Politechnika Poznańska
Wydział Inżynierii Lądowej i Transportu
Instytut Analizy Konstrukcji
Zakład Mechaniki Budowli

Nazwisko i imię studenta: **Gocki Maksymilian**
Nr albumu: **159862**
Rok akad. / Rodzaj studiów: **2024/25 BS-I**
Grupa ćw. / Grupa lab.: **C3 / P5**

MECHANIKA BUDOWLI / SEM.3 / ĆWICZENIE NR 2

METODA SIŁ.
OBLICZANIE PRZEMIESZCZEŃ
W UKŁADACH STATYCZNIE NIEWYZNACZALNYCH.

Termin oddania: 22.01.2025

Data	Uwagi sprawdzającego	Podpis

Dla zadanej **RAMY**: **/ Nr schematu: 1**

1. Przyjąć wstępnie przekroje prętów I_1 i I_2 - różne, zgodnie z oznaczeniem na schemacie, z profili dwuteowych (IN, IPE, HEB, HEA).
2. Obliczyć siły przekrojowe (M, N, T) od zadanego obciążenia i wykonać kontrolę kinematyczną. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2 oraz sformułować wnioski.*
3. Obliczyć siły przekrojowe (M, N, T) od zadanego wpływu temperatury i wykonać kontrolę kinematyczną.
4. Obliczyć siły przekrojowe (M, N, T) od zadanego osiadania podpór i wykonać kontrolę kinematyczną.
5. Korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego obliczyć:
   a) od sił zewnętrznych: **obrót przekroju w p. "E"**
   b) od wpływu temperatury: **obrót przekroju w p. "F"**
   c) od osiadania podpór: **obrót przekroju w p. "F"**

Dla zadanej **BELKI**: **/ Nr schematu: 4**

1. Przyjąć wstępnie przekroje prętów I_1 i I_2 - różne, zgodnie z oznaczeniem na schemacie, z profili dwuteowych (IN, IPE, HEB, HEA).
2. Obliczyć siły przekrojowe (M, N, T) od zadanego obciążenia i wykonać kontrolę kinematyczną. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2 oraz sformułować wnioski.*
3. Korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego obliczyć:
   obrót przekroju w p. "B"

We wszystkich obliczeniach przyjąć: $E = 210$ GPa, $f_y = 235$ MPa.

* UWAGA! W przypadku niespełnienia warunku nośności obliczeń nie trzeba powtarzać.

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DANE DLA RAMY — Nr schematu: 1

Wymiary							
a [m]	b [m]	c [m]	d [m]	e [m]	f [m]	*n*	*m*
5	4,8	4,8	5,6	5,5	6	0,7	0,5

Sztywność liniowa podpór [kN/m]		
k_1	k_2	Sztywność pozostałych więzów
8900	∞	∞

Obciążenie siłami zewnętrznymi						
q_1 [kN/m]	q_2 [kN/m]	P_1 [kN]	P_2 [kN]	P_3 [kN]	M_1 [kNm]	M_2 [kNm]
-	10	13	-	25	-	-31

Obciążenie termiczne			
t_1 [ºC]	t_2 [ºC]	t_3 [ºC]	t_m [ºC]
20	15	50	2

Obciążenie kinematyczne (osiadanie podpór)		
osiadanie kątowe węzła A:	1,8	[º]
osiadanie pionowe węzła E:	1,1	[cm]
osiadanie poziome węzła A:	1	[cm]

DANE DLA BELKI — Nr schematu: 4

L [m]	Sztywność k [kN/m]	Sztywność pozostałych więzów
4,5	1 000	∞

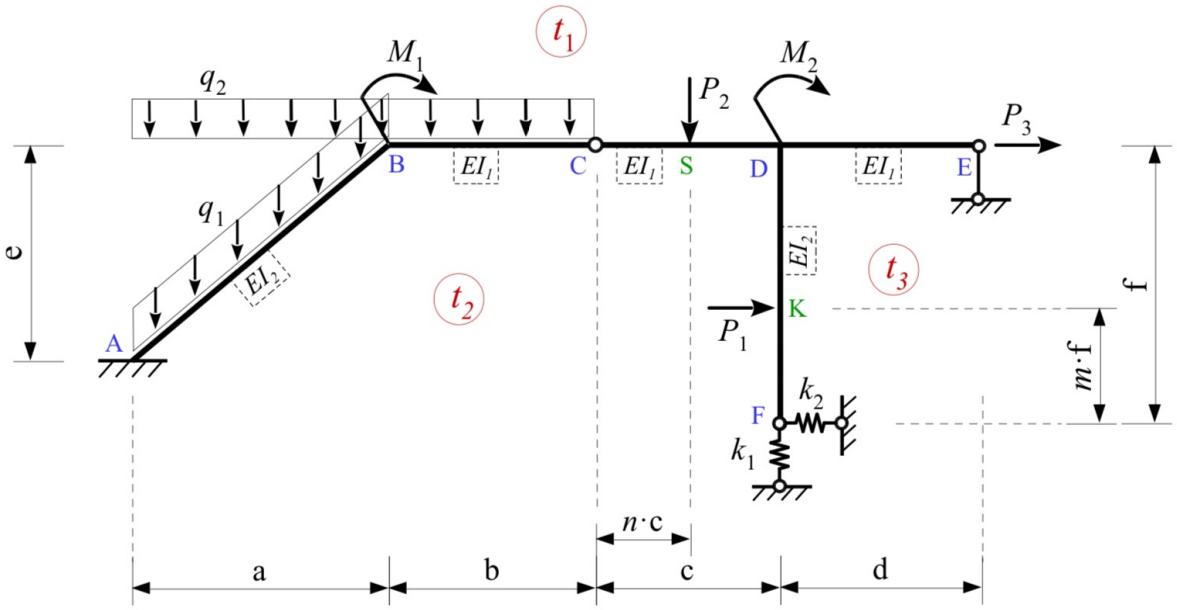
Obciążenie		
q [kN/m]	P [kN]	M [kNm]
15,9	-	18

UWAGA!
Wartości dodatnie obciążeń zadanych w tabeli oznaczają, że obciążenia te mają zwrot zgodny z zadanym na schemacie ramy, ujemne – przeciwny.

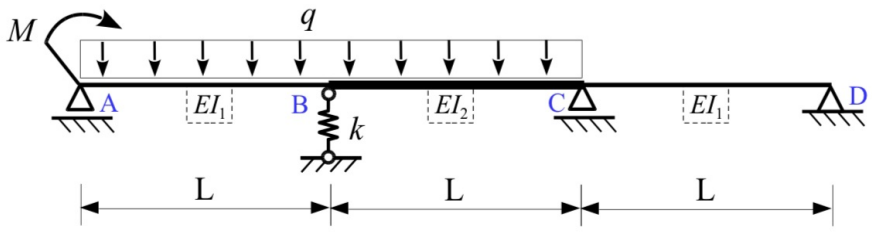
Wartości dodatnie osiadania podpór zadanego w tabeli oznaczają, że: **osiadania liniowe** mają zwrot **w prawo** lub **w dół**, **osiadania kątowe - w prawo**.

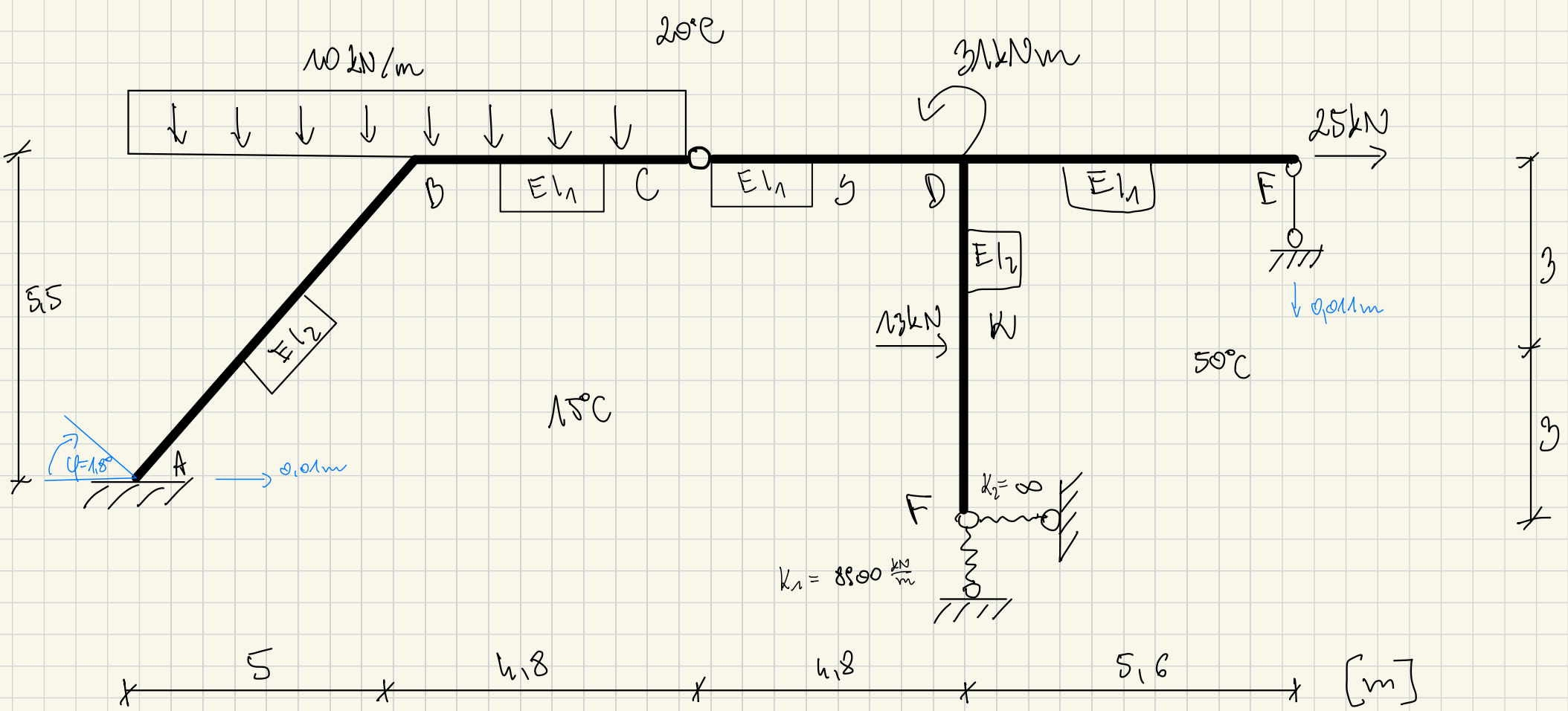
Schematy do projektu

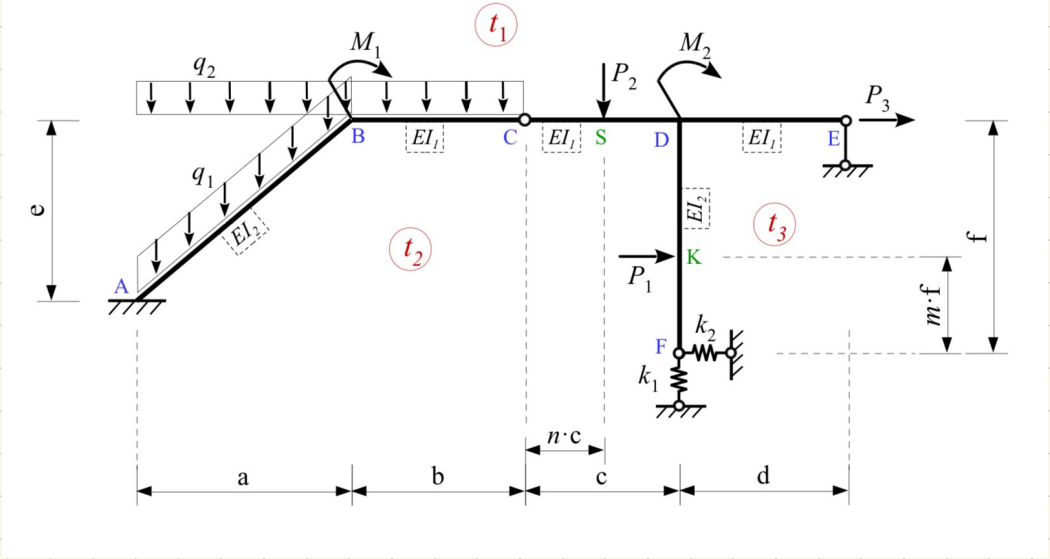
Schemat 1.



Schemat 4.

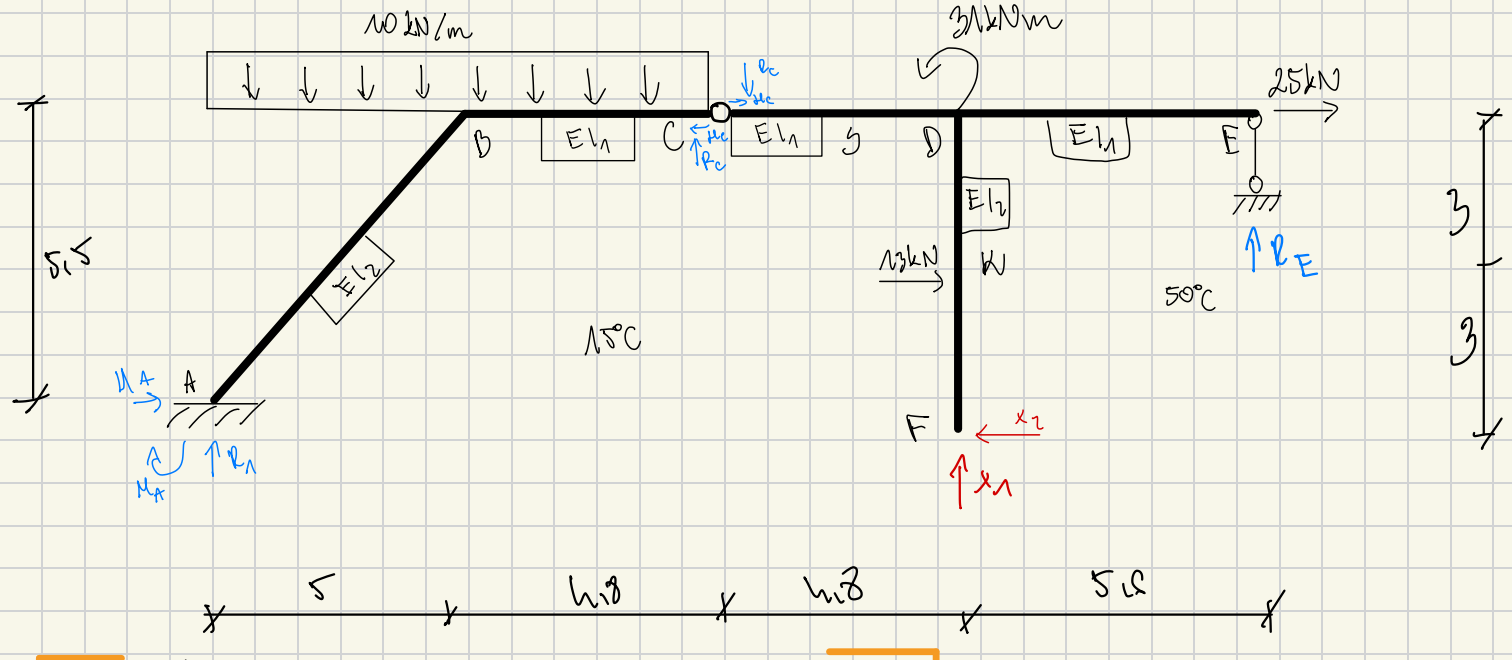



20°C
10 kN/m
31 kNm
25kN
B
EI1
C
EI1
5
D
EI1
E
EI2
5,5
3
3
13kN
W
50°C
15°C
φ=1,8°
A
0,01m
0,01m
F
k2 = ∞
k1 = 8500 kN/m
5
4,8
4,8
5,6
[m]


t1
M1
q2
P2
M2
P3
B
EI1
C
EI1
S
D
EI1
E
q1
EI2
t2
EI2
t3
K
P1
m·f
f
e
A
F
k2
k1
n·c
a
b
c
d

układ podstawowy

$SSN = 6 - 3 - 1 = 2$



wstępne przyjęcie przekrojów

$E = 210\ GPa$

I 140 $I_1 = 573{,}0\ cm^4$

$EI_1 = 210 \cdot 10^6 \cdot 573 \cdot 10^{-8} = 1203{,}3\ kNm^2$

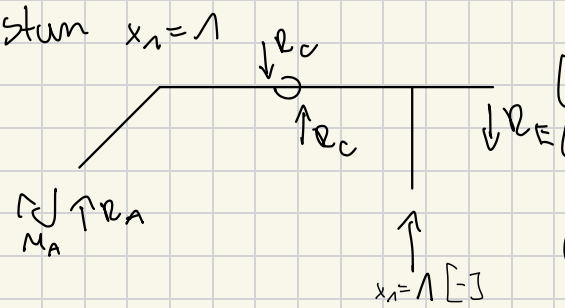
I 200 $I_2 = 2140{,}0\ cm^4$

$EI_2 = 210 \cdot 2140 \cdot 10^6 \cdot 10^{-8} = 4494\ kNm^2$

$\frac{EI_1}{EI_2} = \frac{1203{,}3}{4494} = 0{,}2678$

$EI_1 = 0{,}2678\, EI_2$

stan $x_1 = 1$



$\sum M_C'' = 0$

$-1 \cdot 4{,}8 + R_E \cdot 10{,}4 = 0$

$R_E = 0{,}4615$

$\sum V'' = 0$

$R_A + 1 - R_E = 0$

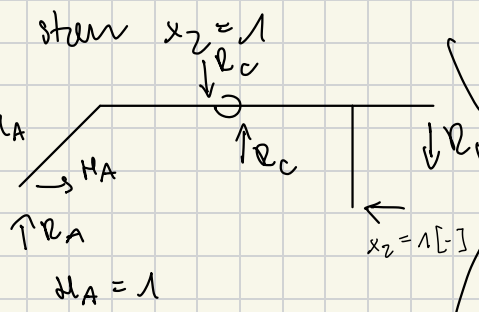
$R_A = 0{,}4615 - 1 = -0{,}5385$

$\sum M_A'' = 0$

$M_A - 1 \cdot 11{,}6 + R_E \cdot 20{,}2 = 0$

$M_A = 5{,}277\ m$

stan $x_2 = 1$



$M_A = 1$

$\sum M_C'' = 0$

$1 \cdot 6 + R_E \cdot 10{,}4 = 0$

$R_E = -0{,}5769$

$\sum V'' = 0$

$R_A - R_E = 0$

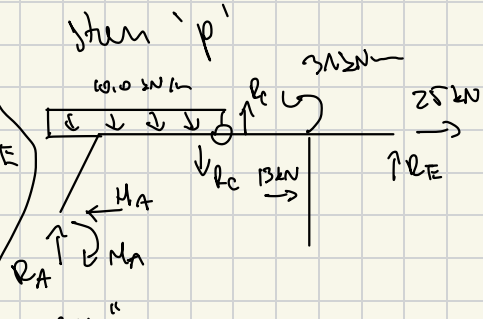
$R_A = -0{,}5769\ kN$

$\sum M_A'' = 0$

$1 \cdot 0{,}5 + R_E \cdot 20{,}2 + M_A = 0$

$M_A = 11{,}15\ m$

stan 'p'



$\sum M_C'' = 0$

$-31 - 13 \cdot 3 - R_E \cdot 10{,}4 = 0$

$R_E = -6{,}731\ kN$

$\sum M_A'' = 0$

$10 \cdot 9{,}8 \cdot 4{,}9 + M_A - 31 + 13 \cdot 2{,}5 + 25 \cdot 5{,}5 - R_E \cdot 20{,}2 = 0$

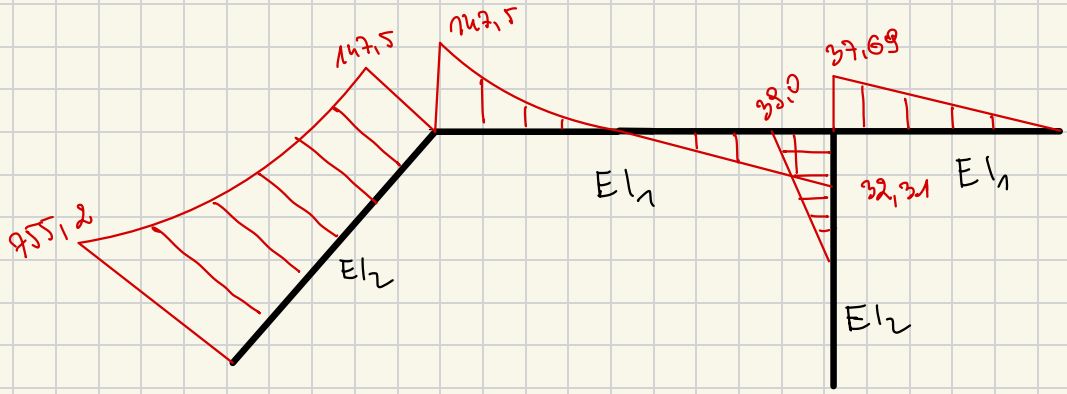
$M_A = -755{,}2\ kNm$

$\sum V'' = 0$

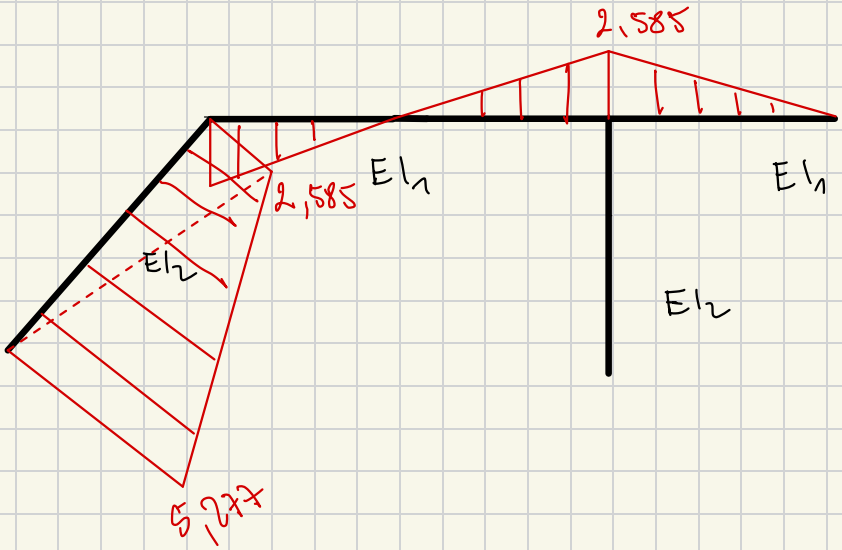
$R_A - W \cdot 9{,}8 + R_E = 0$

$R_A = 104{,}7\ kN$

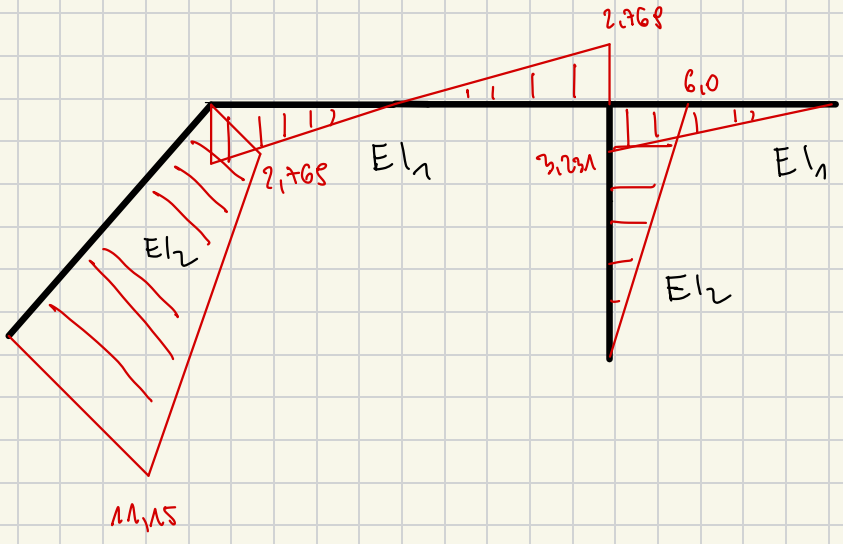
$M_A = 38\ kN$



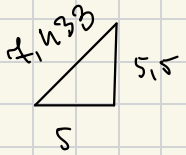
M_p [kNm]



M_1 [m]



M_2 [m]



$$\alpha = \frac{k}{EI_2} = \frac{8500}{4494} = 1,8904$$

$$k = 1,8904\, EI_2$$

$$\delta_{11} = \sum \int_S \frac{M_1 M_1}{EI} ds + \frac{R_1 R_1}{k}$$

$$\delta_{11} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 2{,}585 \cdot 4{,}8 \cdot \left(\frac{2}{3}\cdot 2{,}585\right) + \frac{1}{2}\cdot 2{,}585 \cdot 4{,}8 \cdot \left(\frac{2}{3}\cdot 2{,}585\right) + \right.$$

$$+ \frac{1}{2}\cdot 2{,}585 \cdot 5{,}16 \cdot \left(\frac{2}{3}\cdot 2{,}585\right) + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 7{,}433 \cdot 2{,}585 \cdot \right.$$

$$\cdot \left(\frac{2}{3}\cdot 2{,}585 + \frac{1}{3}\cdot 5{,}277\right) + \frac{1}{2}\cdot 7{,}433 \cdot 5{,}277 \cdot \left(\frac{1}{3}\cdot 2{,}585 + \right.$$

$$\left. + \frac{2}{3}\cdot 5{,}277\right)\Big] + \frac{1\cdot 1}{1{,}8804\, EI_2} = \frac{33{,}86}{EI_1} + \frac{118{,}3}{EI_2} + \frac{0{,}5048}{EI_2} =$$

$$= \frac{126{,}4}{EI_2} + \frac{118{,}8}{EI_2} = \frac{246{,}3}{EI_2}$$

$$\delta_{12} = \sum \int_S \frac{M_1 M_2}{EI} ds + \frac{R_1 R_2}{k}$$

$$\delta_{12} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 2{,}585 \cdot 4{,}8 \cdot \left(\frac{2}{3}\cdot 2{,}768\right) + \frac{1}{2}\cdot 2{,}585 \cdot 4{,}8 \cdot \left(\frac{2}{3}\cdot 2{,}768\right) - \right.$$

$$- \frac{1}{2}\cdot 2{,}585 \cdot 5{,}16 \cdot \left(\frac{2}{3}\cdot 3{,}231\right)\Big] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 2{,}585 \cdot 7{,}433 \cdot \left(\frac{2}{3}\cdot 2{,}768 + \right.\right.$$

$$\left. + \frac{1}{3}\cdot 11{,}15\right) + \frac{1}{2}\cdot 5{,}277 \cdot 7{,}433 \cdot \left(\frac{2}{3}\cdot 11{,}15 + \frac{1}{3}\cdot 2{,}768\right)\Big] =$$

$$= \frac{7{,}314}{EI_1} + \frac{217{,}4}{EI_2} = \frac{27{,}31}{EI_2} + \frac{217{,}4}{EI_2} = \frac{244{,}7}{EI_2}$$

$$\delta_{22} = \sum \int_S \frac{M_2 M_2}{EI} ds + \frac{R_2 R_2}{k}$$

$$\delta_{22} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 2{,}768 \cdot 4{,}8 \cdot \left(\frac{2}{3}\cdot 2{,}768\right) + \frac{1}{2}\cdot 2{,}768 \cdot 4{,}8 \cdot \left(\frac{2}{3}\cdot 2{,}768\right) + \right.$$

$$\left. + \frac{1}{2}\cdot 3{,}231 \cdot 5{,}16 \cdot \left(\frac{2}{3}\cdot 3{,}231\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 6\cdot 6\cdot \left(\frac{2}{3}\cdot 6\right) + \frac{1}{2}\cdot 2{,}768 \cdot 7{,}433 \cdot \right.$$

$$\cdot \left(\frac{2}{3}\cdot 2{,}768 + \frac{1}{3}\cdot 11{,}15\right) + \frac{1}{2}\cdot 11{,}15 \cdot 7{,}433 \cdot \left(\frac{2}{3}\cdot 11{,}15 + \frac{1}{3}\cdot 2{,}768\right)\Big] =$$

$$= \frac{44{,}02}{EI_1} + \frac{475{,}8}{EI_2} = \frac{164{,}4}{EI_2} + \frac{475{,}8}{EI_2} = \frac{640{,}2}{EI_2}$$

$$\delta_{1P} = \varepsilon \int_S \frac{M_1 M_P}{EI} ds + \frac{R_1 R_P}{k}$$

$$\delta_{1P} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 147{,}5 \cdot 4{,}8 \cdot \left(-\frac{2}{3}\cdot 2{,}585\right) + \frac{2}{3}\cdot 4{,}8 \cdot \frac{10\cdot 4{,}8^2}{8}\cdot\left(\frac{1}{2}\cdot 2{,}585\right)\right.$$

$$\left. + \frac{1}{2}\cdot 4{,}8 \cdot 32{,}31 \cdot \left(-\frac{2}{3}\cdot 2{,}585\right) + \frac{1}{2}\cdot 37{,}68\cdot 5{,}16\cdot\left(\frac{2}{3}\cdot 2{,}585\right)\right] +$$

$$+ \frac{1}{EI_2}\left[\frac{1}{2}\cdot 147{,}5 \cdot 7{,}433\cdot\left(-\frac{2}{3}\cdot 2{,}585 - \frac{1}{3}\cdot 5{,}277\right) + \frac{1}{2}\cdot 755{,}2\cdot 7{,}433\cdot\right.$$

$$\left.\cdot\left(-\frac{2}{3}\cdot 5{,}277 - \frac{1}{3}\cdot 2{,}585\right) + \frac{2}{3}\cdot 7{,}433\cdot\frac{10\cdot 5^2}{8}\cdot\left(\frac{1}{2}\cdot 2{,}585 + \frac{1}{2}\cdot 5{,}277\right)\right]$$

$$= -\frac{442{,}7}{EI_1} - \frac{13582}{EI_2} = -\frac{1653}{EI_2} - \frac{13582}{EI_2} =$$

$$= -\frac{15235}{EI_2}$$

$$\delta_{2P} = \varepsilon \int_S \frac{M_2 M_P}{EI} ds + \frac{R_2 R_P}{k}$$

$$\delta_{2P} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 147{,}5 \cdot 4{,}8\cdot\left(-\frac{2}{3}\cdot 2{,}768\right) + \frac{2}{3}\cdot 4{,}8\cdot\frac{10\cdot 4{,}8^2}{8}\cdot\left(\frac{1}{2}\cdot 2{,}768\right)\right.$$

$$\left. + \frac{1}{2}\cdot 32{,}31\cdot 4{,}8\cdot\left(-\frac{2}{3}\cdot 2{,}768\right) + \frac{1}{2}\cdot 37{,}68\cdot 5{,}16\cdot\left(-\frac{2}{3}\cdot 3{,}231\right)\right]$$

$$+ \frac{1}{EI_2}\left[\frac{1}{2}\cdot 38\cdot 3\cdot\left(-\frac{1}{3}\cdot 3 - \frac{2}{3}\cdot 6\right) + \frac{1}{2}\cdot 147{,}5\cdot 7{,}433\cdot\left(-\frac{2}{3}\cdot 2{,}768 -\right.\right.$$

$$\left.\left. - \frac{1}{3}\cdot 11{,}15\right) + \frac{1}{2}\cdot 755{,}2\cdot 7{,}433\cdot\left(-\frac{2}{3}\cdot 11{,}15 - \frac{1}{3}\cdot 2{,}768\right) +\right.$$

$$\left. + \frac{2}{3}\cdot 7{,}433\cdot\frac{10\cdot 5^2}{8}\cdot\left(\frac{1}{2}\cdot 2{,}768 + \frac{1}{2}\cdot 11{,}15\right)\right] =$$

$$= -\frac{886{,}4}{EI_1} - \frac{25724}{EI_2} = -\frac{3347}{EI_2} - \frac{25724}{EI_2} =$$

$$= -\frac{29071}{EI_2}$$

$$\begin{cases} \delta_{11} x_1 + \delta_{12} x_2 + \delta_{1P} = 0 \\ \delta_{21} x_1 + \delta_{22} x_2 + \delta_{2P} = 0 \end{cases}$$

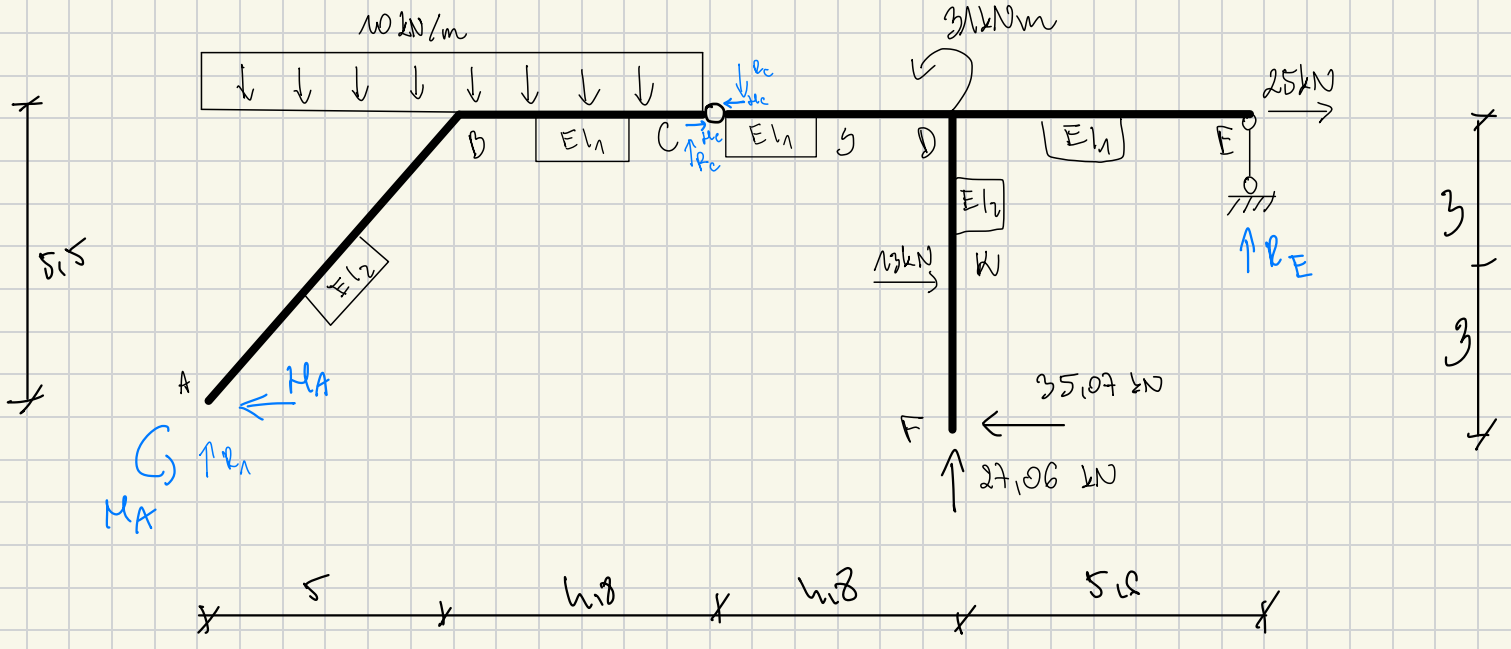
$$\delta_{11} = \frac{246,3}{EI_2}$$

$$\delta_{22} = \frac{640,2}{EI_2}$$

$$\delta_{12} = \frac{244,7}{EI_2}$$

$$\begin{cases} 246,3\, x_1 + 244,7\, x_2 - 15245 = 0 \\ 244,7\, x_1 + 640,2\, x_2 - 23071 = 0 \end{cases}$$

$$\begin{cases} x_1 = 27,06 \text{ kN} \\ x_2 = 35,07 \text{ kN} \end{cases}$$



$$\sum M_C = 0$$

$$-27{,}06 \cdot 4{,}8 + 35{,}07 \cdot 6 - 13 \cdot 3 - 31 - R_E \cdot 10{,}4 = 0$$

$$R_E = 1{,}014 \text{ kN}$$

$$\sum M_A = 0$$

$$10 \cdot 8{,}8 \cdot 4{,}5 - 31 + 13 \cdot 2{,}5 + 25 \cdot 5{,}5 - R_E \cdot 20{,}2 + 35{,}07 \cdot 0{,}5 - 27{,}06 \cdot 14{,}6 - M_A = 0$$

$$M_A = 221{,}2 \text{ kNm}$$

$$\sum X = 0$$

$$R_A + 27{,}06 - 10 \cdot 8{,}8 + 2{,}812 = 0$$

$$R_A = 69{,}93 \text{ kN}$$

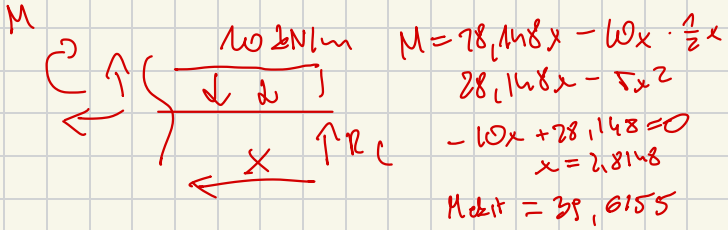
$$\sum X = 0$$

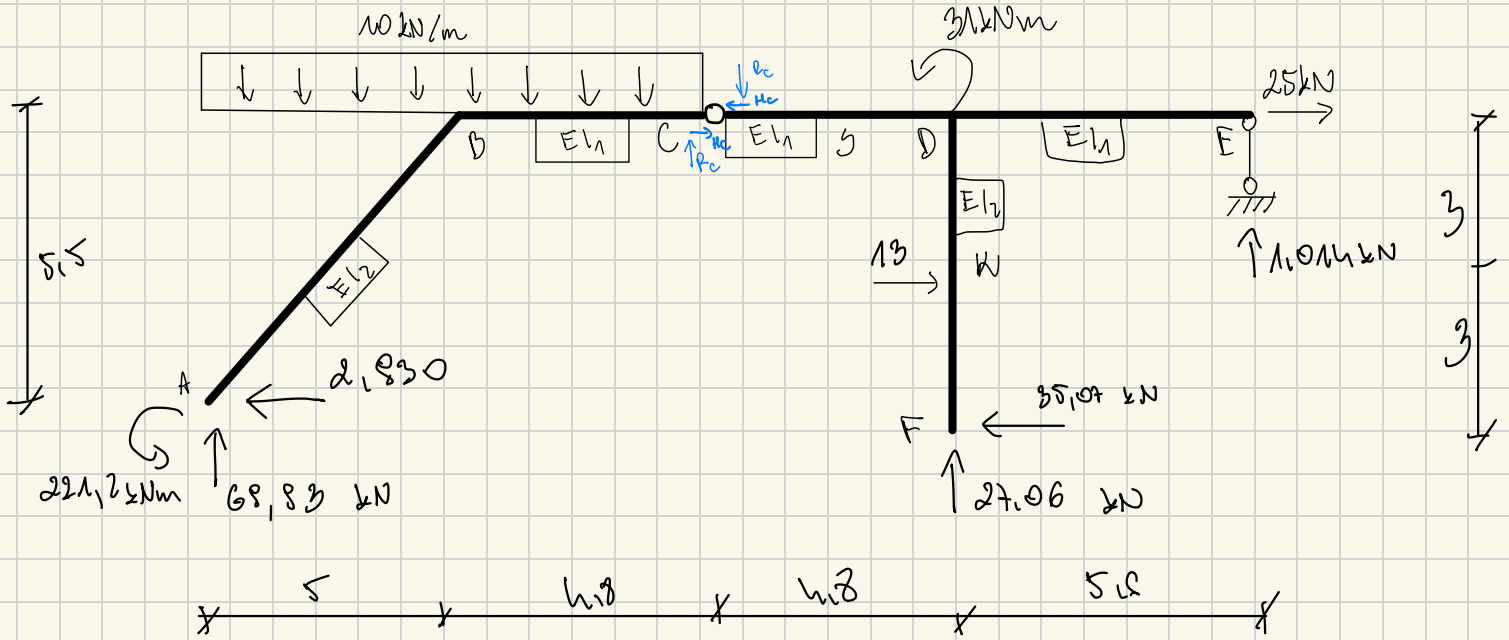
$$-H_A + 13 + 25 - 35{,}07 = 0$$

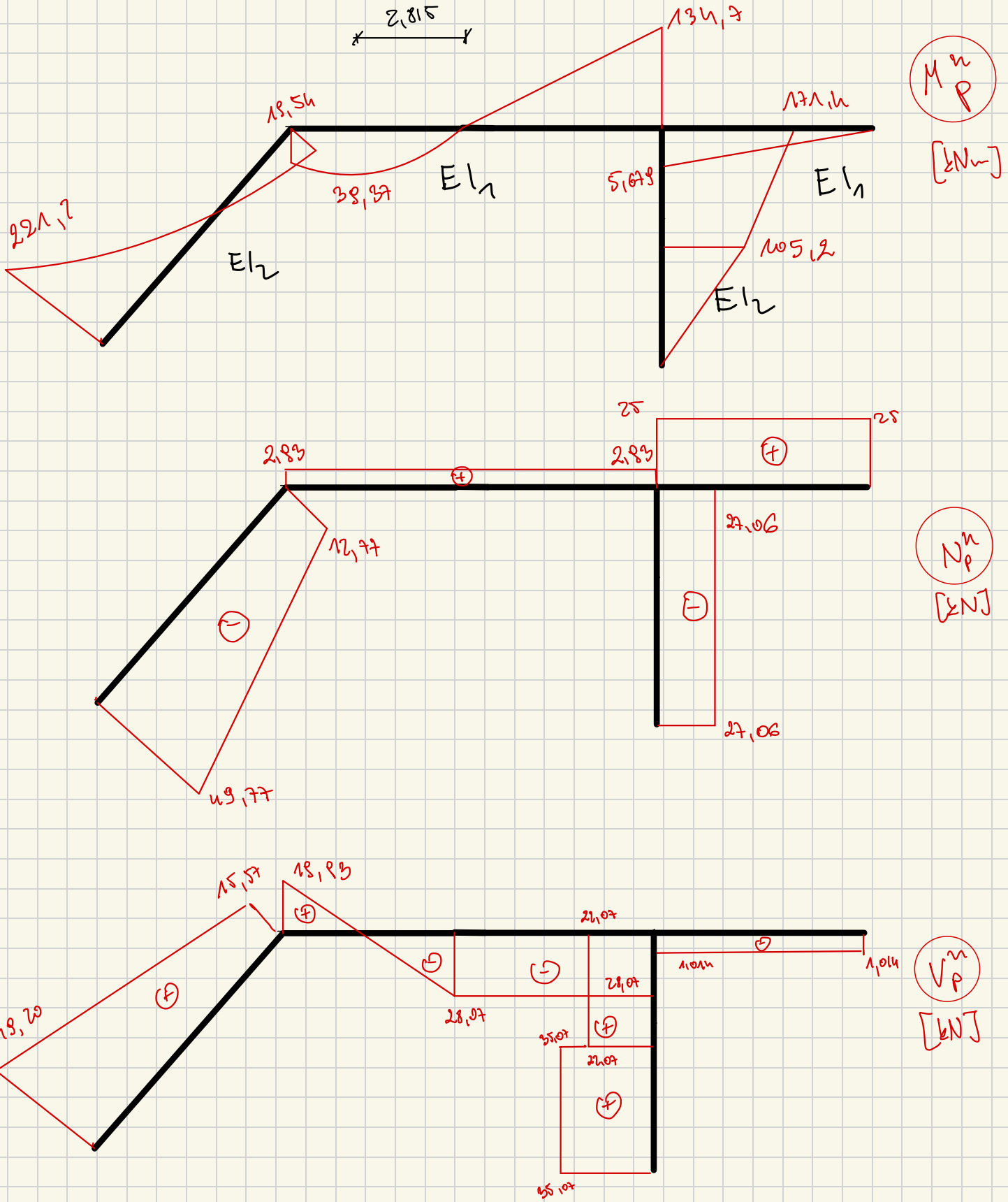
$$H_A = 2{,}930 \text{ kN}$$

$$R_C = 28{,}07 \text{ kN}$$

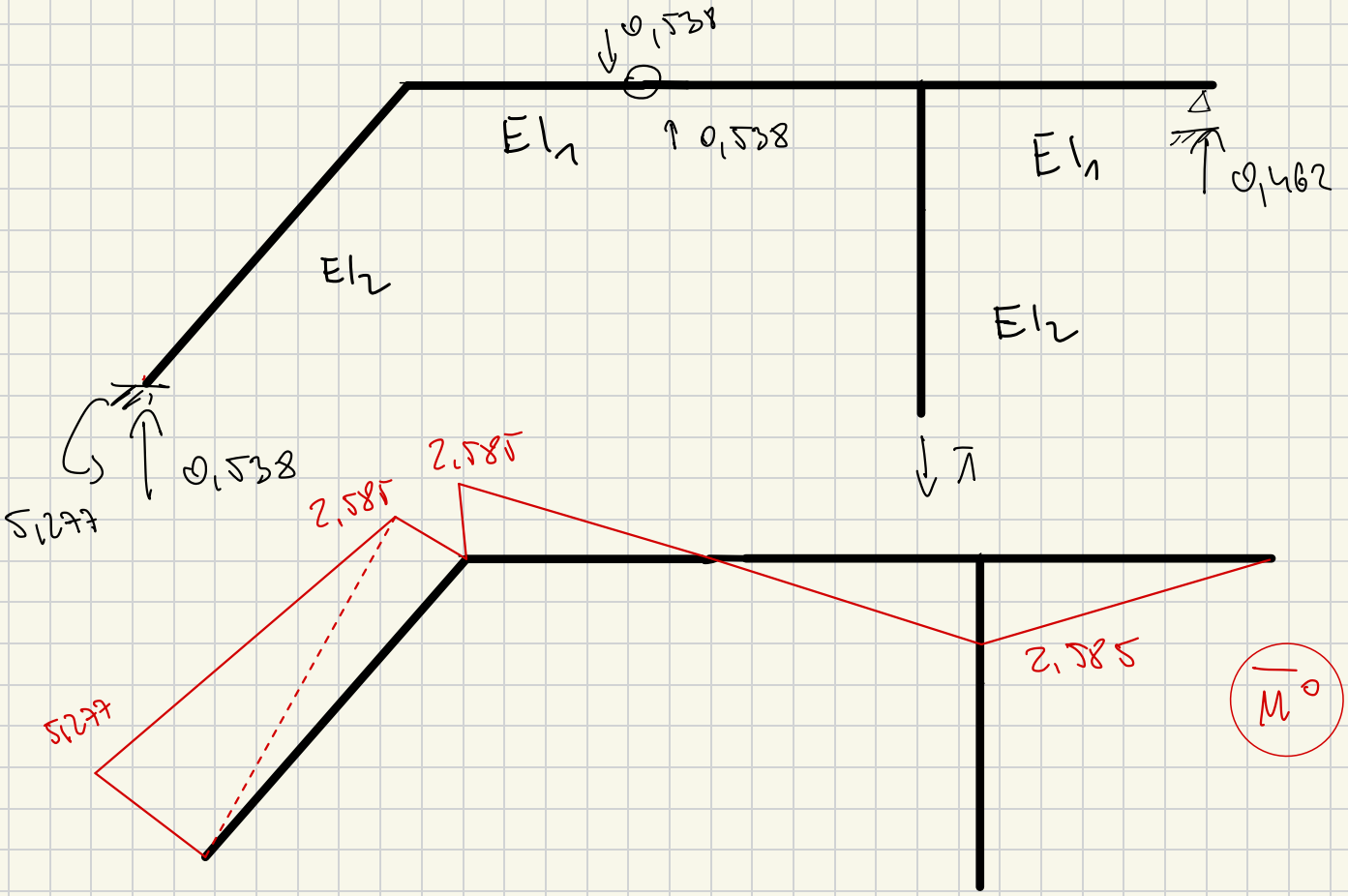
$$H_C = 2{,}830 \text{ kN}$$

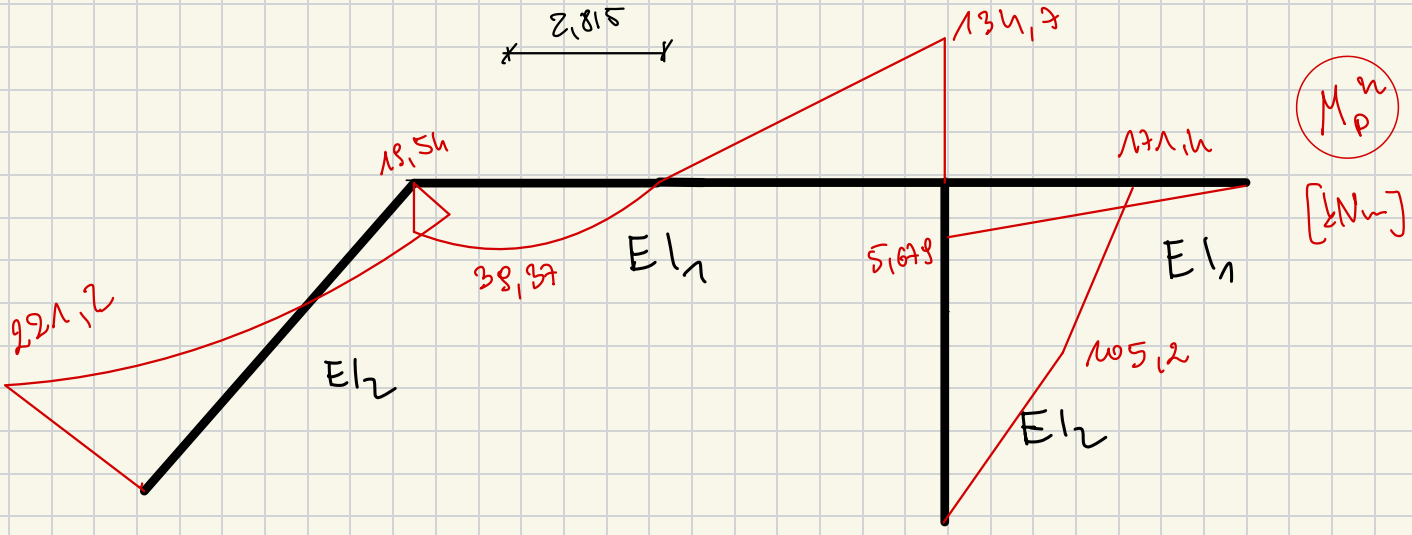





2,815
134,7
18,54
171,14
38,37
EI1
5,078
EI1
221,7
EI2
105,2
EI2
M n p
[kNm]
25
25
2,83
2,83
27,06
12,77
49,77
27,06
N n p
[kN]
15,57
18,83
21,07
1,014
1,014
48,20
28,07
28,07
35,07
22,07
55,10
V n p
[kN]

kontrola kinematyczna





$$V_F = \varepsilon \int_S \frac{M_p^n \bar{M}^0}{EI} ds + \frac{R_p^n \bar{R}^0}{k}$$

$$V_F = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}8\cdot 18{,}54\cdot\left(-\frac{2}{3}\cdot 2{,}585\right) + \frac{2}{3}\cdot 4{,}8\cdot\frac{10\cdot 4{,}8^2}{8}\cdot\left(-\frac{1}{2}\cdot 2{,}585\right)\right.$$

$$+\frac{1}{2}\cdot 134{,}74\cdot 4{,}8\cdot\left(-\frac{2}{3}\cdot 2{,}585\right) + \frac{1}{2}\cdot 5{,}679\cdot 5{,}6\cdot\left(\frac{2}{3}\cdot 2{,}585\right)\Big] +$$

$$\frac{1}{EI_2}\left[\frac{1}{2}\cdot 7{,}433\cdot 221{,}2\cdot\left(\frac{2}{3}\cdot 5{,}277 + \frac{1}{3}\cdot 2{,}585\right) +\right.$$

$$+\frac{1}{2}\cdot 18{,}54\cdot 7{,}433\cdot\left(-\frac{2}{3}\cdot 2{,}585 - \frac{1}{3}\cdot 5{,}277\right) +$$

$$\left.+\frac{2}{3}\cdot 7{,}433\cdot\frac{10\cdot 5^2}{8}\cdot\left(-\frac{1}{2}\cdot 5{,}277 - \frac{1}{2}\cdot 2{,}585\right)\right] - \frac{1\cdot 27{,}096}{1{,}9804\,EI_2} =$$

$$= \frac{-728{,}8}{EI_1} + \frac{2738}{EI_2} - \frac{13{,}66}{EI_2} = -\frac{2725}{EI_2} + \frac{2738}{EI_2} - \frac{13{,}66}{EI_2} =$$

$$= \frac{0{,}2187}{EI_2}$$

$$|V_F| < \frac{1}{EI_2} \Rightarrow V_F \cong 0$$

sprawdzenie doboru przekrojów

Przekrój I_1 I 140

$h = 140\,mm$

$\sigma_{dop} = 235\,MPa$

$$\sigma = \frac{|M|}{W} + \frac{|N|}{A} = \frac{1347}{81{,}9} + \frac{2{,}85}{18{,}3} = 164{,}7\ \frac{kN}{cm^2} = 1647\,MPa$$

$1647\,MPa > \sigma_{dop}$

Przekrój I_2 I 200

$h = 200\,mm$

$\sigma_{dop} = 235\,MPa$

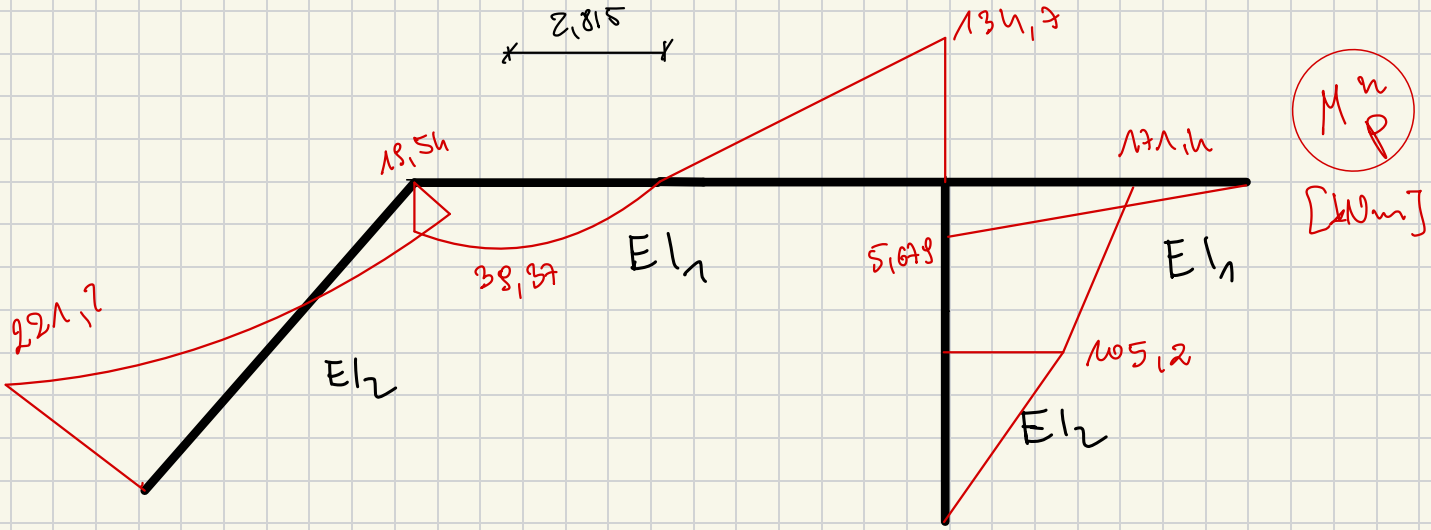
$$\sigma = \frac{2212{,}0}{214} + \frac{48{,}772}{33{,}5} = 104{,}9\ \frac{kN}{cm^2} = 1049\,MPa$$

$1049\,MPa > \sigma_{dop}$

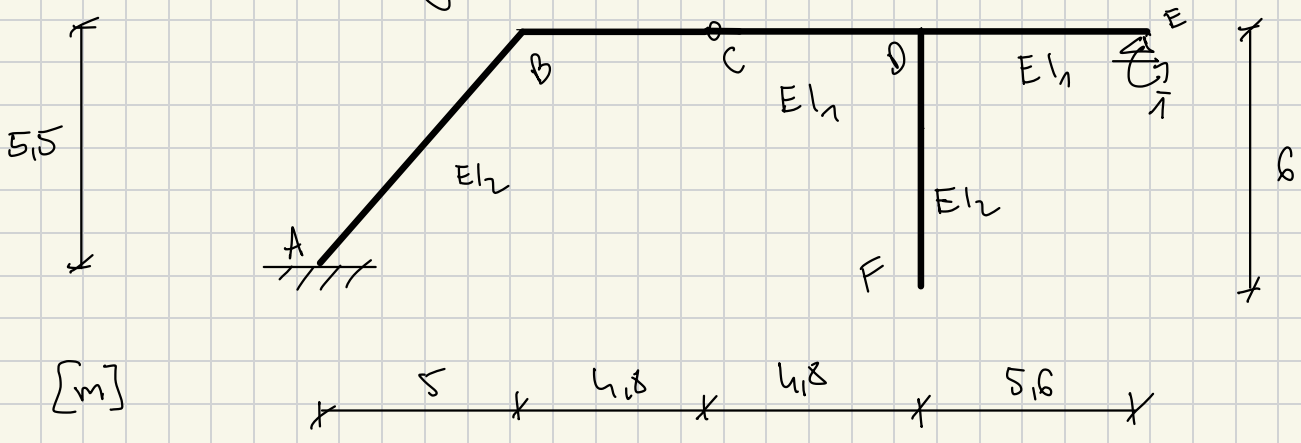
Wnioski: Naprężenia w obu przekrojach są większe niż dopuszczalne. Należy przyjąć większe. Zmiana momentów bezwładności J_1 i J_2 spowoduje zmianę współczynnika $n = \frac{J_1}{J_2}$, a więc całe zadanie trzeba policzyć dla nowego „n" i ponownie sprawdzić naprężenia.

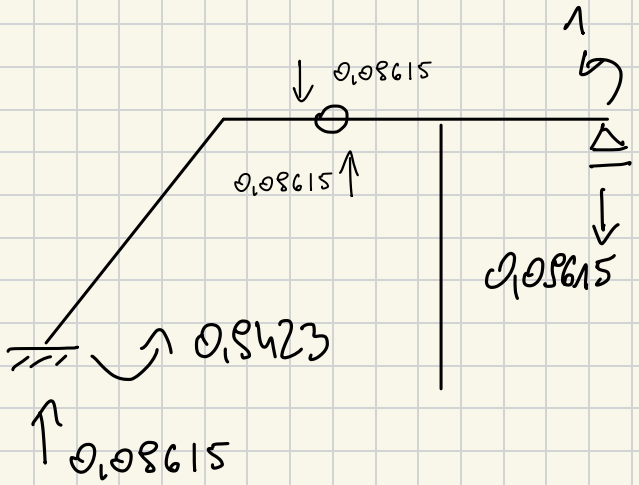
obrót przekroju w pkt E

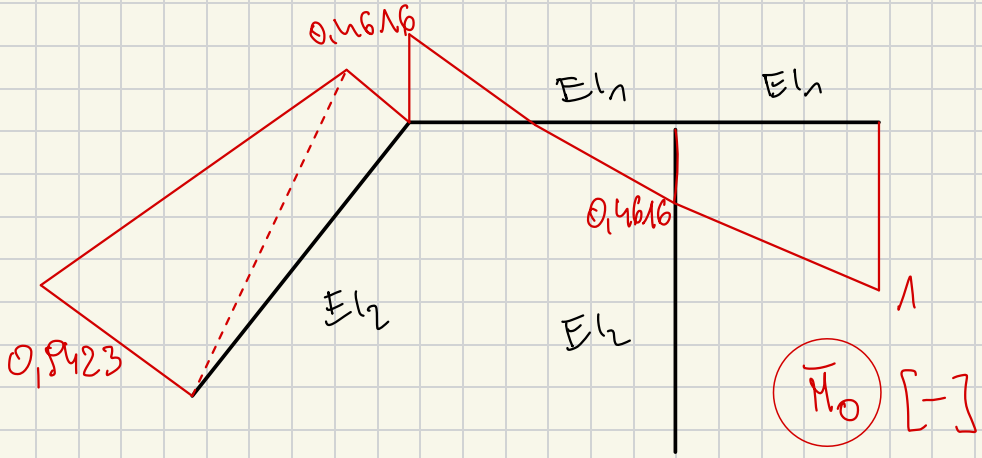
$$\varphi_E = \varepsilon \int_S \frac{M_p^n \bar{M}^0}{EI} ds + \frac{P_p^n \bar{R}^0}{k}$$

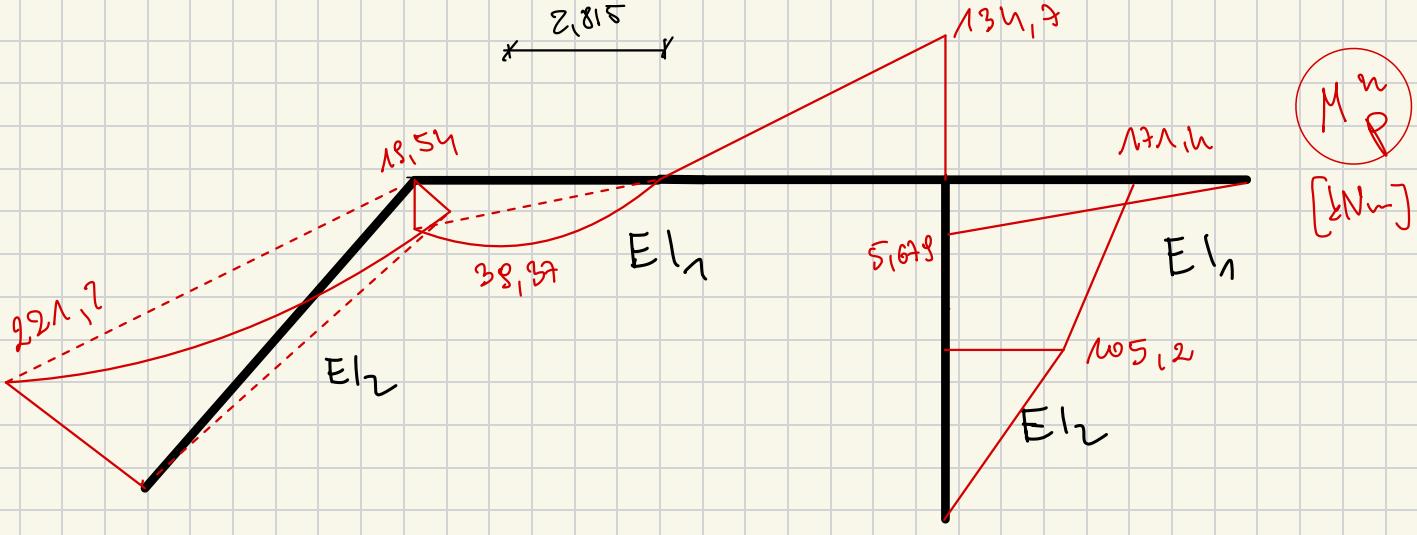


Stan wirtualny – w układzie statycznie wyznaczalnym









$$\varphi_E = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}8\cdot 18{,}54\cdot\left(-\frac{2}{3}\cdot 0{,}4616\right)+\frac{2}{3}\cdot 4{,}8\cdot\frac{10\cdot 4{,}8^2}{8}\cdot\right.$$

$$\cdot\left(-\frac{1}{2}\cdot 0{,}4616\right)+\frac{1}{2}\cdot 4{,}8\cdot 13{,}7\cdot\left(-\frac{2}{3}\cdot 0{,}4616\right)+$$

$$\left.+\frac{1}{2}\cdot 5{,}6\cdot 5{,}679\cdot\left(\frac{2}{3}\cdot 0{,}4616+\frac{1}{3}\cdot 1\right)\right]+\frac{1}{EI_2}\left[\frac{1}{2}\cdot 7{,}433\cdot 22{,}12\right.$$

$$\cdot\left[\frac{2}{3}\cdot 0{,}8423+\frac{1}{3}\cdot 0{,}4616\right)+\frac{1}{2}\cdot 18{,}54\cdot 7{,}433\cdot\left(-\frac{2}{3}\cdot 0{,}4616-\right.$$

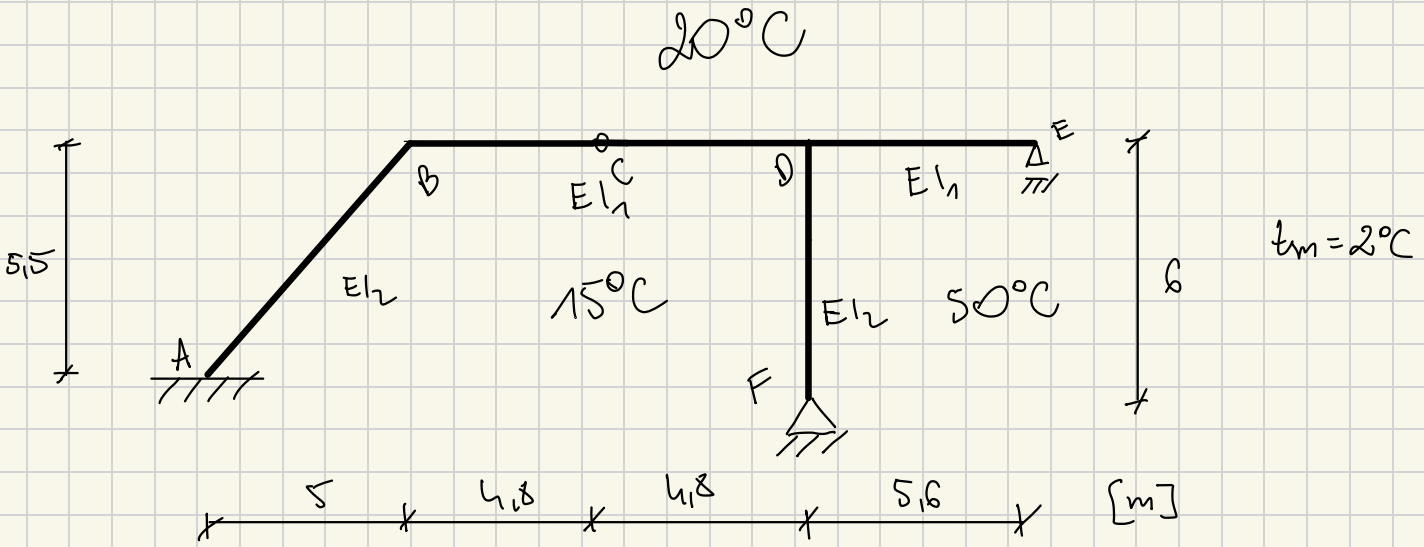
$$\left.-\frac{1}{3}\cdot 0{,}8423\right)+\frac{2}{3}\cdot 7{,}433\cdot\frac{10\cdot 5^2}{8}\cdot\left(-\frac{1}{2}\cdot 0{,}8423-\frac{1}{2}\cdot 0{,}4616\right)\Bigg]$$

$$= -\frac{125{,}10}{EI_1}+\frac{488{,}3}{EI_2}=-\frac{125{,}10}{1203{,}3}+\frac{488{,}3}{4484}=$$

$$= -0{,}1038+0{,}1089=0{,}005\ \text{rad}$$

(doprót w lewo)

Obliczanie sił przekrojowych od zadanych wpływów temperatury



$t_m = 2°C$

$\Delta t = |t_d - t_g|$

$t_0 = \frac{t_d + t_g}{2} - t_m$

EI_1: I140 $h = 0{,}14m$

EI_2: I200 $h = 0{,}2m$

$$t_{0(AB)} = t_{BC} = t_{CD} = \frac{20+15}{2} - 2 = 15{,}5°C$$

$$t_{0(DE)} = \frac{20+50}{2} - 2 = 33°C$$

$$t_{0(DF)} = \frac{15+50}{2} - 2 = 30{,}5°C$$

$$\Delta t_{AB} = \Delta t_{BC} = \Delta t_{CD} = |20-15| = 5°C$$

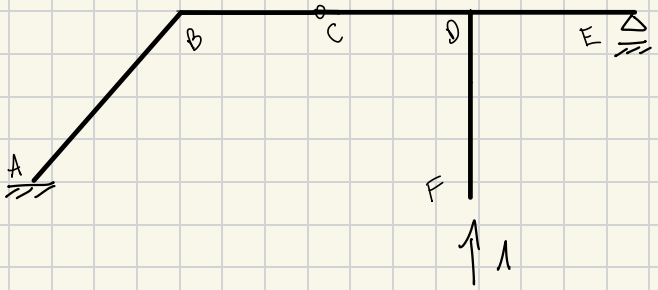
$$\Delta t_{DE} = |20-50| = 30°C$$

$$\Delta t_{DF} = |15-50| = 35°C$$

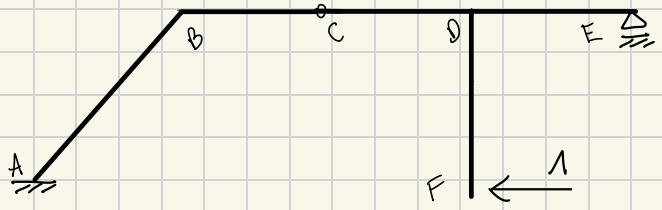
$$\text{UKK} \begin{cases} \delta_1 x_1 + \delta_{12} x_2 + \delta_{1T} = 0 \\ \delta_{21} x_1 + \delta_{22} x_2 + \delta_{2T} = 0 \end{cases}$$

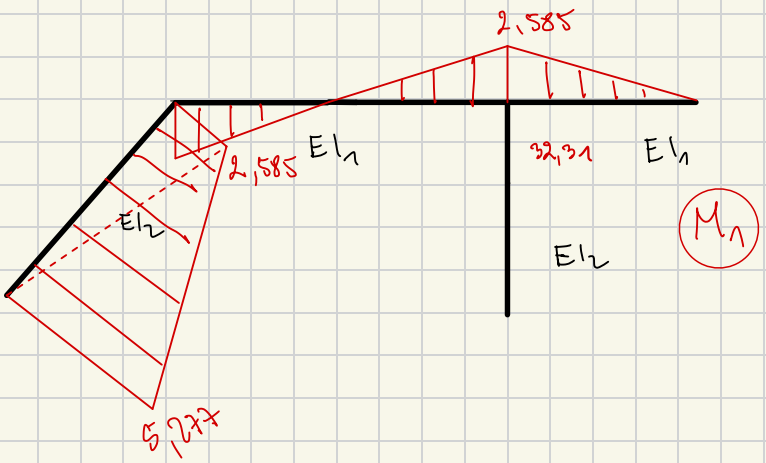
$$\delta_{iT} = \sum \int_x M_i \alpha_t \cdot \frac{\Delta t}{h} dx + \sum \int_x N_i \alpha_t t_0 dx$$

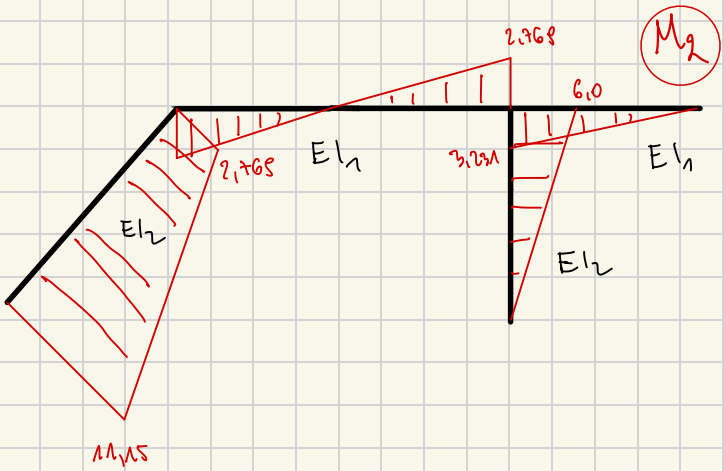
stan $x_1 = 1$

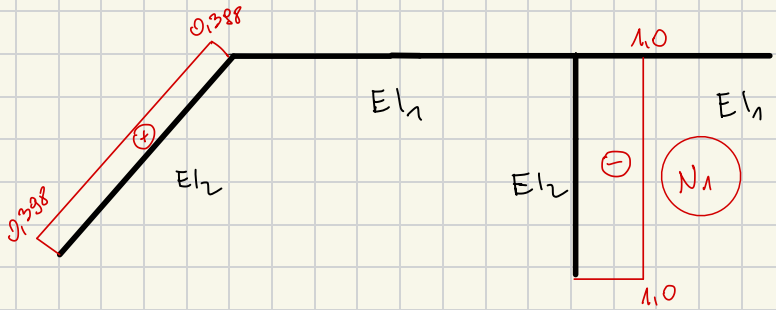


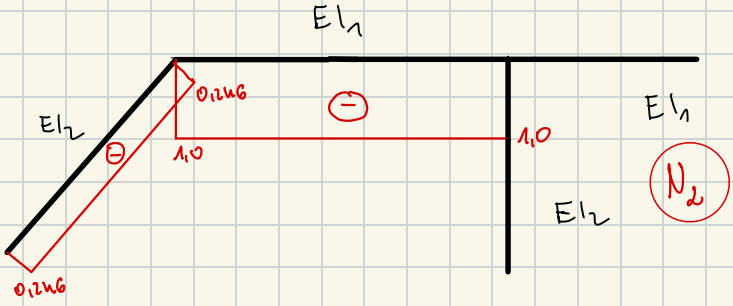
stan $x_2 = 1$











$$\delta_{11} = \frac{246,3}{EI_2}$$

$$\delta_{12} = \frac{244,7}{EI_2}$$

$$\delta_{22} = \frac{640,2}{EI_2}$$

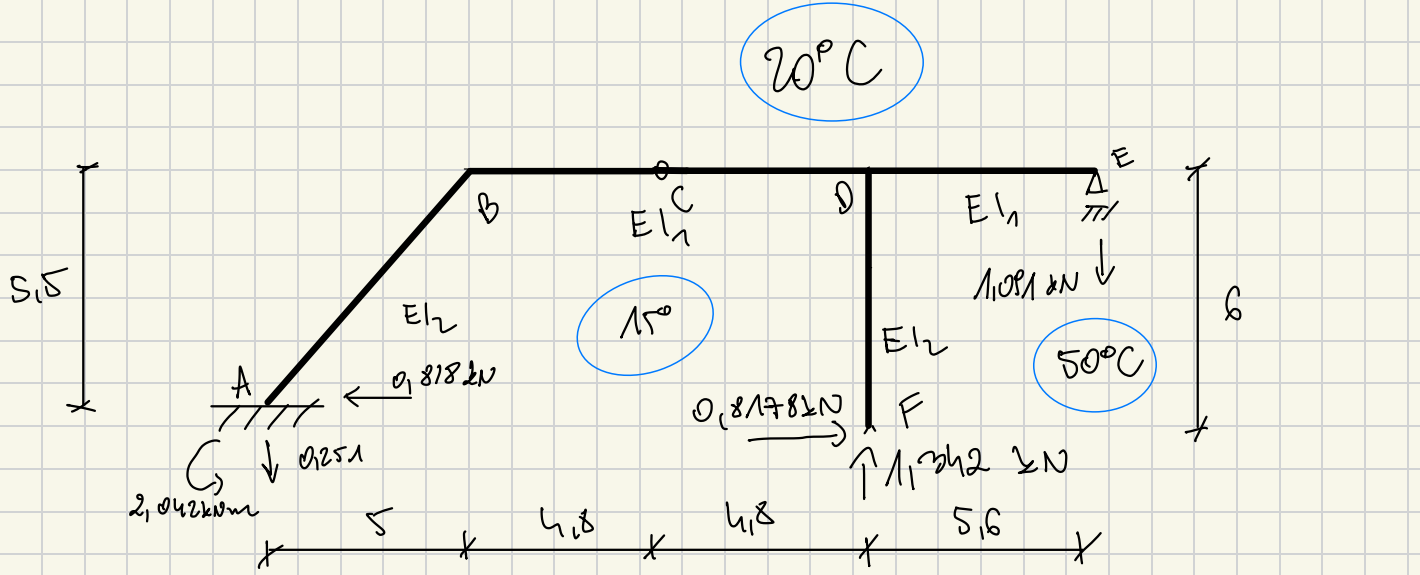
$$\delta_{1T} = \frac{1,2 \cdot 10^{-5}}{0,14}\left[\left(-\frac{1}{2}\cdot 2,585 \cdot 4,8 + \frac{1}{2}\cdot 2,585 \cdot 4,8\right)\cdot 5 - \frac{1}{2}\cdot 5,6 \cdot 2,585 \cdot 30\right] + \frac{1,2\cdot 10^{-5}}{0,2}\left[\left(-\frac{1}{2}\cdot 2,585 \cdot 7,433 - \frac{1}{2}\cdot 5,277 \cdot 7,433\right)\cdot 5\right] + 1,2\cdot 10^{-5}\left[0,388 \cdot 7,433 \cdot 15,5 - 1\cdot 6 \cdot 30,5\right] =$$

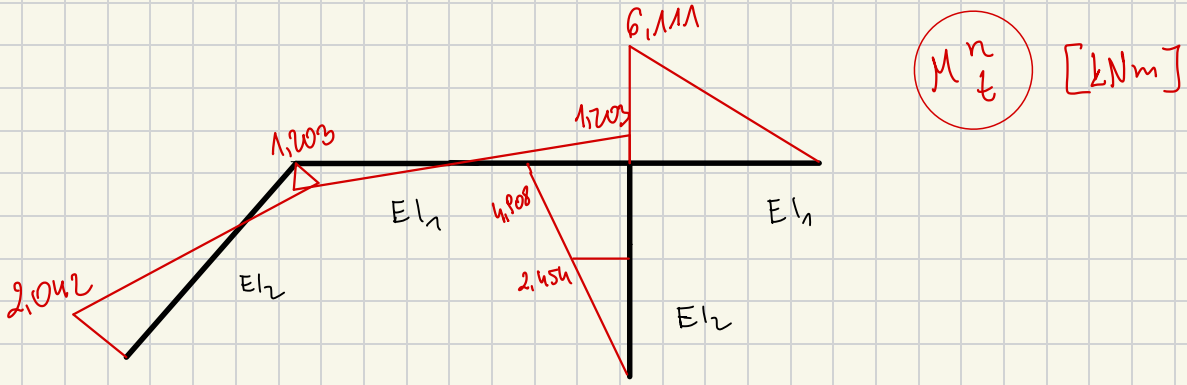
$$= -0,01861 - 0,008765 - 0,001645 = -0,02902$$

$$\delta_{2T} = \frac{1,2\cdot 10^{-5}}{0,14}\left[\left(\frac{1}{2}\cdot 2,769 \cdot 4,8 + \frac{1}{2}\cdot 2,768\cdot 4,8\right)\cdot 5 + \frac{1}{2}\cdot 5,6 \cdot 3,231 \cdot 30\right] + \frac{1,2\cdot 10^{-5}}{0,2}\left[\left(-\frac{1}{2}\cdot 2,768\cdot 7,433 - \frac{1}{2}\cdot 7,433\cdot 11,15\right)\cdot 5 + \frac{1}{2}\cdot 6\cdot 6\cdot 35\right] + 1,2\cdot 10^{-5}\left[-0,246\cdot 7,433\cdot 15,5 - 1\cdot 5,6\cdot 15,5\right] = 0,104341$$

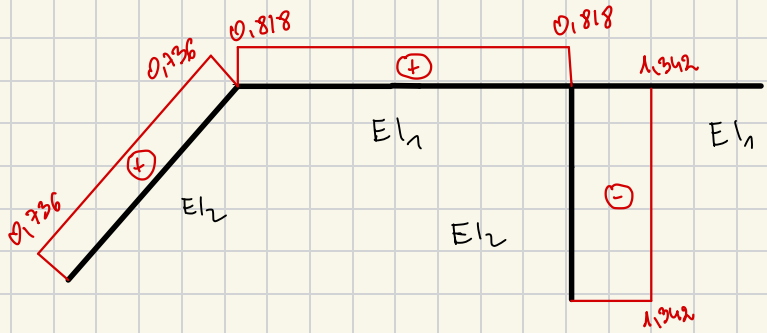
$$\begin{cases} \frac{246,3}{44,84}x_1 + \frac{244,7}{44,84}x_2 - 0,02902 = 0 \\ \frac{244,7}{44,84}x_1 + \frac{640,2}{44,84}x_2 + 0,104341 = 0 \end{cases}$$

$$\begin{cases} x_1 = 1,342\ kN \\ x_2 = -0,8178\ kN \end{cases}$$

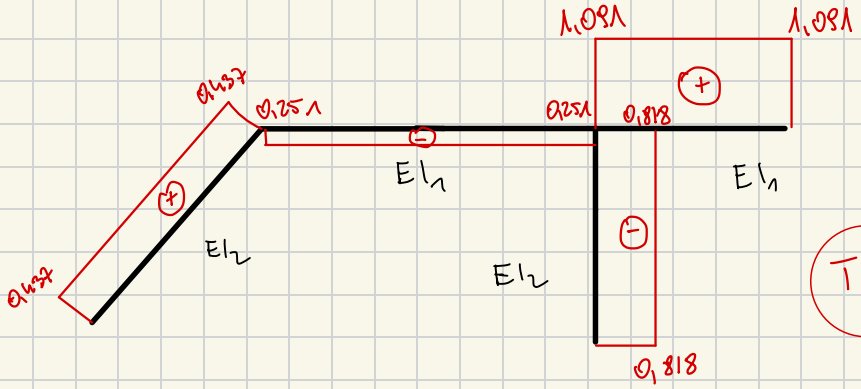




M^n_t [kNm]

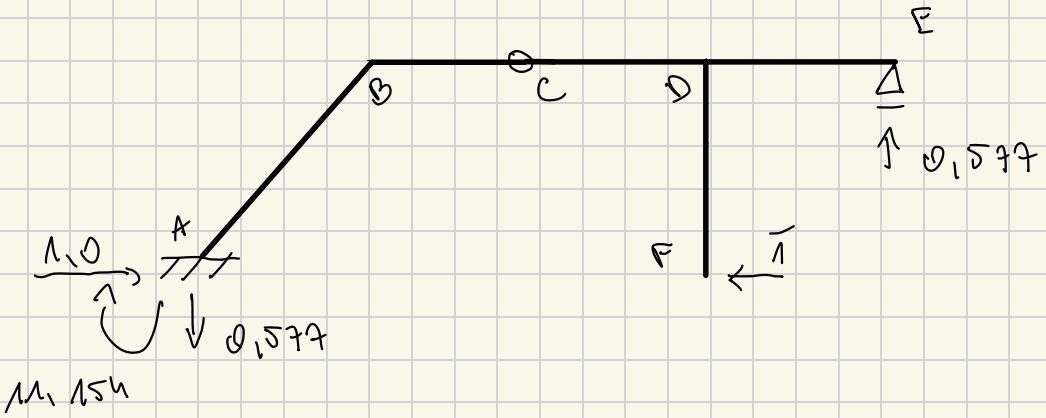


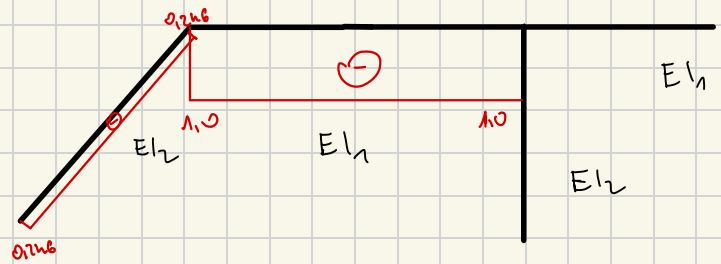
N^n_t [kN]



T^n_t [kN]

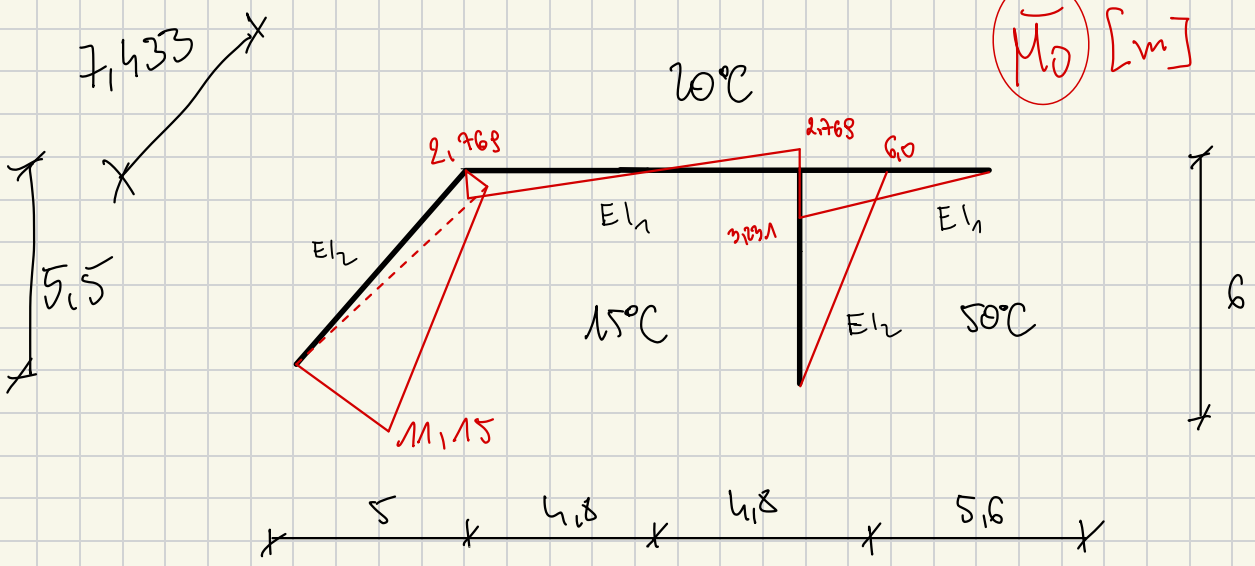
Kontrola kinematyczna

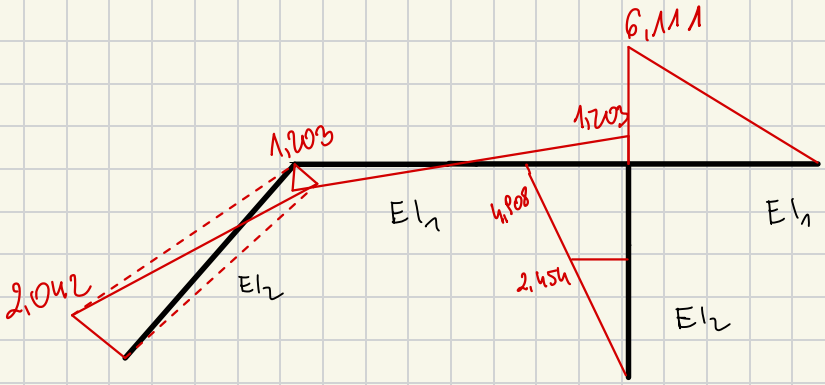




$\bar{N}_0$ [-]

$\bar{M}_0$ [m]





M_t^n [kNm]

$$\delta_F = \sum \int \frac{M_t^n \bar{M}_0}{EI} dx + \sum \frac{\bar{R}^0 \cdot R_t^n}{k} + \sum \int \bar{M}^0 \alpha_t \frac{\Delta t}{h} dx + \sum \int \bar{N}^0 \alpha_t t_0 dx$$

$$\delta_F = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}8\cdot 2{,}768\cdot\frac{2}{3}\cdot 1{,}203 + \frac{1}{2}\cdot 4{,}8\cdot 2{,}768\cdot\frac{2}{3}\cdot 1{,}203 + \right.$$

$$\left. + \frac{1}{2}\cdot 5{,}6\cdot 6{,}111\cdot\left(-\frac{2}{3}\cdot 3{,}231\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 2{,}042\cdot 7{,}433\cdot\right.$$

$$\left.\left(-\frac{1}{3}\cdot 2{,}768 - \frac{2}{3}\cdot 11{,}15\right) + \frac{1}{2}\cdot 1{,}203\cdot 7{,}433\cdot\left(\frac{1}{3}\cdot 11{,}15 + \right.\right.$$

$$\left.\left.\frac{2}{3}\cdot 2{,}768\right) + \frac{1}{2}\cdot 4{,}808\cdot 6\cdot\left(-\frac{2}{3}\cdot 6\right)\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}14}\cdot$$

$$\left[5\cdot\left(-\frac{1}{2}\cdot 2{,}768\cdot 4{,}8 + \frac{1}{2}\cdot 2{,}768\cdot 4{,}8\right) + 30\cdot\frac{1}{2}\cdot 3{,}231\cdot 5{,}6\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}2}$$

$$\cdot\left[\left(-\frac{1}{2}\cdot 2{,}768\cdot 7{,}433 - \frac{1}{2}\cdot 11{,}15\cdot 7{,}433\right)\cdot 5 + \frac{1}{2}\cdot 6\cdot 6\cdot 35\right] +$$

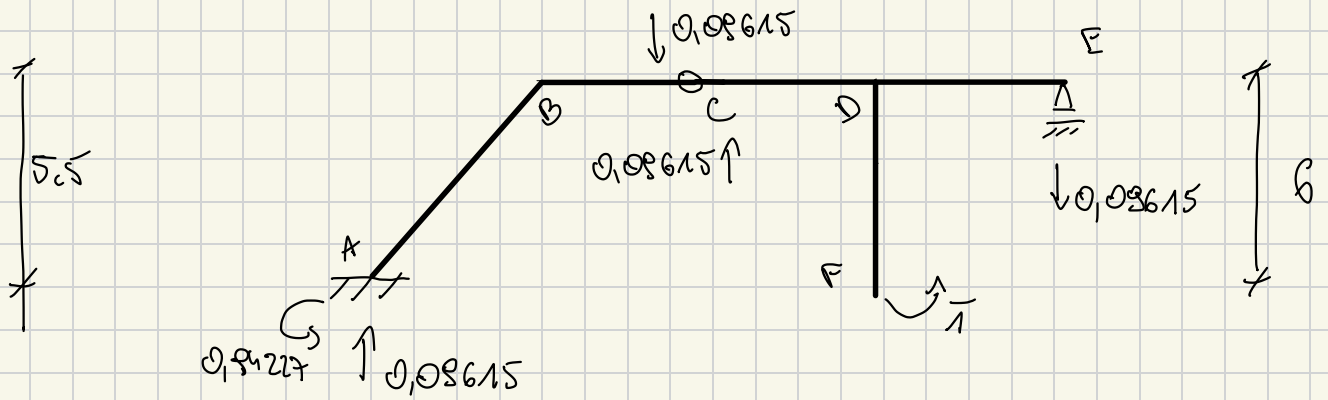
$$+ 1{,}2\cdot 10^{-5}\left[-1\cdot 8{,}6\cdot 15{,}5 - 0{,}246\cdot 7{,}433\cdot 15{,}5\right] =$$

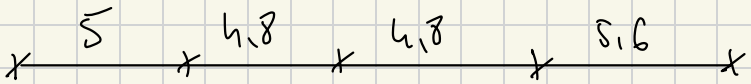
$$= \frac{-26{,}18}{EI_1} - \frac{87{,}45}{EI_2} + 0{,}02326 + 0{,}02228 - 0{,}002126 =$$

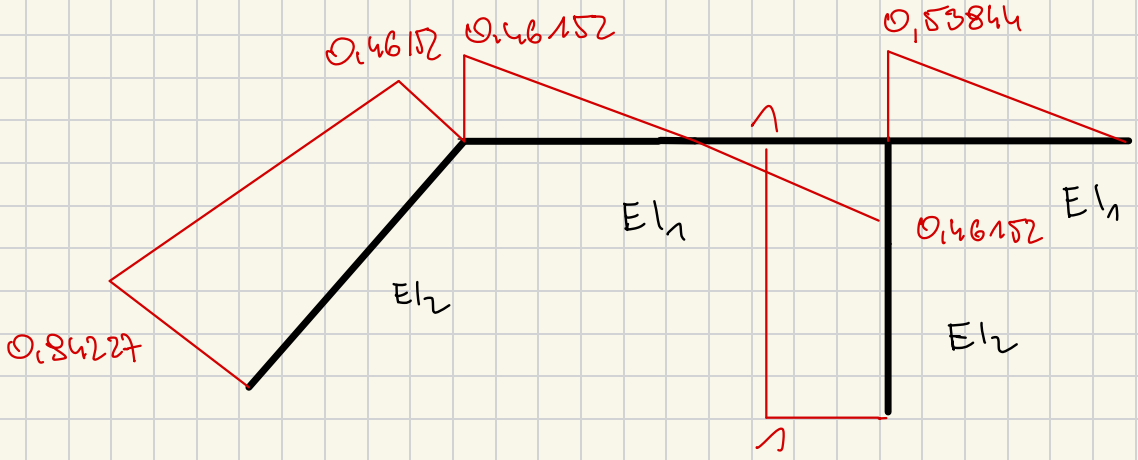
$$= -\frac{87{,}82}{EI_2} - \frac{87{,}45}{EI_2} + 0{,}04341\cdot\frac{4484}{EI_2} = -\frac{0{,}1888}{EI_2}$$

$$|\delta_F| < \frac{1}{EI_2} \Rightarrow \delta_F \cong 0$$

obrót przekroju w pkt F

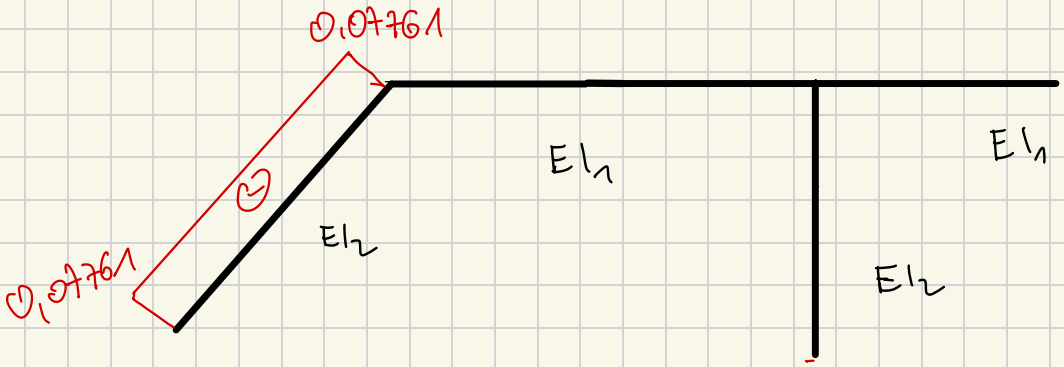




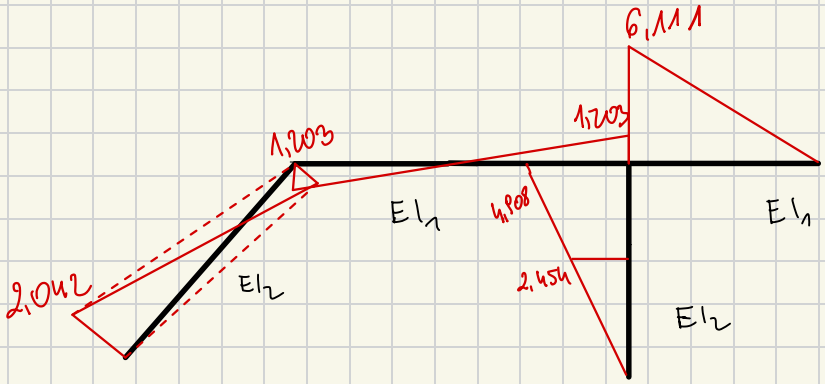


$\overline{M^0}$

[-]



$\overline{N^0}$ $\left[\frac{1}{m}\right]$



M^n_t [kNm]

$$\varphi_F = \Sigma \int \frac{M_t^n M^0}{EJ} dx + \Sigma \frac{\bar{R}^0 \cdot R_t^n}{k} + \Sigma \int \bar{M}^0 \cdot \alpha_t \cdot \frac{\Delta t}{h} dx + \Sigma \int \bar{N}^0 \cdot \alpha_t t_0 dx$$

$$\varphi_F = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 1{,}203 \cdot 4{,}8 \cdot \left(-\frac{2}{3}\cdot 0{,}46152\right) + \frac{1}{2}\cdot 1{,}203 \cdot 4{,}8 \cdot \left(-\frac{2}{3}\cdot 0{,}46152\right)\right.$$

$$\left. + \frac{1}{2}\ 5{,}6 \cdot 6{,}111 \cdot \left(\frac{2}{3}\cdot 0{,}53844\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 6 \cdot 4{,}808 \cdot 1 + \right.$$

$$+ \frac{1}{2}\cdot 7{,}433 \cdot 1{,}203 \cdot \left(-\frac{2}{3}\cdot 0{,}46152 - \frac{1}{3}\cdot 0{,}84227\right) +$$

$$\left. + \frac{1}{2}\cdot 2{,}042 \cdot 7{,}433 \cdot \left(\frac{2}{3}\cdot 0{,}84227 + \frac{1}{3}\cdot 0{,}46152\right)\right] +$$

$$+ \frac{1{,}2\cdot 10^{-5}}{0{,}14}\left[\left(\frac{1}{2}\cdot 4{,}8 \cdot 0{,}46152 - \frac{1}{2}\cdot 0{,}46152 \cdot 4{,}8\right)\cdot 5 - \frac{1}{2}\cdot\right.$$

$$\left. \cdot 5{,}6 \cdot 0{,}53844 \cdot 30\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}2}\left[-1\cdot 6 \cdot 35 + \right.$$

$$\left. + \left(\frac{1}{2}\cdot 7{,}433 \cdot 0{,}46152 + \frac{1}{2}\cdot 7{,}433 \cdot 0{,}84227\right)\cdot 5\right] +$$

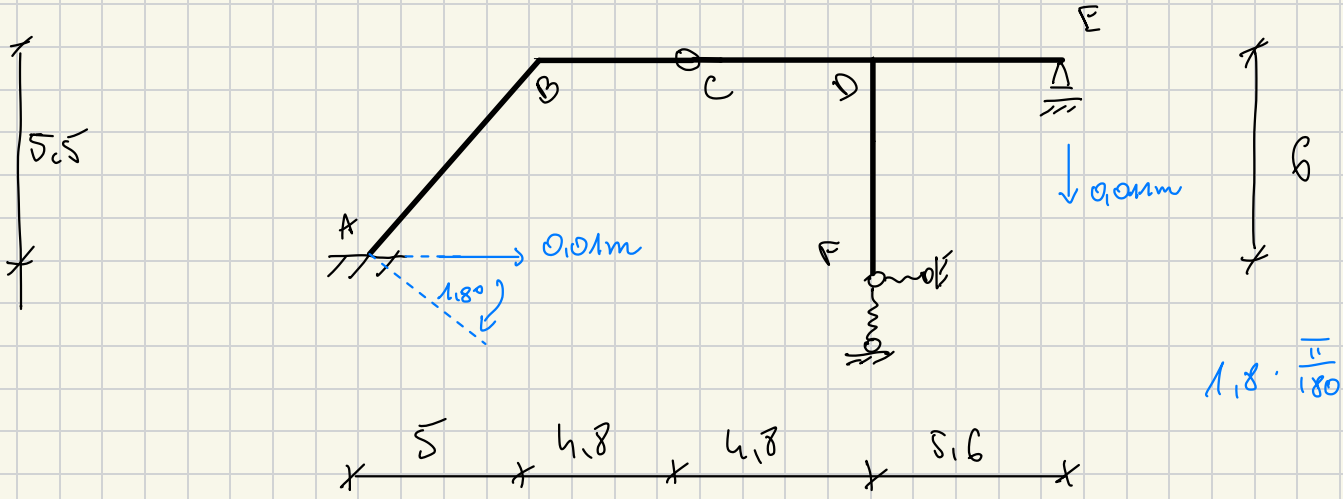
$$+ 1{,}2\cdot 10^{-5}\cdot\left[-0{,}07761 \cdot 7{,}433 \cdot 15{,}5\right] =$$

$$= \frac{4{,}36543}{EI_1} + \frac{17{,}8789}{EI_2} - 0{,}00038768 - 0{,}011035 - 0{,}000107289 =$$

$$= 0{,}003628 + 0{,}003878 - 0{,}015016 = -0{,}007409 \text{ rad}$$

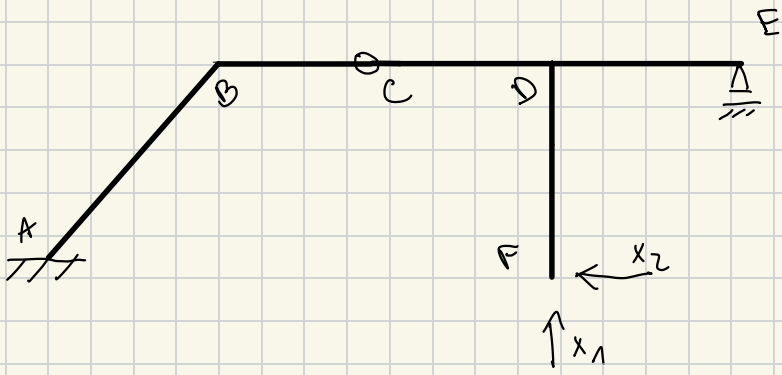
(obrót w prawo)

obliczenie sił przekrojowych od osiadania podpór



$1{,}8 \cdot \frac{\pi}{180} = 0{,}031416\ rad$

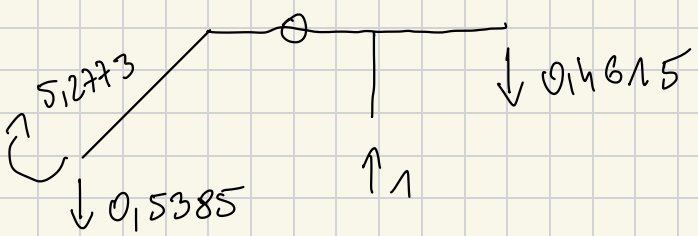
układ podstawowy



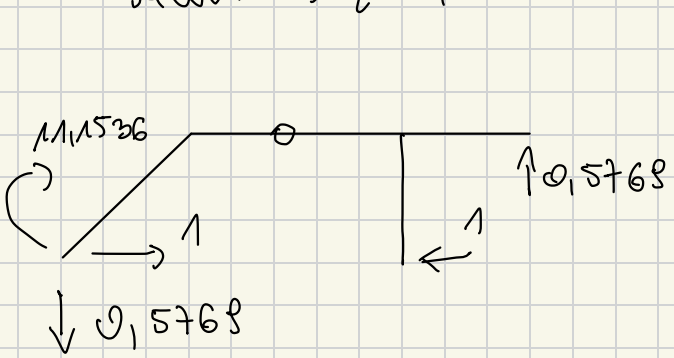
URK

$$\begin{cases} \delta_{11} x_1 + \delta_{12} x_2 + \delta_{1\Delta} = 0 \\ \delta_{21} x_1 + \delta_{22} x_2 + \delta_{2\Delta} = 0 \end{cases}$$

stan $x_1 = 1$



stan $x_2 = 1$



$$\delta_{11} = \frac{246{,}3}{EI_2}$$

$$\delta_{22} = \frac{640{,}2}{EI_2}$$

$$\delta_{12} = \frac{244{,}17}{EI_2}$$

$$\delta_{1\Delta} = -\sum R_1 \cdot \Delta = -(5{,}2773 \cdot 0{,}031416 + 0{,}1461 5 \cdot 0{,}01) = -0{,}1709\ m$$

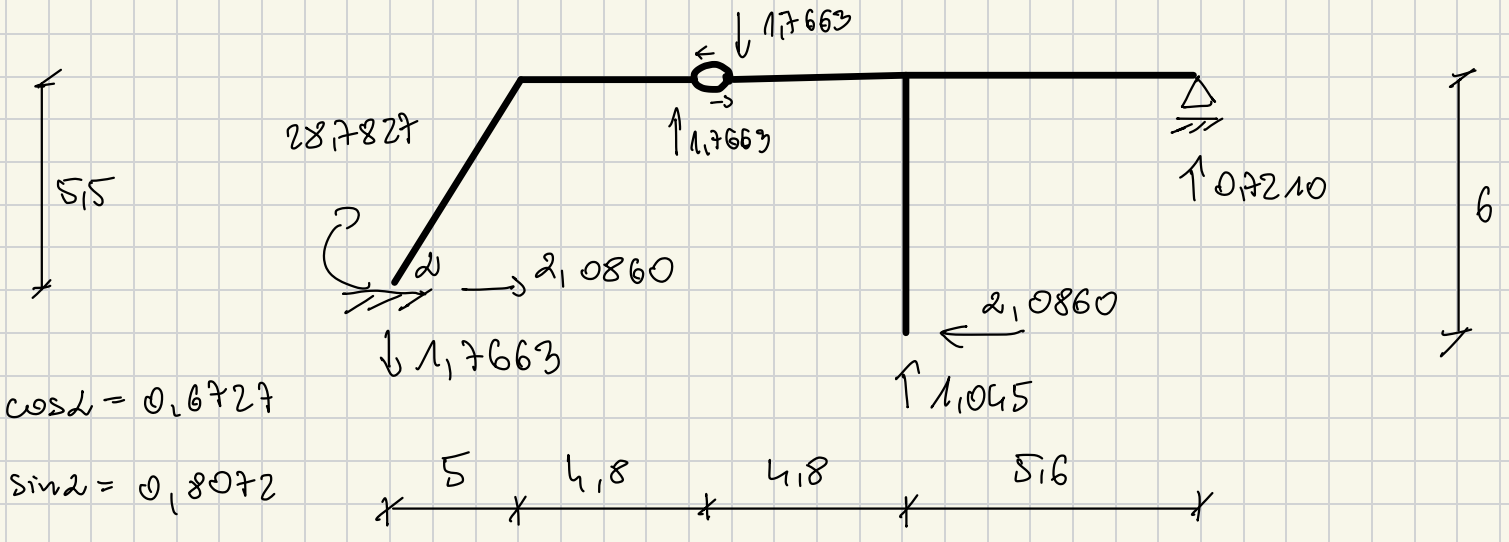
$$\delta_{2\Delta} = -(1 \cdot 0{,}01 - 0{,}1576 \cdot 0{,}01 + 11{,}1536 \cdot 0{,}031416) = -0{,}3541\ m$$

$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{1\Delta} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{2\Delta} = 0 \end{cases}$$

$$\begin{cases} \frac{2461{,}3}{4484}x_1 + \frac{2444{,}7}{4484}x_2 - 0{,}1709 = 0 \\ \frac{2444{,}7}{4484}x_1 + \frac{6402}{4484}x_2 - 0{,}3541 = 0 \end{cases}$$

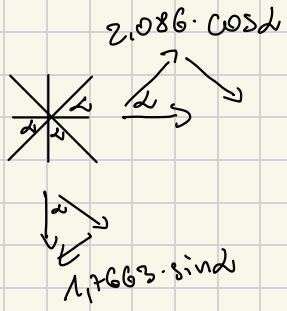
$$\begin{cases} 0{,}0548\,02\,x_1 + 0{,}0544\,51\,x_2 - 0{,}17087 = 0 \\ 0{,}0544\,51\,x_1 + 0{,}1424\,47\,x_2 - 0{,}35406 = 0 \end{cases}$$

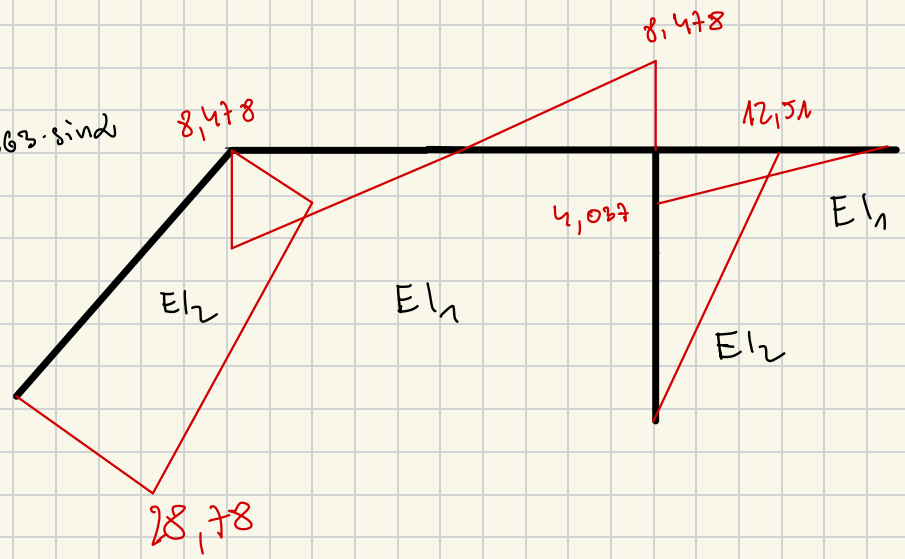
$$\begin{cases} x_1 = 1{,}045 \text{ kN} \\ x_2 = 2{,}086 \text{ kN} \end{cases}$$



$\cos\alpha = 0{,}6727$

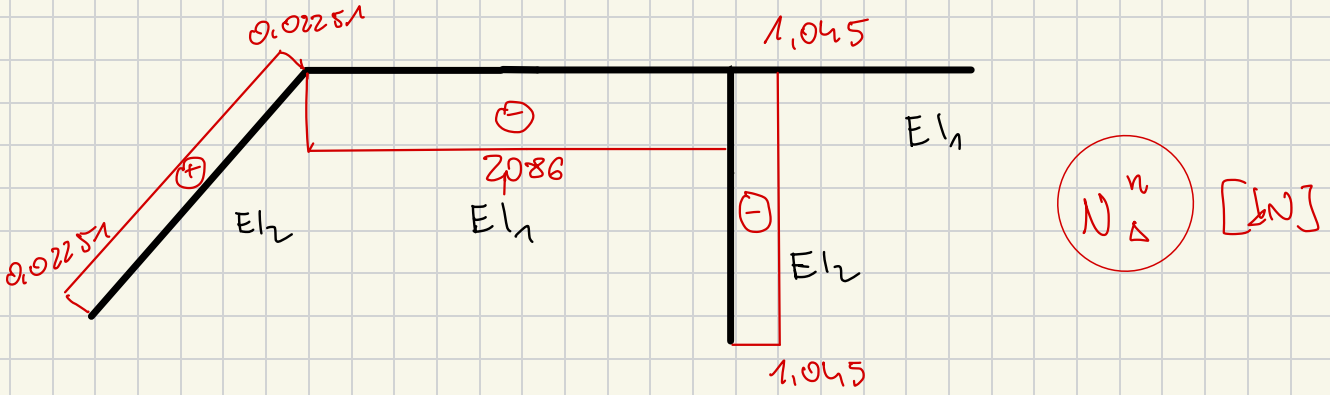
$\sin\alpha = 0{,}8072$

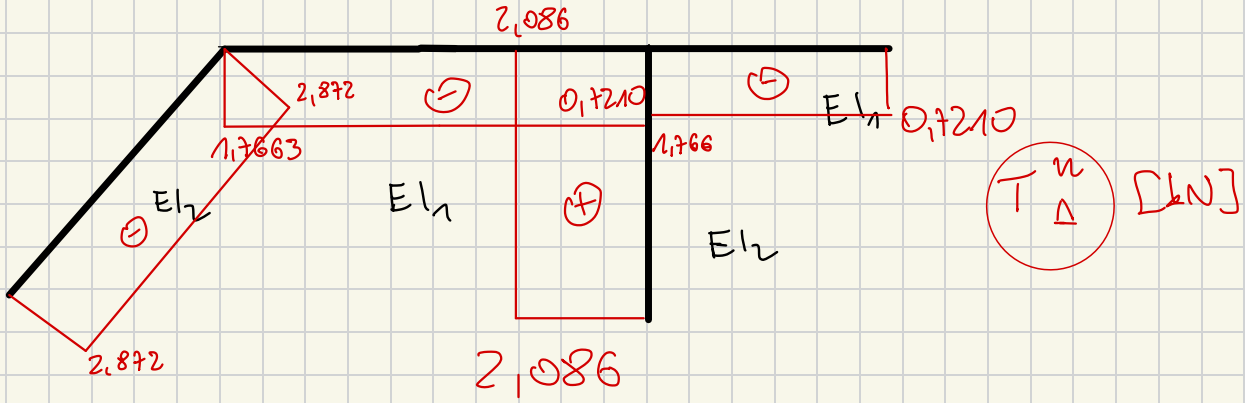




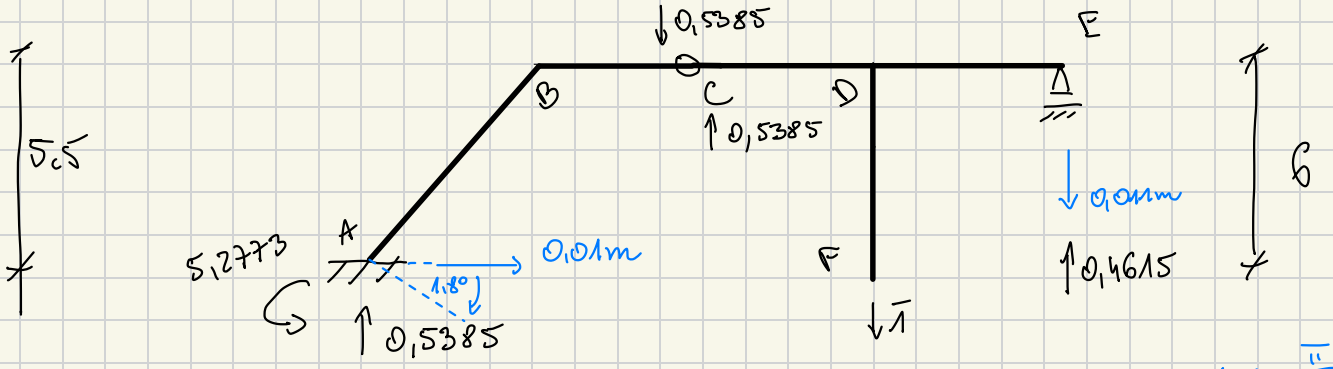
M^n_Δ

[kNm]

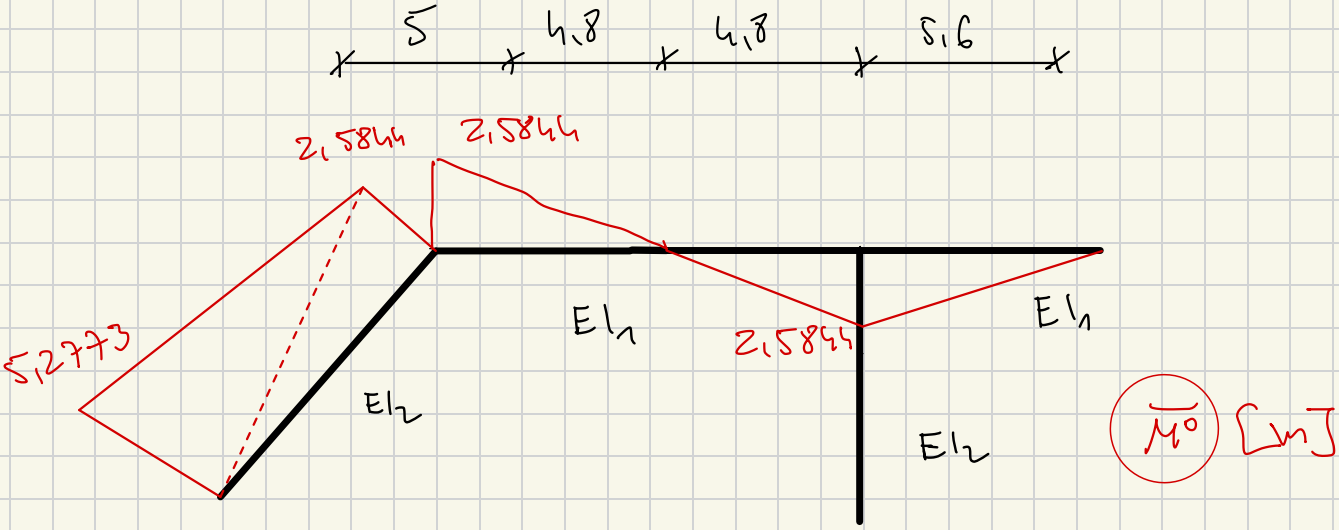


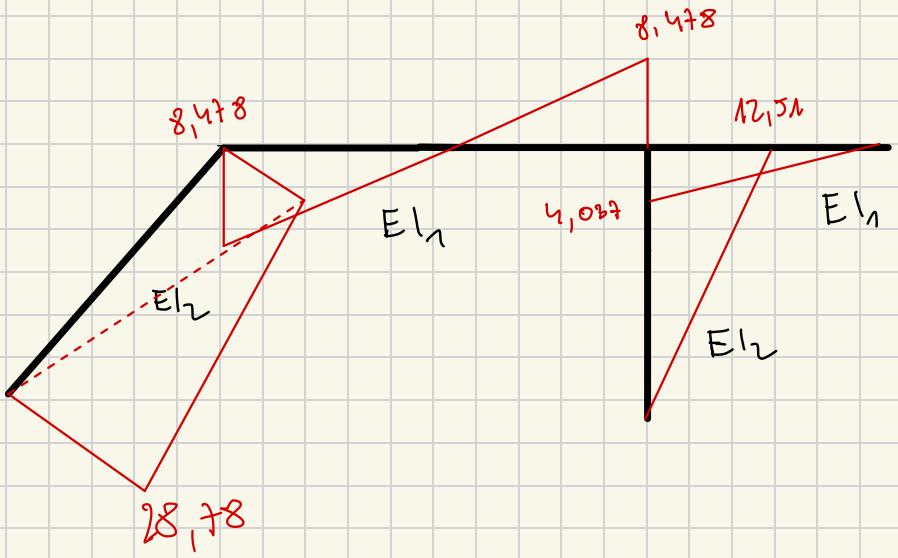


Kontrola kinematyczna



$1{,}8 \cdot \frac{\pi}{180} = 0{,}031416\ rad$





M_Δ^n [kNm]

$$\delta_F = \sum \int \frac{M_\Delta^n \overline{M}^0}{EI} dx + \sum \frac{\overline{R}^0 \cdot R_\Delta^n}{k} - \sum \overline{R}^0 \Delta$$

$$\delta_F = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}8 \cdot 8{,}4782 \cdot \left(-\frac{2}{3}\cdot 2{,}5844\right) + \frac{1}{2}\cdot 8{,}4782 \cdot 4{,}8 \cdot \left(-\frac{2}{3}\cdot 2{,}5844\right)\right.$$

$$\left. + \frac{1}{2}\cdot 4{,}0376 \cdot 5{,}6 \cdot \left(\frac{2}{3}\cdot 2{,}5844\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 7{,}433 \cdot 2{,}5844 \cdot\right.$$

$$\left.\cdot\left(-\frac{2}{3}\cdot 8{,}4782 - \frac{1}{3}\cdot 28{,}7827\right) + \frac{1}{2}\cdot 7{,}433 \cdot 5{,}2773 \cdot \left(-\frac{2}{3}\cdot 28{,}7827\right.\right.$$

$$\left.\left. - \frac{1}{3}\cdot 8{,}4782\right)\right] - \frac{1 \cdot 1{,}0453}{1{,}9804\, EI_2} - \left(-0{,}4615 \cdot 0{,}011 -\right.$$

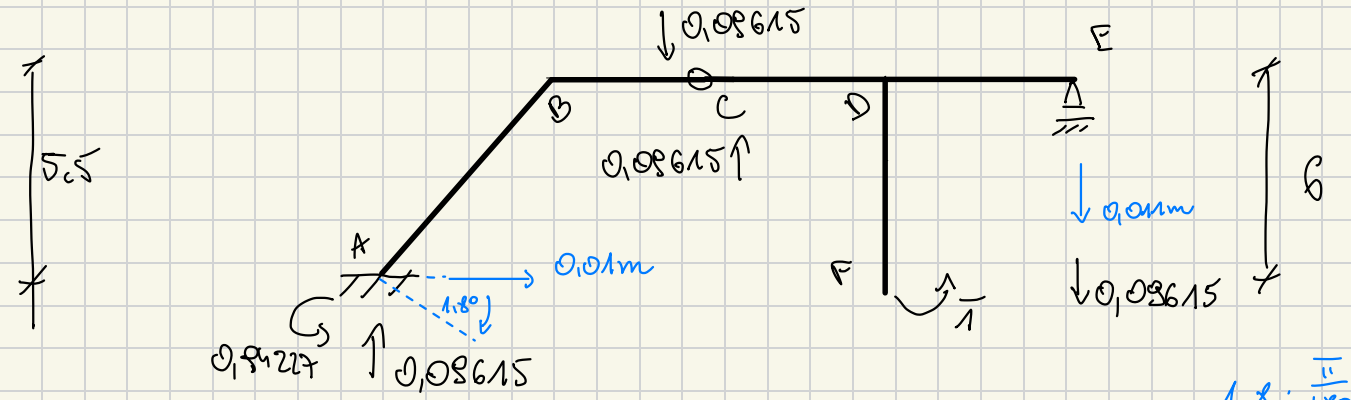
$$\left. - 5{,}2773 \cdot 0{,}031416\right) \cdot \frac{4484}{EI_2} =$$

$$= -\frac{50{,}63}{EI_1} - \frac{578{,}2}{EI_2} + \frac{767{,}8}{EI_2} - \frac{0{,}5278}{EI_2} =$$

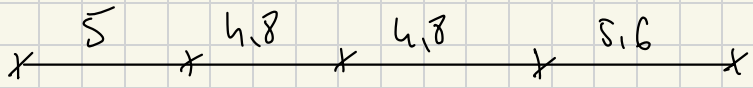
$$= \frac{-188}{EI_2} - \frac{578{,}2}{EI_2} + \frac{767{,}8}{EI_2} - \frac{0{,}5278}{EI_2} = \frac{0{,}0556}{EI_2}$$

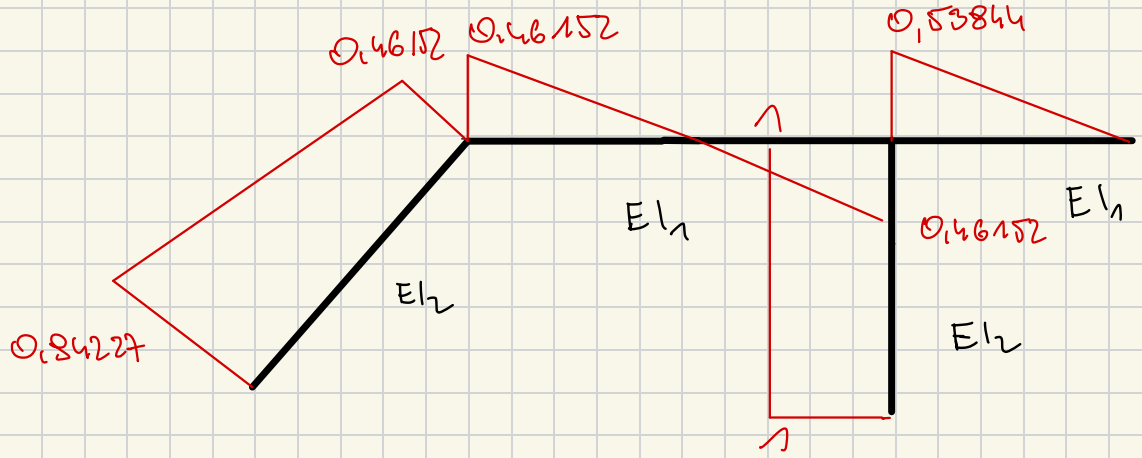
$$|\delta_F| < \frac{1}{EI_2} \Rightarrow \delta_F \cong 0$$

obrót przekroju w pkt F

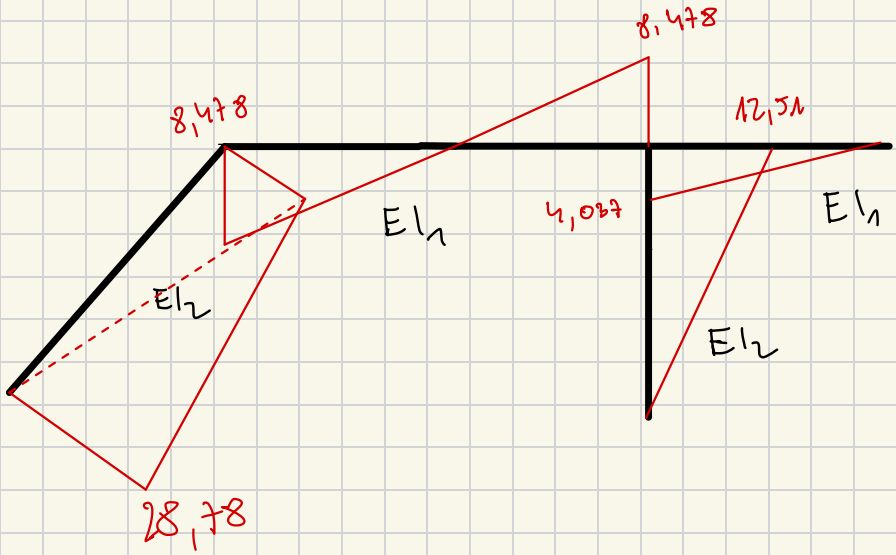


$1{,}8 \cdot \frac{\pi}{180} = 0{,}031416$ rad





$\overline{M^0}$ [-]



M^n_Δ [kNm]

$$\varphi_F = \sum \int \frac{M_\Delta^n \overline{M}^0}{EI} dx + \sum \frac{\overline{R}^0 \cdot R_\Delta^n}{k} - \sum \overline{R}^0 \Delta$$

$$\varphi_F = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}8 \cdot 8{,}4782 \cdot \left(-\frac{2}{3}\cdot 0{,}46152\right) + \frac{1}{2}\cdot 4{,}8 \cdot 8{,}4782 \cdot \left(-\frac{2}{3}\cdot 0{,}46152\right) + \frac{1}{2}\cdot 4{,}0376 \cdot 5{,}6\left(-0{,}53844 \cdot \frac{2}{3}\right)\right]$$

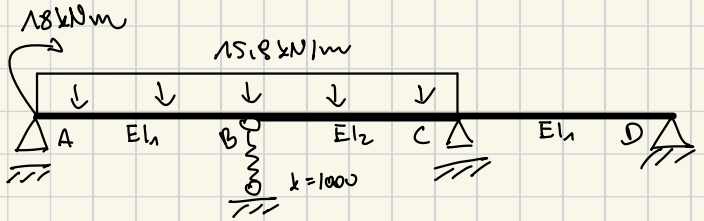
$$+ \frac{1}{EI_2}\left[\frac{1}{2}\cdot 12{,}516 \cdot 6 \cdot (-1) + \frac{1}{2}\cdot 7{,}433 \cdot 0{,}46152 \cdot \left(-\frac{2}{3}\cdot 8{,}4782 - \frac{1}{3}\cdot 28{,}7827\right) + \frac{1}{2}\cdot 0{,}84227 \cdot 7{,}433 \cdot \left(-\frac{2}{3}\cdot 28{,}7827 - \frac{1}{3}\cdot 8{,}4782\right)\right] - \left(0{,}09615 \cdot 0{,}011 - 0{,}84227 \cdot 0{,}031416\right) = -\frac{16{,}5783}{EI_1} - \frac{140{,}7828}{EI_2} +$$

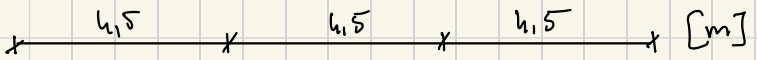
$$+ 0{,}02854 \quad = -0{,}01377 - 0{,}03133 + 0{,}02854 =$$

$$= -0{,}01656 \text{ rad}$$

(obrót w prawo)

Belka





$ssn = 5 - 3 = 2$

Przyjęte przekroje

$E = 210\ GPa$

I 140 $\quad I_1 = 573,0\ cm^4$

$EI_1 = 210 \cdot 10^6 \cdot 573 \cdot 10^{-8} = 1203,3\ kNm^2$

I 200 $\quad I_2 = 2140,0\ cm^4$

$EI_2 = 210 \cdot 2140 \cdot 10^6 \cdot 10^{-8} = 4494\ kNm^2$

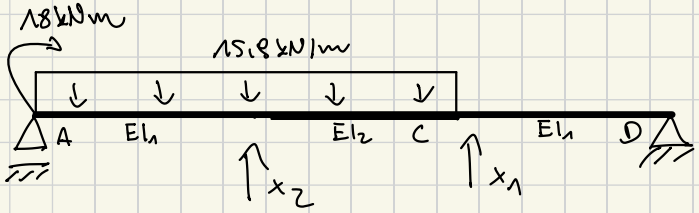
$$\frac{EI_1}{EI_2} = \frac{1203,3}{4494} = 0,2678$$

$$EI_1 = 0,2678\ EI_2 = 0,2678\ EI_0$$

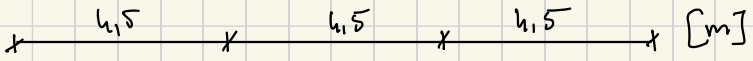
$k = 1000\ kN/m$ $\qquad \alpha = \frac{k}{EI_2} = \frac{1000}{4494} = 0,2225$

$EI_2 = 4494\ kNm^2$ $\qquad k = 0,2225\ EI_0$

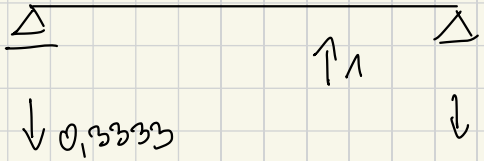
Układ podstawowy

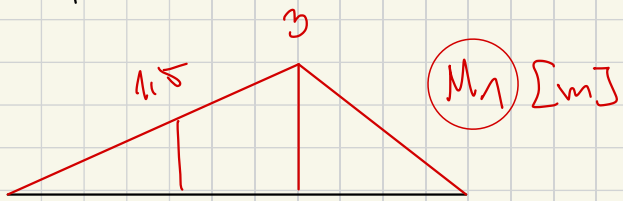


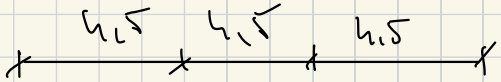
$$\begin{cases} \delta_{11} x_1 + \delta_{12} x_2 + \delta_{1P} = 0 \\ \delta_{21} x_1 + \delta_{22} x_2 + \delta_{2P} = 0 \end{cases}$$



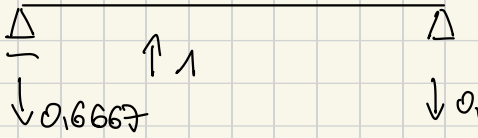
stan $x_1 = 1$

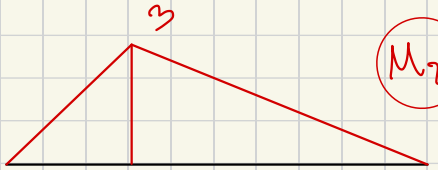




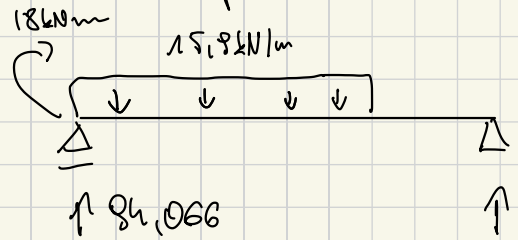


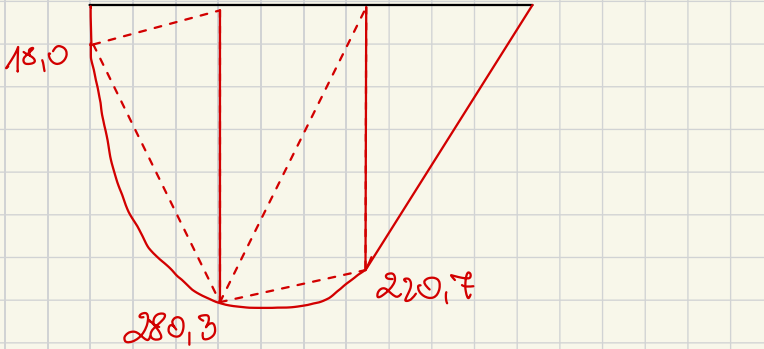
stan $x_2 = 1$





stan p





$$\delta_{11} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}5\cdot 1{,}5\cdot\frac{2}{3}\cdot 1{,}5 + \frac{1}{2}\cdot 4{,}5\cdot 3\cdot\frac{2}{3}\cdot 3\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 4{,}5\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 1{,}5+\frac{1}{3}\cdot 3\right)\right.$$

$$\left. + \frac{1}{2}\cdot 3\cdot 4{,}5\cdot\left(\frac{2}{3}\cdot 3+\frac{1}{3}\cdot 1{,}5\right)\right] = \frac{16{,}875}{0{,}2678\,E10} + \frac{23{,}625}{E10} = \frac{86{,}6384}{E10}$$

$$\delta_{12} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 1{,}5\cdot 4{,}5\cdot\frac{2}{3}\cdot 3 + \frac{1}{2}\cdot 3\cdot 4{,}5\cdot\frac{2}{3}\cdot 1{,}5\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 1{,}5\cdot 4{,}5\cdot\left(\frac{2}{3}\cdot 3+\right.\right.$$

$$\left.\left. + \frac{1}{3}\cdot 1{,}5\right) + \frac{1}{2}\cdot 3\cdot 4{,}5\cdot\left(\frac{1}{3}\cdot 3+\frac{2}{3}\cdot 1{,}5\right)\right] = \frac{13{,}5}{0{,}2678\,E10} + \frac{21{,}8375}{E10} = \frac{72{,}3483}{E10}$$

$$\delta_{22} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}5\cdot 3\cdot\frac{2}{3}\cdot 3 + \frac{1}{2}\cdot 4{,}5\cdot 1{,}5\cdot\frac{2}{3}\cdot 1{,}5\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 4{,}5\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 1{,}5+\right.\right.$$

$$\left.\left. + \frac{1}{3}\cdot 3\right) + \frac{1}{2}\cdot 3\cdot 4{,}5\cdot\left(\frac{2}{3}\cdot 3+\frac{1}{3}\cdot 1{,}5\right)\right] + \frac{1\cdot 1}{0{,}2225\,E10} = \frac{16{,}875}{0{,}2678\,E10} + \frac{23{,}625}{E10} +$$

$$+ \frac{4{,}4484}{E10} = \frac{81{,}13}{E10}$$

$$\delta_{1P} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}5\cdot 18\cdot\left(-\frac{1}{3}\cdot 1{,}5\right) + \frac{1}{2}\cdot 280{,}31\cdot 4{,}5\left(-\frac{2}{3}\cdot 1{,}5\right) + \frac{2}{3}\cdot 4{,}5\cdot\frac{15{,}8\cdot 4{,}5^2}{8}\right.$$

$$\left.\cdot\left(-\frac{1}{2}\cdot 1{,}5\right) + \frac{1}{2}\cdot 4{,}5\cdot 220{,}653\cdot\left(-\frac{2}{3}\cdot 3\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 220{,}653\cdot 4{,}5\cdot\left(-\frac{2}{3}\cdot 3\right.\right.$$

$$\left.\left. - \frac{1}{3}\cdot 1{,}5\right) + \frac{1}{2}\cdot 280{,}31\cdot 4{,}5\cdot\left(-\frac{2}{3}\cdot 1{,}5 - \frac{1}{3}\cdot 3\right) + \frac{2}{3}\cdot 4{,}5\cdot\frac{15{,}8\cdot 4{,}5^2}{8}\cdot\left(-\frac{1}{2}\cdot 1{,}5 - \frac{1}{2}\cdot 3\right)\right.$$

$$= \frac{-1734}{0{,}2678\,EI_0} - \frac{2774}{EI_0} = -\frac{8251}{EI_0}$$

$$\delta_{2P} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 18\cdot 4{,}5\cdot\left(-\frac{1}{3}\cdot 3\right) + \frac{1}{2}\cdot 280{,}31\cdot 4{,}5\cdot\left(-\frac{2}{3}\cdot 3\right) + \frac{2}{3}\cdot 4{,}5\cdot\frac{15{,}8\cdot 4{,}5^2}{8}\cdot\right.$$

$$\left.\cdot\left(-\frac{1}{2}\cdot 3\right) + \frac{1}{2}\cdot 4{,}5\cdot 220{,}653\cdot\left(-\frac{2}{3}\cdot 1{,}5\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 220{,}653\cdot 4{,}5\cdot\right.$$

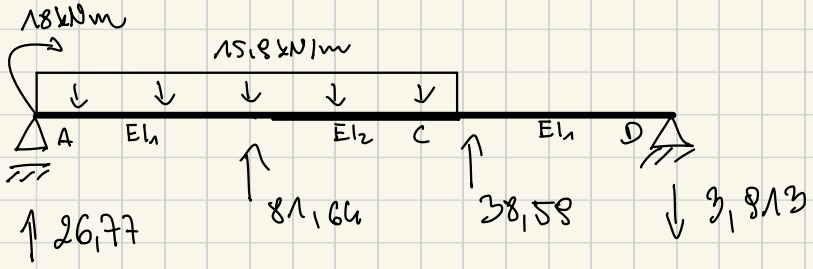
$$\cdot\left(-\frac{2}{3}\cdot 1{,}5 - \frac{1}{3}\cdot 3\right) + \frac{1}{2}\cdot 280{,}31\cdot 4{,}5\cdot\left(-\frac{2}{3}\cdot 3 - \frac{1}{3}\cdot 1{,}5\right) + \frac{2}{3}\cdot 4{,}5\cdot\frac{15{,}8\cdot 4{,}5^2}{8}\cdot$$

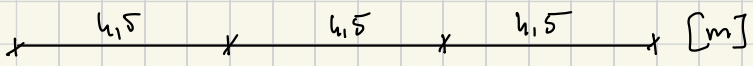
$$\left.\cdot\left(-\frac{1}{2}\cdot 3 - \frac{1}{2}\cdot 1{,}5\right)\right] = -\frac{1978}{0{,}2678\,EI_0} - \frac{2841}{EI_0} = -\frac{10232}{EI_0}$$

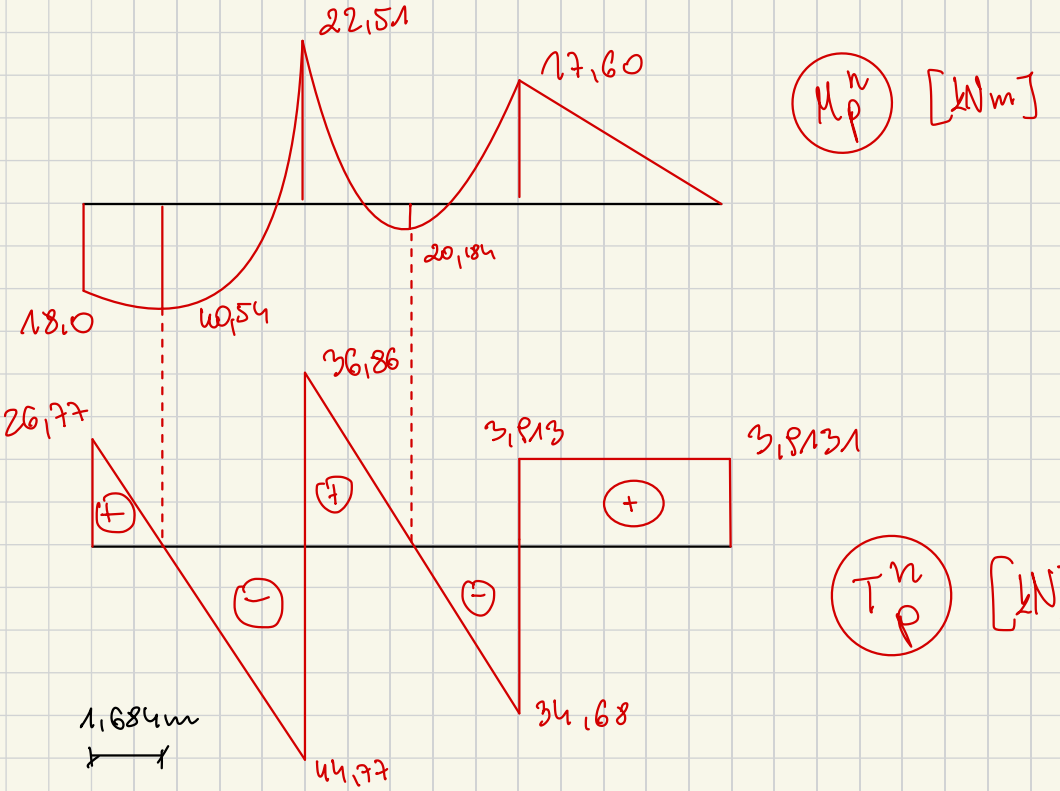
$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{1p} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{2p} = 0 \end{cases}$$

$$\begin{cases} \frac{86,63}{EI_0}x_1 + \frac{72,34}{EI_0}x_2 - \frac{8250}{EI_0} = 0 \\ \frac{72,34}{EI_0}x_1 + \frac{81,13}{EI_0}x_2 - \frac{10232}{EI_0} = 0 \end{cases}$$

$$\begin{cases} x_1 = 38,59 \text{ kN} \\ x_2 = 81,64 \text{ kN} \end{cases}$$





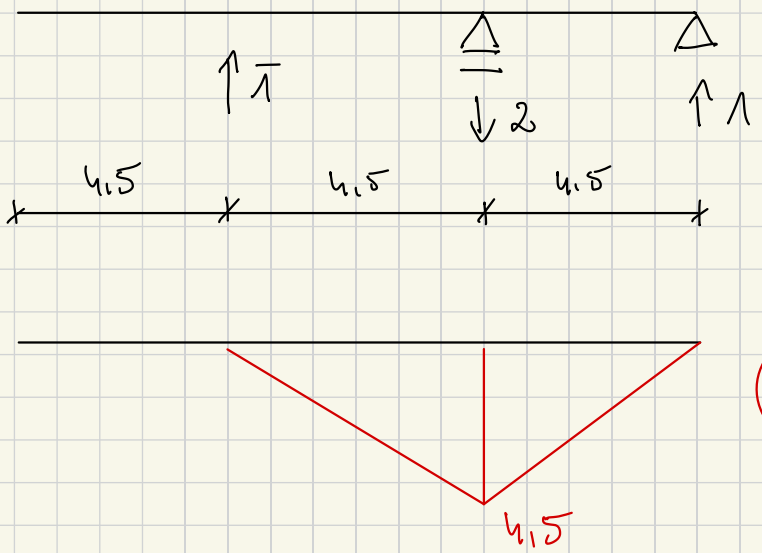


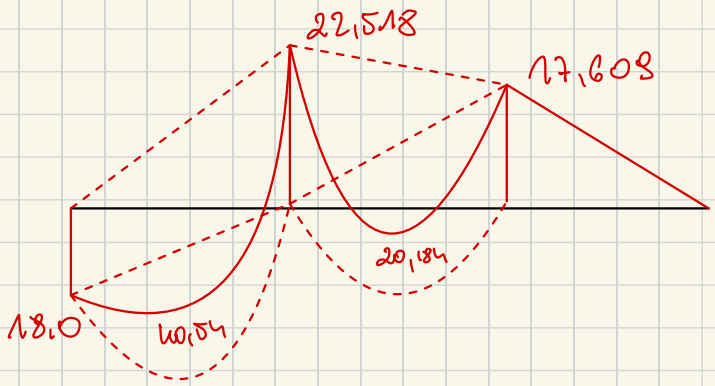
M_p^n [kNm]

T_p^n [kN]

N_p^n [kN]

Kontrola kinematyczna





M_p^n [kNm]

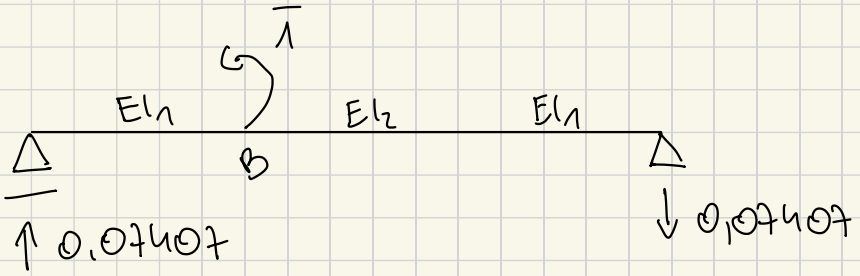
$\overline{M}^0$ [m]

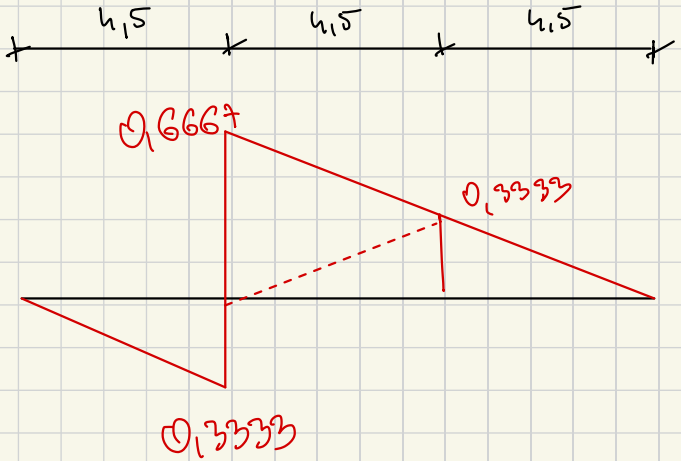
$$V_B = \sum \int \frac{M_p^n \cdot \overline{M}^0}{EI} dx + \sum \frac{\overline{R}^0 \cdot R_p^n}{k} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 4{,}5 \cdot 17{,}608 \cdot \left(-\frac{2}{3}\cdot 4{,}5\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 4{,}5 \cdot 22{,}518 \cdot \left(-\frac{1}{3}\cdot 4{,}5\right) + \frac{1}{2}\cdot 17{,}608 \cdot 4{,}5 \cdot \left(-\frac{2}{3}\cdot 4{,}5\right) + \frac{2}{3}\cdot 4{,}5 \cdot \frac{15{,}8 \cdot 4{,}5^2}{8} \cdot \frac{1}{2}\cdot 4{,}5\right] +$$

$$+ \frac{1 \cdot 81{,}64}{0{,}2225\, EI_0} = -\frac{118{,}8}{0{,}2678\, EI_0} + \frac{76{,}180}{EI_0} + \frac{366{,}8}{EI_0} = -\frac{443{,}8}{EI_0} + \frac{443{,}7}{EI_0} =$$

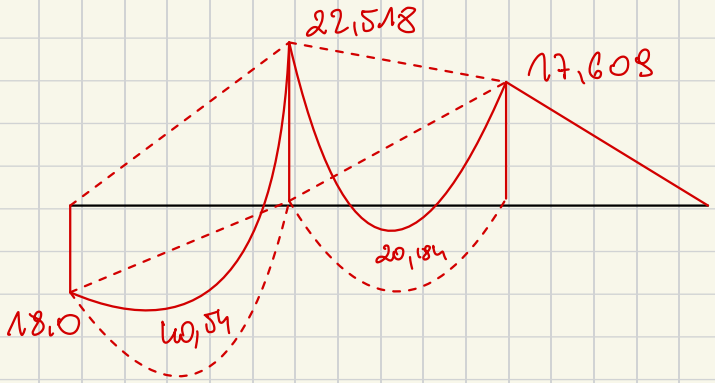
$$= \frac{-0{,}0813}{EI_0} \qquad |V_B| < \frac{1}{EI_0} \Rightarrow V_B \cong 0$$

obrót w punkcie B





$\overline{M^0}$ [-]



M_p^n [kNm]

$$\overline{1} \cdot \varphi_B = \sum \int \frac{M_p^n \cdot \overline{M^0}}{EI} dx + \sum \frac{R_p^w \cdot \overline{R^0}}{k} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 22{,}518 \cdot 4{,}5 \cdot \left(-\frac{2}{3}\cdot 0{,}3333\right) + \frac{1}{2}\cdot 18 \cdot 4{,}5 \cdot \frac{1}{3}\cdot 0{,}3333 + \right.$$

$$\left. + \frac{2}{3}\cdot 4{,}5 \cdot \frac{15{,}8 \cdot 4{,}5^2}{8} \cdot \frac{1}{2}\cdot 0{,}3333 + \frac{1}{2}\cdot 17{,}608 \cdot 4{,}5 \cdot \frac{2}{3}\cdot 0{,}3333\right] + \frac{1}{EI_2}\left[\frac{1}{2}\, 22{,}518 \cdot 4{,}5 \cdot \right.$$

$$\left(\frac{2}{3}\cdot 0{,}6667 + \frac{1}{3}\cdot 0{,}3333\right) + \frac{1}{2}\cdot 4{,}5 \cdot 17{,}608 \cdot \left(\frac{2}{3}\cdot 0{,}3333 + \frac{1}{3}\cdot 0{,}6667\right) +$$

$$\left. + \frac{2}{3}\cdot 4{,}5 \cdot \frac{15{,}8 \cdot 4{,}5^2}{8} \cdot \left(-\frac{1}{2}\cdot 0{,}6667 - \frac{1}{2}\cdot 0{,}3333\right)\right] = \frac{22{,}1667}{EI_1} - \frac{14{,}6137}{EI_2} = 0{,}01517 \text{ rad}$$

(obrót w lewo)

sprawdzenie doboru przekrojów

przekrój $\mathcal{D}_1$ I140

$h = 140\,mm$

$\sigma_{dop} = 235\,MPa$

$$\sigma = \frac{|M|}{W} + \frac{|N|}{A} = \frac{4054,0}{81,8} = 49,50\ \frac{kN}{cm^2} = 495,0\,MPa > \sigma_{dop}$$

przekrój $\mathcal{D}_2$ I 200

$h = 200\,mm$

$\sigma_{dop} = 235\,MPa$

$$\sigma = \frac{2252}{214} = 10,52\ \frac{kN}{cm^2} = 105,2\,MPa < \sigma_{dop}$$

Wnioski: Naprężenia w pierwszym przekroju są większe niż dopuszczalne. Należy przyjąć większy przekrój. Naprężenia w drugim przekroju są mniejsze niż dopuszczalne ze sporym zapasem nośności. Należy zatem przyjąć mniejszy przekrój, który będzie bardziej ekonomiczny. Zmiana przekrojów spowoduje zmianę współczynnika $n = \frac{\mathcal{D}_1}{\mathcal{D}_2}$, trzeba więc całe zadanie przeliczyć od nowa i ponownie sprawdzić naprężenia.