 = 3{,}520\ kN \\ X_3 = 27{,}764\ kNm \end{cases}$$

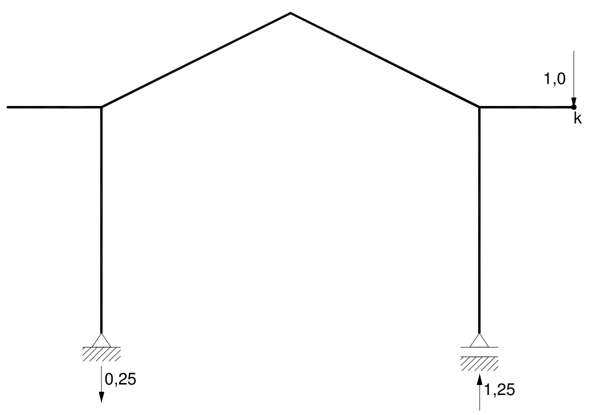
Ostateczny wykres momentów (wpływ obc. zewn.):

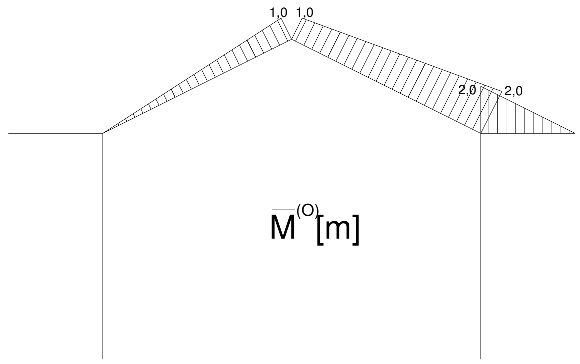




Sprawdzenie kinematyczne:

$$1{,}0 \cdot V_k = \sum \int_S \frac{\overline{M}^{(0)} M_P^{(n)}}{EI} ds$$



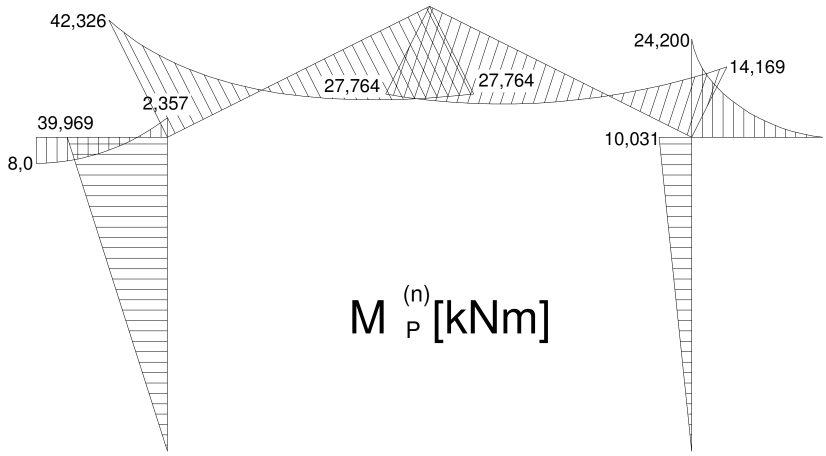


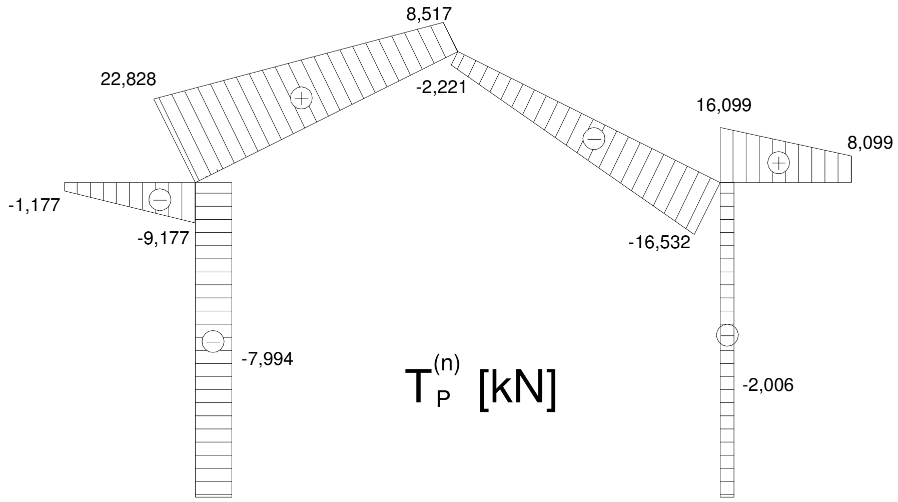
$$1{,}0 \cdot V_k = \frac{1}{EI_2}\left[\frac{2\sqrt{5}}{6}\left(-2\cdot 27{,}764\cdot 1 + 1\cdot 42{,}326\right) - \frac{2}{3}\cdot 2\sqrt{5}\cdot\frac{4\cdot 4^2}{8}\cdot\frac{1}{2}\cdot 1 + \frac{2\sqrt{5}}{6}\left(-2\cdot 27{,}764\cdot 1 + 2\cdot 14{,}169\cdot 2 - 27{,}764\cdot 2 + 14{,}169\cdot 1\right) - \frac{2}{3}\cdot 2\sqrt{5}\cdot\frac{4\cdot 4^2}{8}\cdot 1{,}5\right] +$$

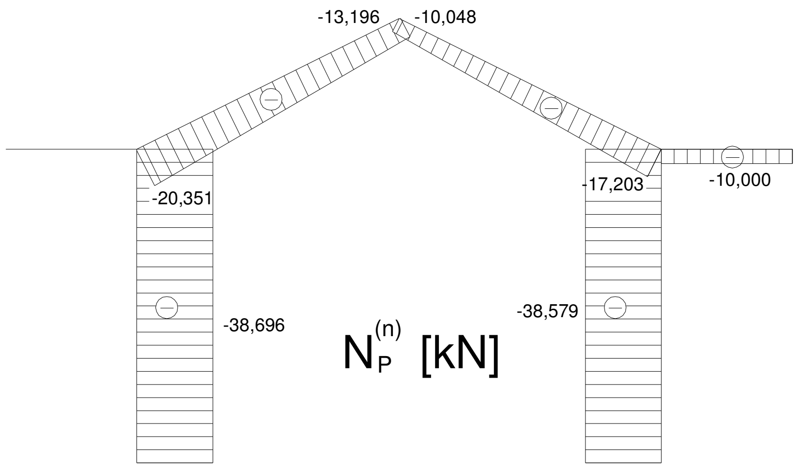
$$+\frac{1}{2}\cdot 2\cdot 24{,}200\cdot\frac{2}{3}\cdot 2 - \frac{2}{3}\cdot 2\cdot\frac{4\cdot 2^2}{8}\cdot\frac{1}{2}\cdot 2\Bigg] = -0{,}0092321\,\text{m}$$ (przemieszczenie przeciwnie do kierunku siły jednostkowej -> w górę)

$$V_k = R_P^{(n)}\cdot\frac{1}{k} = 8{,}100\cdot\frac{1}{877{,}4} = 0{,}0092321\,\text{m}$$ (ugięcie sprężyny; siła rozciągająca –> przemieszczenie w górę)

Ostateczne wykresy *N*, *M*, *T* (wpływ obc. zewn.):

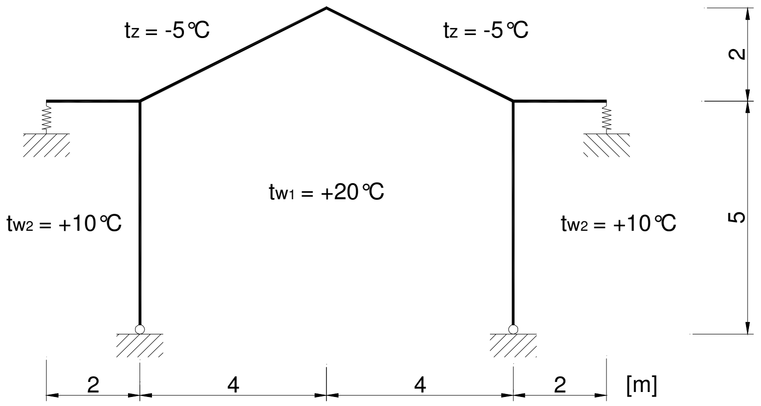






Siły wewnętrzne od wpływu temperatur:

Schemat obciążenia układu:



$t_{W1} = +20^\circ C$

$t_{W2} = +10^\circ C$

$t_Z = -5^\circ C$

$t_m = +8^\circ C$

$|\Delta t_1| = |t_{w1} - t_{w2}| = 10^\circ C$

$|\Delta t_2| = |t_{w1} - t_z| = 25^\circ C$

$|\Delta t_3| = |t_{w2} - t_z| = 15^\circ C$

$t_{01} = 15 - 8 = +7^\circ C$

$t_{02} = 7{,}5 - 8 = -0{,}5^\circ C$

$t_{03} = 2{,}5 - 8 = -5{,}5^\circ C$

$\alpha_t = 1{,}2 \cdot 10^{-5} \left[\frac{1}{^\circ C}\right]$; $h_1 = 0{,}20m$; $h_2 = 0{,}22m$

Przyjęto układ podstawowy jak w poprzednim zadaniu.

Układ równań kanonicznych:

$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{13}x_3 + \delta_{1t} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{23}x_3 + \delta_{2t} = 0 \\ \delta_{31}x_1 + \delta_{32}x_2 + \delta_{33}x_3 + \delta_{3t} = 0 \end{cases} \qquad \delta_{it} = \sum_S \int M_i \cdot \alpha_t \cdot \frac{|\Delta t|}{h} ds + \sum_S \int N_i \cdot \alpha_t \cdot t_0 ds$$

$$\delta_{1t} = 2 \cdot \left[\left(\frac{1}{2} \cdot 5 \cdot 5 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{10}{0{,}20} \right) - \left(\frac{1}{2} \cdot 2 \cdot 7 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{15}{0{,}22} \right) - \left(\frac{1}{2} \cdot 2\sqrt{5} \cdot 2 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{25}{0{,}22} \right) \right] +$$
$$+ 2 \cdot \left[\left(5 \cdot (-3{,}5) \cdot 1{,}2 \cdot 10^{-5} \cdot 7 \right) + \left(2\sqrt{5} \cdot \frac{2}{\sqrt{5}} \cdot 1{,}2 \cdot 10^{-5} \cdot (-0{,}5) \right) \right] = \underline{-0{,}0116393} \quad [m]$$

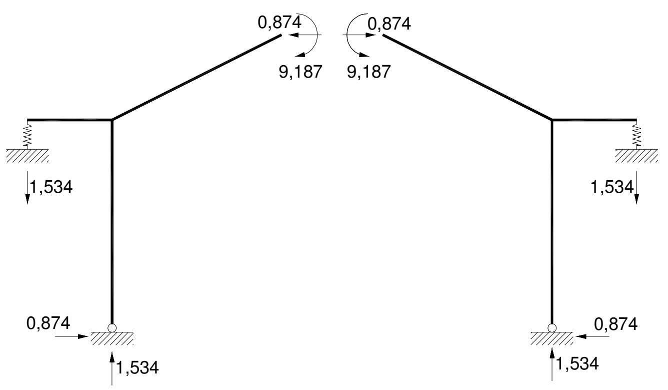
$\delta_{2t} = 0 \quad [m]$

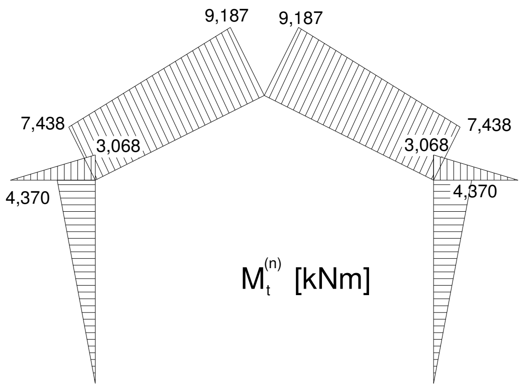
$$\delta_{3t} = 2 \cdot \left[\left(\frac{1}{2} \cdot 2 \cdot 1 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{15}{0{,}22} \right) + \left(2\sqrt{5} \cdot 1 \cdot 1{,}2 \cdot 10^{-5} \cdot \frac{25}{0{,}22} \right) \right] + 2 \cdot \left[\left(5 \cdot 0{,}5 \cdot 1{,}2 \cdot 10^{-5} \cdot 7 \right) \right] = \underline{0{,}0142531} \quad [-]$$

$$\begin{cases} 0{,}0592350 X_1 + 0 X_2 - 0{,}0069028 X_3 = 0{,}0116393 \\ 0 X_1 + 0{,}020123 X_2 + 0 X_3 = 0 \\ -0{,}0069028 X_1 + 0 X_2 + 0{,}0022083 X_3 = -0{,}0142531 \end{cases}$$

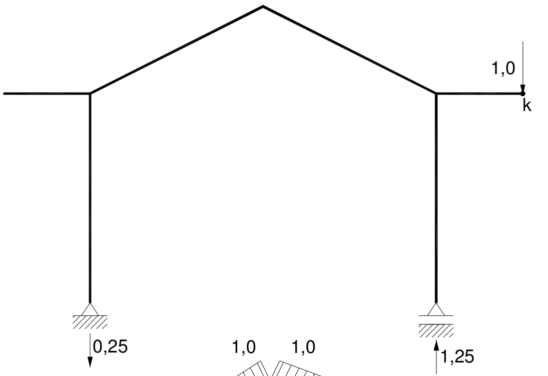
$$\begin{cases} X_1 = -0{,}874 kN \\ X_2 = 0{,}000 kN \\ X_3 = -9{,}187 kNm \end{cases}$$

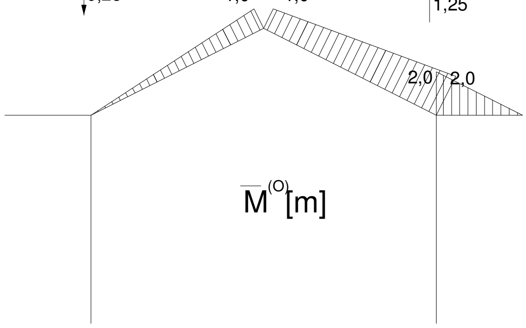
Ostateczny wykres momentów (wpływ temperatury):

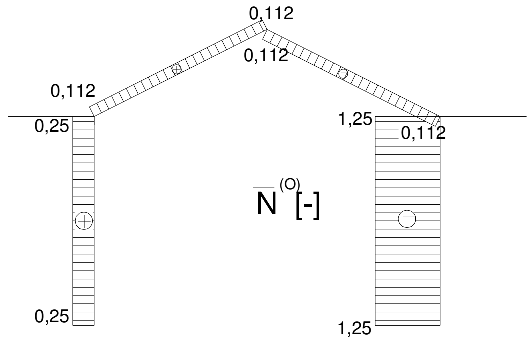




Sprawdzenie kinematyczne:







$$\overline{1{,}0}\cdot V_k=\sum\int_S\frac{\overline{M}^{(o)}M_t^{(n)}}{EI}ds+\sum\int_S\overline{M}^{(0)}\cdot\alpha_t\cdot\frac{\Delta t}{h}ds+\sum\int_S\overline{N}^{(0)}\cdot\alpha_t\cdot t_0 ds$$

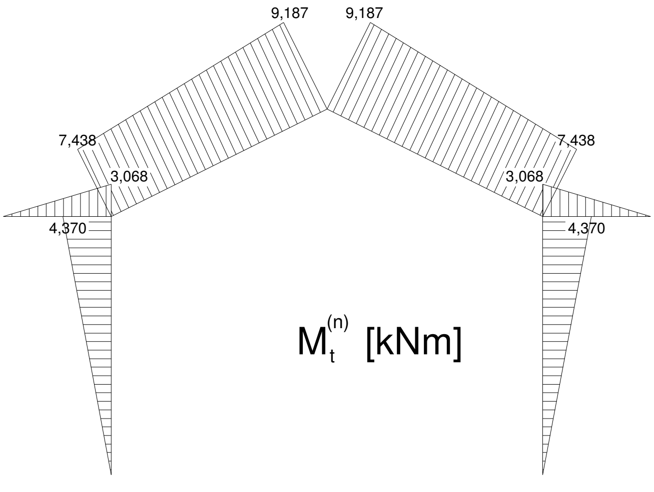
$$\overline{1{,}0}\cdot V_k=\frac{1}{EI_2}\left[\frac{2\sqrt{5}}{6}(2\cdot 9{,}187\cdot 1+1\cdot 7{,}439)+\frac{2\sqrt{5}}{6}(2\cdot 9{,}187\cdot 1+2\cdot 7{,}438\cdot 2+9{,}187\cdot 2+7{,}438\cdot 1)+\frac{1}{2}\cdot 2\cdot 3{,}068\cdot\frac{2}{3}\cdot 2\right]+$$

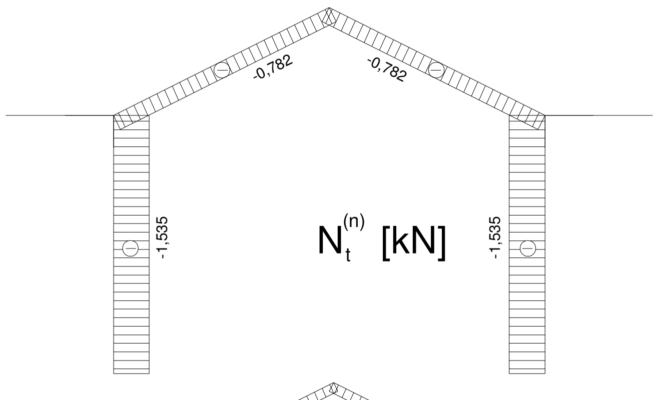
$$+1{,}2\cdot 10^{-5}\cdot\left(-\frac{1}{2}\cdot 2\sqrt{5}\cdot 1\cdot\frac{25}{0{,}22}-1{,}5\cdot 2\sqrt{5}\cdot\frac{25}{0{,}22}-\frac{1}{2}\cdot 2\cdot 2\cdot\frac{15}{0{,}22}\right)+1{,}2\cdot 10^{-5}\cdot 7\cdot(0{,}25\cdot 5-1{,}25\cdot 5)=\frac{78{,}4410}{EI_2}-0{,}014253=$$

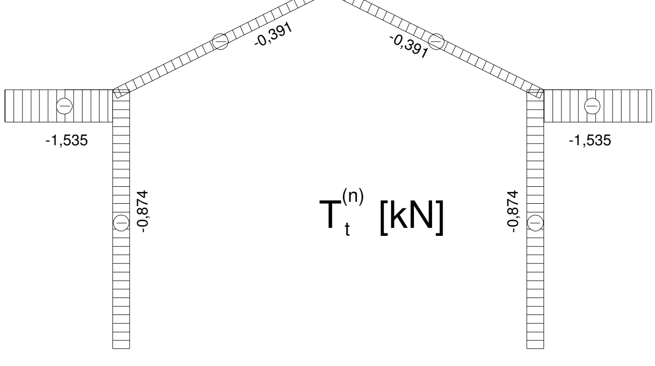
$$=0{,}012505-0{,}014253=-0{,}001748m$$ (przemieszczenie przeciwnie do kierunku siły jednostkowej -> w górę)

$$V_k=R_t^{(n)}\frac{1}{k}=1{,}534\cdot\frac{1}{877{,}4}=0{,}001748m$$ (ugięcie sprężyny; siła rozciągająca –> przemieszczenie w górę)

Ostateczne wykresy *N*, *M*, *T* (wpływ temperatury):

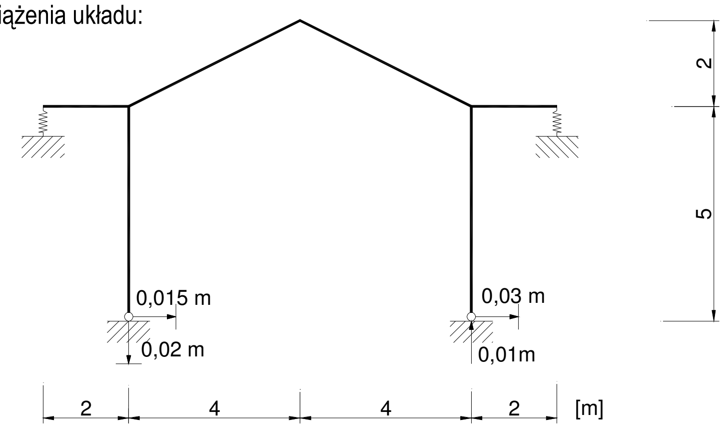






Siły wewnętrzne - wpływ osiadania podpór:

Schemat obciążenia układu:



Przyjęto układ podstawowy jak w poprzednim zadaniu.
Układ równań kanonicznych:

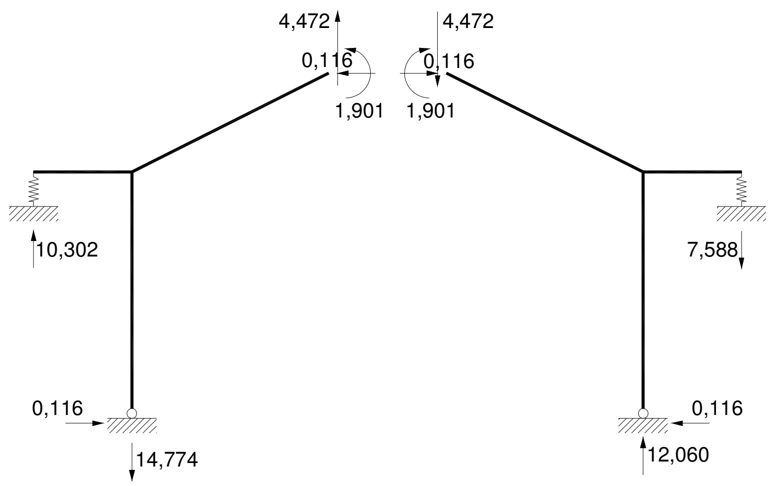
$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{13}x_3 + \delta_{1\Delta} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{23}x_3 + \delta_{2\Delta} = 0 \\ \delta_{31}x_1 + \delta_{32}x_2 + \delta_{33}x_3 + \delta_{3\Delta} = 0 \end{cases} \qquad \delta_{i\Delta} = -\sum R_i \Delta$$

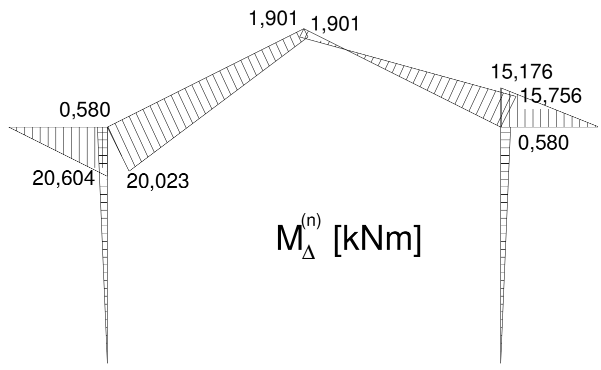
$$\delta_{1\Delta} = -[(-0{,}02 \cdot 3{,}5) + (-0{,}015 \cdot 1{,}0) + (0{,}01 \cdot 3{,}5) + (0{,}03 \cdot 1{,}0)] = \underline{0{,}020} \quad [m]$$
$$\delta_{2\Delta} = -[(-0{,}02 \cdot 3{,}0) + (-0{,}01 \cdot 3{,}0)] = \underline{0{,}090} \quad [m]$$
$$\delta_{3\Delta} = -[(0{,}02 \cdot 0{,}5) + (-0{,}01 \cdot 0{,}5)] = \underline{-0{,}005} \quad [-]$$

$$\begin{cases} 0{,}0592350\,x_1 + 0x_2 - 0{,}0069028\,x_3 = -0{,}0200 \\ 0x_1 + 0{,}0201231x_2 + 0x_3 = -0{,}0900 \\ -0{,}0069028\,x_1 + 0x_2 + 0{,}0022083\,x_3 = 0{,}0050 \end{cases}$$

$$\begin{cases} X_1 = -0{,}116kN \\ X_2 = -4{,}472kN \\ X_3 = 1{,}901kNm \end{cases}$$

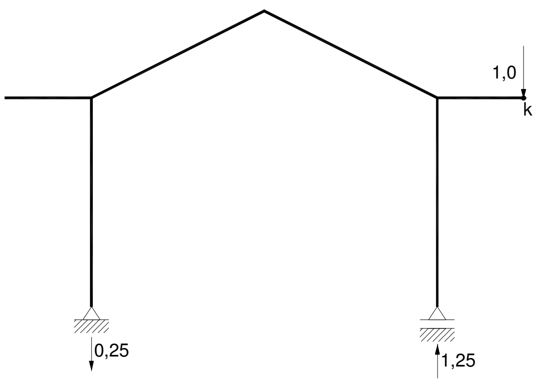
Ostateczny wykres momentów (wpływ osiadania podpór):

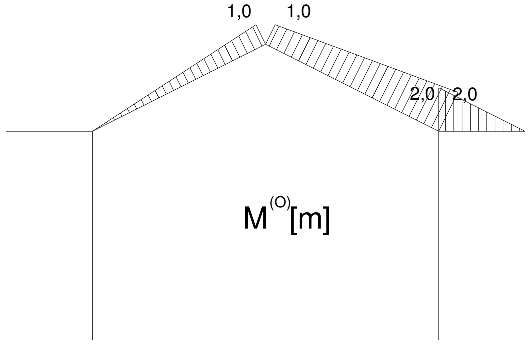




Sprawdzenie kinematyczne:

$$1{,}0 \cdot V_k = \sum \int_S \frac{\overline{M}^{(o)} M_{\Delta}^{(n)}}{EI} ds - \sum_j \overline{R}_i^{(0)} \Delta_j$$





$$\overline{1{,}0} \cdot V_k = \frac{1}{EI_2}\left[\frac{2\sqrt{5}}{6}(-2 \cdot 1{,}901 \cdot 1 - 20{,}023 \cdot 1) + \frac{2\sqrt{5}}{6}(-2 \cdot 1{,}901 \cdot 1 + 2 \cdot 15{,}756 \cdot 2 - 1{,}901 \cdot 2 + 15{,}756 \cdot 1) + \frac{1}{2} \cdot 2 \cdot 15{,}176 \cdot \frac{2}{3} \cdot 2\right]$$

$$-\left[(0{,}02 \cdot 0{,}25) + (0{,}01 \cdot 1{,}25)\right] = \frac{55{,}52684}{EI_2} - 0{,}0175 = \frac{55{,}52684}{6273{,}0} - 0{,}0175 = 0{,}008851 - 0{,}0175 = -0{,}00865 m$$

(przemieszczenie przeciwnie do kierunku siły jednostkowej -> w górę)

$$V_k = R_{\Delta}^{(n)} \cdot \frac{1}{k} = 7{,}588 \cdot \frac{1}{877{,}4} = 0{,}00865 m$$ (ugięcie sprężyny; siła rozciągająca –> przemieszczenie w górę)

Ostateczne wykresy *N*, *M*, *T* (wpływ osiadania podpór):