

MECHANIKA BUDOWLI / SEM.3

ĆWICZENIE NR 1

**OBLICZANIE PRZEMIESZCZEŃ W UKŁADACH STATYCZNIE
WYZNACZALNYCH - METODA PRAC WIRTUALNYCH**

Termin oddania: 13.11.2024

Data	Uwagi sprawdzającego	Podpis

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Dla zadanej **RAMY**: / Nr schematu: 4

1. Przyjąć wstępnie jednakowy dla wszystkich prętów przekrój dwuteowy (IN, IPE, HEB, HEA), tak aby pod działaniem podanego obciążenia powstałe w prętach naprężenia normalne spełniały warunek $\sigma \leq f_y$.
2. Obliczyć wywołane zadanymi siłami zewnętrznymi:
 - a) **przemieszczenie wypadkowe w punkcie "D"**
 - b) **obrót przekroju w punkcie "S"**UWAGA! W obliczeniach pominąć wpływ sił tnących i normalnych.
3. Obliczyć **przemieszczenie pionowe w punkcie "S"** wywołane zadanymi wpływami termicznymi oraz kinematycznymi (osiadaniem podpór).

Dla zadanej **KRATOWNICY**: / Nr schematu: 4

1. Przyjąć wstępnie jednakowy dla wszystkich prętów przekrój - z kształtowników zamkniętych kwadratowych (SHS), tak aby pod działaniem podanego obciążenia powstałe w prętach naprężenia normalne spełniały warunek $\sigma \leq f_y$.
2. Obliczyć wywołane zadanymi siłami zewnętrznymi **przemieszczenie pionowe węzła nr: 6**

W obliczeniach przyjąć: $E = 210$ GPa, $f_y = 235$ MPa.

DANE DLA RAMY

Nr schematu: 4

wymiary						
a [m]	b [m]	c [m]	d [m]	e [m]	n	m
5	2,8	4	4,8	5	0,7	0,7

sztywność podpór [kN/m]	
szt. liniowa podpory w punkcie: "A", po kierunku "x"	szt. pozostałych podpór:
2000	∞

obciążenie siłami zewnętrznymi					
q ₁ [kN/m]	q ₂ [kN/m]	P ₁ [kN]	P ₂ [kN]	P ₃ [kN]	M [kNm]
-	12,4	-25	-	-5	-14

obciążenie termiczne		
t ₁ [°C]	t ₂ [°C]	t _m [°C]
-5	15	-10

obciążenie kinematyczne (osiadanie podpór)				
δ ₁ [cm]	δ ₂ [cm]	δ ₃ [cm]	δ ₄ [cm]	φ [°]
2	-	-2,3	-	-1,7

UWAGA!

Wartości dodatnie obciążeń zadanych w tabeli oznaczają, że obciążenia te mają zwrot zgodny z zadaniem na schemacie ramy, ujemne – przeciwny.

DANE DLA KRATOWNICY

Nr schematu: 4

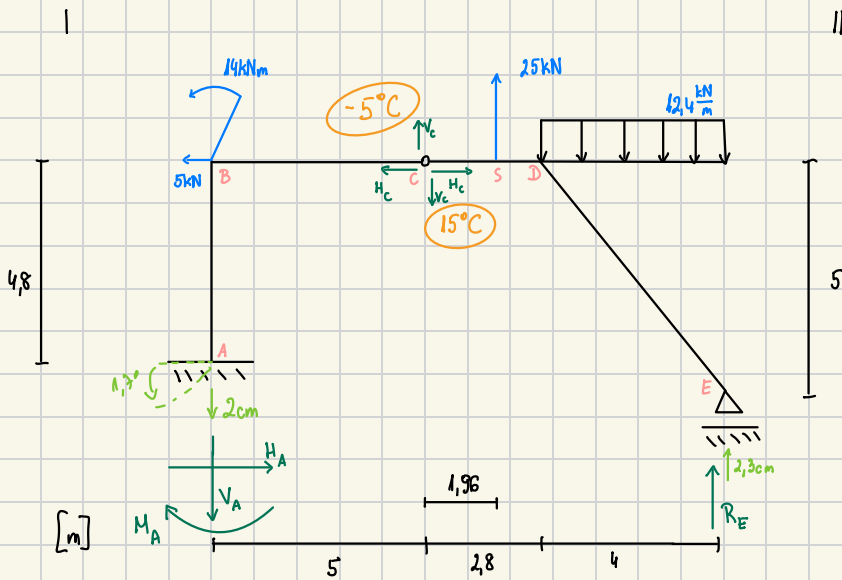
wymiary		położenie podpory "B":	sztywność podpór [kN/m]
a [m]	h [m]	nr węzła	
2,6	2,6	1	∞

obciążenie		
nr węzła	kierunek siły	wartość siły [kN]
12	pionowy	-26
9	pionowy	-32
12	poziomy	-40

UWAGA!

Wartość dodatnia sił zadanych w tabeli oznacza zwrot zgodny z osią układu współrzędnych, który jest zaznaczony przy schemacie kratownicy.

nr schematu 4



$$t_m = -10^\circ\text{C}$$

wyznaczenie reakcji:

$$\sum X'' : H_c = 0 \text{ kN}$$

$$\begin{aligned} \sum M^C : -R_E \cdot (2,8 + 4) + 12,4 \cdot 4 \cdot (2,8 + 2) - 25 \cdot 1,96 &= 0 \\ -6,8R_E + 238,08 - 49 &= 0 \\ 6,8R_E &= 189,08 \\ R_E &= 27,81 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M^E : -V_c(2,8 + 4) + (0,84 + 4) \cdot 25 - 12,4 \cdot 4 \cdot 2 &= 0 \\ -6,8V_c + 121 - 99,2 &= 0 \\ 6,8V_c &= 21,8 \\ V_c &= 3,206 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y' : -V_A + V_c &= 0 \\ V_A &= 3,206 \text{ kN} \end{aligned}$$

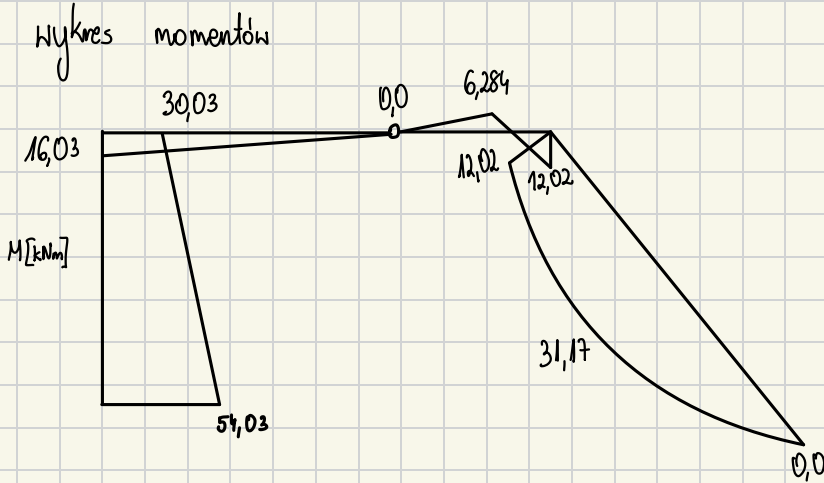
$$\sum X': H_A - 5 = 0$$

$$H_A = 5 \text{ kN}$$

$$\sum M^O = M_A - 5 \cdot 3,206 - 5 \cdot 4,8 - 14 = 0$$

$$M_A - 16,03 - 24 - 14 = 0$$

$$M_A = 54,03 \text{ kNm}$$



$$M_{A^a} = 54,03 \text{ kNm}$$

$$M_{B^a} = 54,03 - (5 \cdot 4,8) = 30,03 \text{ kNm}$$

$$M_{B^c} = -14 + 54,03 - (5 \cdot 4,8) = 16,03 \text{ kNm}$$

$$M_{C^c} = 0,0 \text{ kNm}$$

$$M_{C^f} = 0,0 \text{ kNm}$$

$$M_K = -6,284 \text{ kNm}$$

$$M_D = 12,02 \text{ kNm}$$

$$M_{DE}^L = 12,02 \text{ kNm}$$

$$M_{DE}^P = 0,0 \text{ kNm}$$

$$M_A = 31,17 \text{ kNm}$$

projektowanie przekroju

$$\sigma_x = \frac{M_{\max}}{W_x} \Rightarrow N_x = \frac{M_{\max}}{\sigma_x}$$

$$M_{\max} = 54,03 \text{ kNm} = 5403 \text{ kNm}$$

$$f_y = 235 \text{ MPa}$$

$$\sigma_{\text{dop}} = 235 \text{ MPa} = 23,5 \frac{\text{kN}}{\text{cm}^2}$$

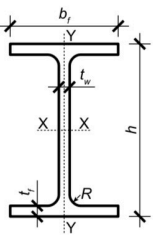
$$\sigma \leq \sigma_{\text{dop}}$$

$$N \geq \frac{5403}{23,5}$$

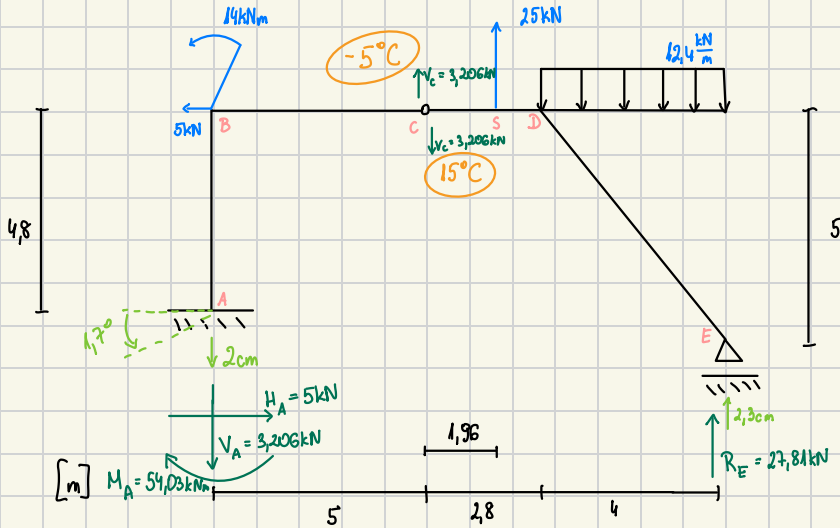
$$N \geq 229,9 \text{ cm}^3$$

przyjęto dwuteownik 160 HEB o wskaźniku wytrzymałości $n = 3 \text{ M cm}^3$

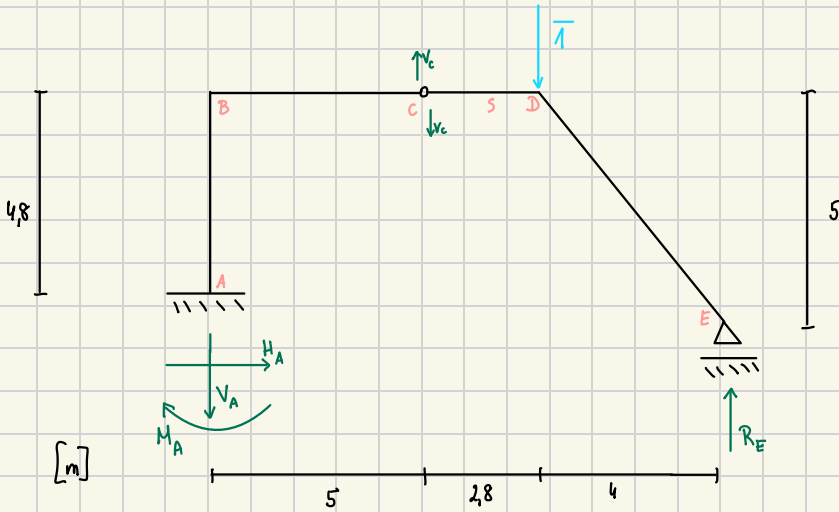
Tabela 1.3. Dane dla dwuteowników szerokostopowych HEB



HEB	h [mm]	b _f [mm]	t _w [mm]	t _f [mm]	R [mm]	A [cm ²]	J _x [cm ⁴]	J _y [cm ⁴]	W _x [cm ³]
100	100,0	100,0	6,0	10,0	12,0	26,00	450,00	167,00	89,90
120	120,0	120,0	6,5	11,0	12,0	34,00	864,00	318,00	144,00
140	140,0	140,0	7,0	12,0	12,0	43,00	1510,00	550,00	216,00
160	160,0	160,0	8,0	13,0	15,0	54,30	2490,00	889,00	311,00
180	180,0	180,0	8,5	14,0	15,0	65,30	3830,00	1360,00	426,00
200	200,0	200,0	9,0	15,0	18,0	78,10	5700,00	2000,00	570,00
220	220,0	220,0	9,5	16,0	18,0	91,00	8090,00	2840,00	736,00
240	240,0	240,0	10,0	17,0	21,0	106,00	11260,00	3920,00	938,00
260	260,0	260,0	10,0	17,5	24,0	118,00	14920,00	5130,00	1150,00
280	280,0	280,0	10,5	18,0	24,0	131,00	19270,00	6590,00	1380,00
300	300,0	300,0	11,0	19,0	27,0	149,00	25170,00	8560,00	1680,00
320	320,0	300,0	11,5	20,5	27,0	161,00	30820,00	9240,00	1930,00
340	340,0	300,0	12,0	21,5	27,0	171,00	36660,00	9690,00	2160,00



$t_m = -10^\circ\text{C}$



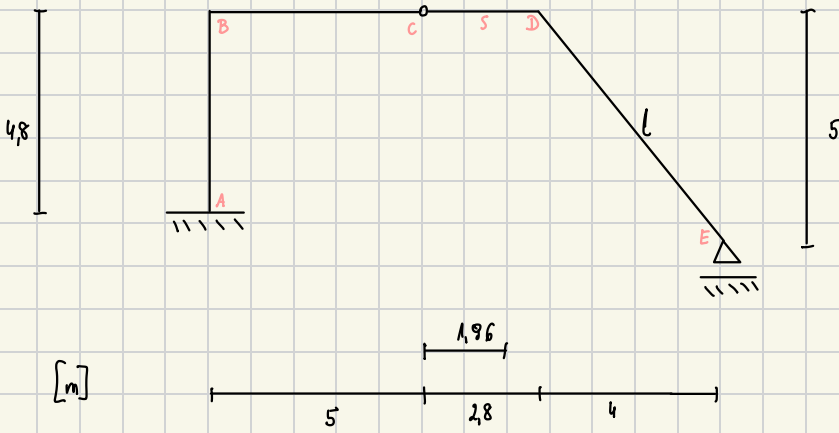
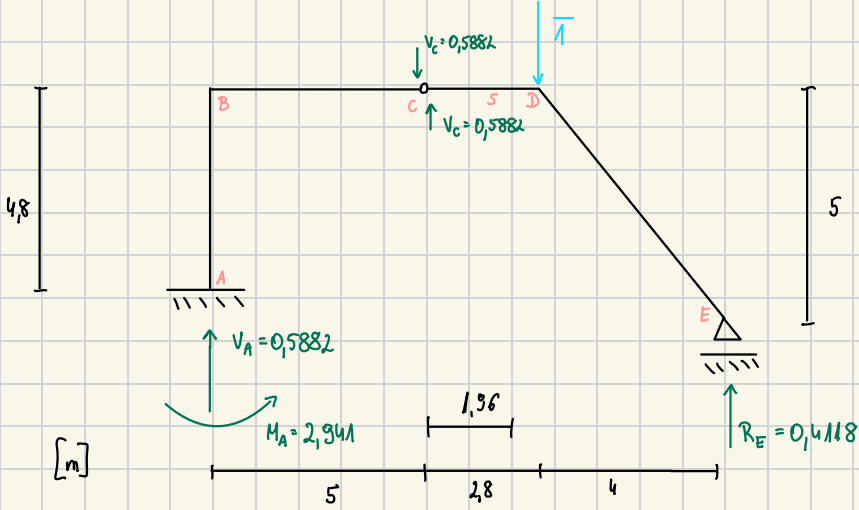
$$\begin{aligned} \sum M_C'' &= -R_E(2.8+4) + 1 \cdot 2.8 = 0 \\ &= -6.8R_E + 2.8 = 0 \\ &6.8R_E = 2.8 \\ &R_E = 0.4118 [-] \end{aligned}$$

$$\sum X = H_A = 0 [-]$$

$$\begin{aligned} \sum Y'' &= R_E - 1 - V_c = 0 \\ 0.4118 - 1 &= V_c \\ -0.5882 &= V_c \end{aligned}$$

$$\begin{aligned} \sum Y' &= -0.5882 - V_A = 0 \\ -0.5882 &= V_A \end{aligned}$$

$$\begin{aligned} \sum M_C' &= M_A + 0.5882 \cdot 5 = 0 \\ M_A &= -2.941 \text{ m} \end{aligned}$$



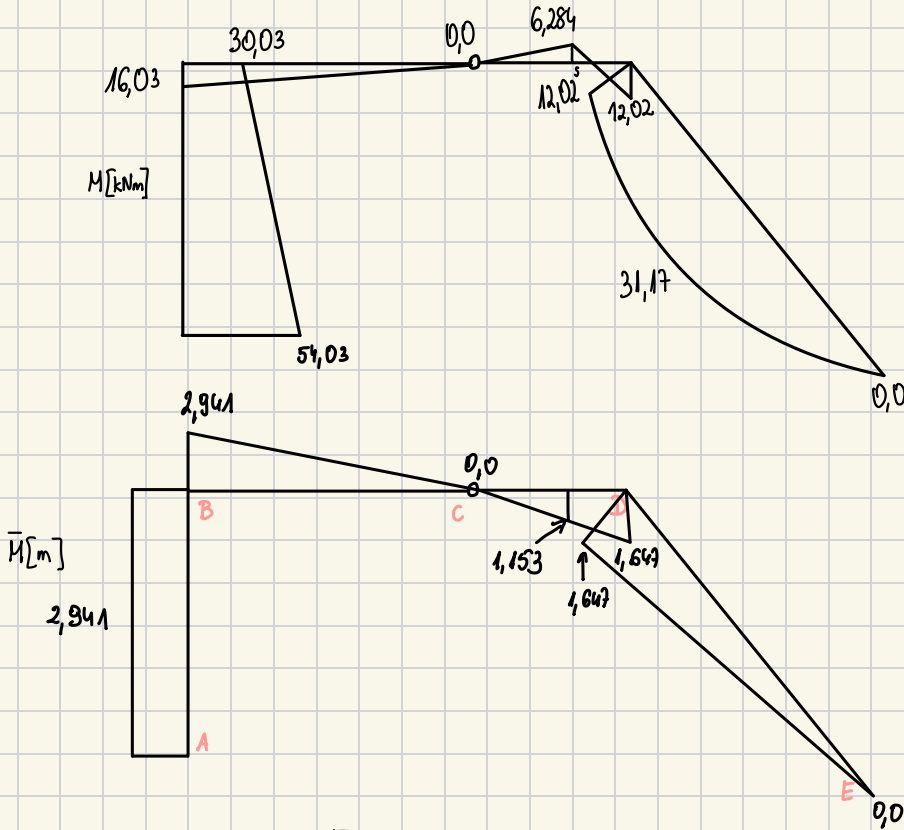
$$l^2 = 5^2 + 4^2$$

$$l^2 = 25 + 16$$

$$l^2 = 41$$

$$l = 6,403 \text{ m}$$

a) przemieszczenie pionowe w punkcie „D”



$$V_D^M \cdot \bar{1} = \int \frac{M\bar{M}}{EI} dx + \sum \frac{R\bar{R}}{k}$$

$$EI = 210 \cdot 10^6 \cdot 2490 \cdot 10^{-8} = 5229 \text{ kNm}^2$$

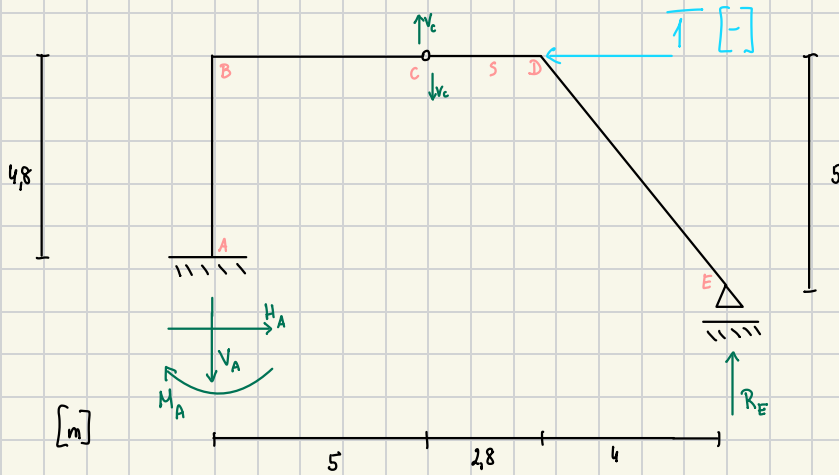
$$\begin{aligned} V_D^M \cdot \bar{1} = & \frac{1}{5229} \left[-\frac{1}{2} \cdot 4,8 \cdot 54,03 \cdot 2,941 - \frac{1}{2} \cdot 4,8 \cdot 30,03 \cdot 2,941 - \frac{1}{2} \cdot 5 \cdot 16,03 \cdot \frac{2}{3} \cdot 2,941 \right. \\ & - \frac{1}{2} \cdot 1,96 \cdot 6,284 \cdot \frac{2}{3} \cdot 1,153 - \frac{1}{2} \cdot 0,84 \cdot 6,284 \left(\frac{1}{3} \cdot 1,647 + \frac{2}{3} \cdot 1,153 \right) \\ & + \frac{1}{2} \cdot 0,84 \cdot 12,02 \left(\frac{2}{3} \cdot 1,647 + \frac{1}{3} \cdot 1,153 \right) + \frac{1}{2} \cdot 6,403 \cdot 12,02 \cdot \frac{2}{3} \cdot 1,647 \\ & \left. + \frac{2}{3} \cdot \frac{12,4 \cdot 4^2}{8} \cdot 6,403 \cdot \frac{1}{2} \cdot 1,647 \right] + 0 \end{aligned}$$

$$V_D^M \cdot \bar{\Gamma} = \frac{1}{5229} [-381,4 - 212,0 - 78,57 - 4,734 - 3,478 + 7,483 + 42,25 + 87,18]$$

$$V_D^M \cdot \bar{\Gamma} = - \frac{543,2}{5229} \checkmark$$

$$V_D^M \cdot \bar{\Gamma} = -0,1039 \text{ m} \checkmark$$

premieszczenie poziome w pkt. D



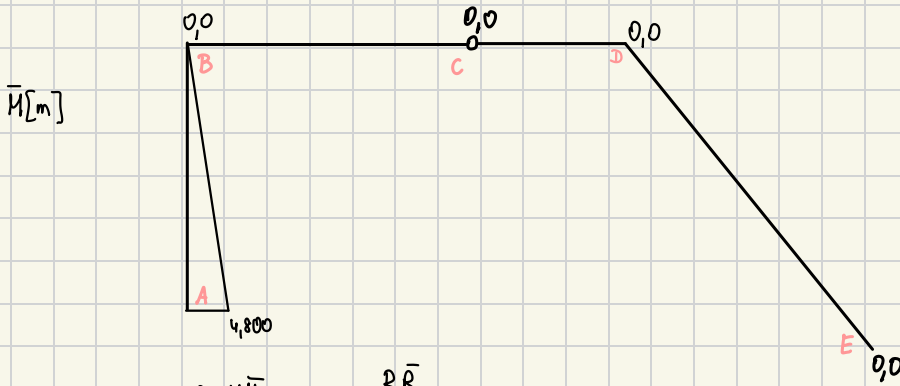
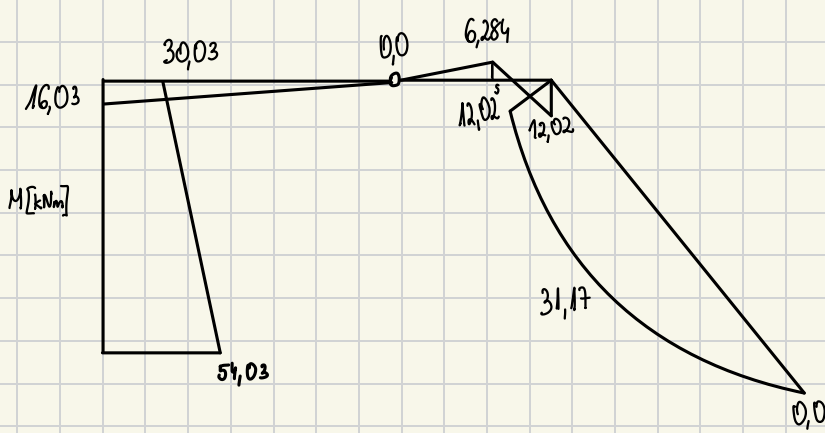
$$R_E = 0 [-]$$

$$V_C = 0 [-]$$

$$V_A = 0 [-]$$

$$H_A = 1 [-]$$

$$M_A = 4,8 \text{ m}$$



$$H_D^M \cdot \bar{1} = \sum \int \frac{M \bar{M}}{EI} dx + \sum \frac{R \bar{R}}{k_i}$$

$$H_D^M \cdot \bar{1} = \frac{1}{5229} \left[\frac{1}{2} \cdot 4.8 \cdot 57.03 \cdot \frac{2}{3} \cdot 4.8 + \frac{1}{2} \cdot 4.8 \cdot 30.03 \cdot \frac{1}{3} \cdot 4.8 \right] + \frac{5.1}{2000}$$

$$H_D^M \cdot \bar{1} = \frac{530.3}{5229} + 0.0025$$

$$H_D^M \cdot \bar{1} = 0.1039 \text{ m}$$

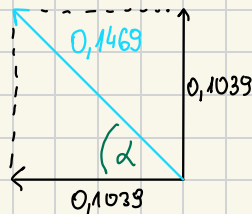


przemieszczenie wypadkowe

$$W_D^M = \sqrt{(V_k^M)^2 + (H_k^M)^2}$$

$$W_D^M = \sqrt{(0.1039)^2 + (0.1039)^2}$$

$$W_D^M = 0.1469 \text{ m}$$

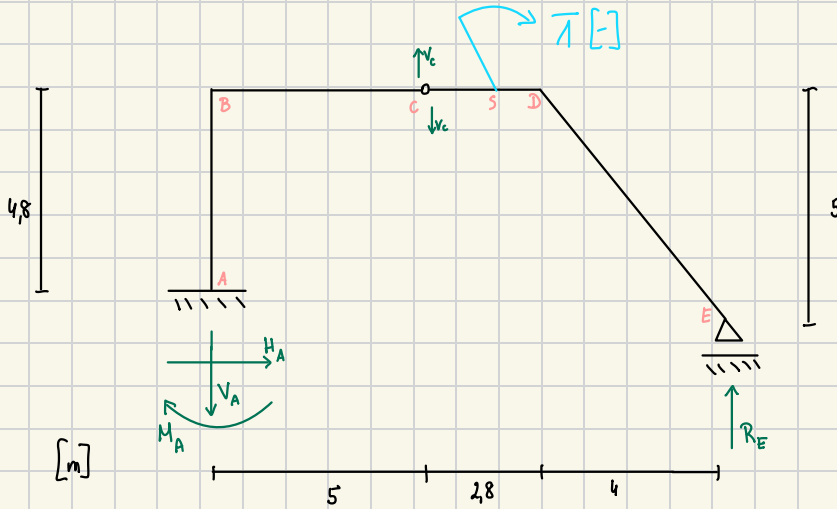


$$\text{tg } \alpha = \frac{0.1039}{0.1039}$$

$$\text{tg } \alpha = 1$$

$$\alpha = 45^\circ$$

b) obrót przekroju w punkcie S



$$\sum M_C'' = -6,8R_E + 1 = 0$$

$$-6,8R_E = -1$$

$$R_E = 0,1471 \frac{1}{m}$$

$$\sum X: H_A = 0 \frac{1}{m}$$

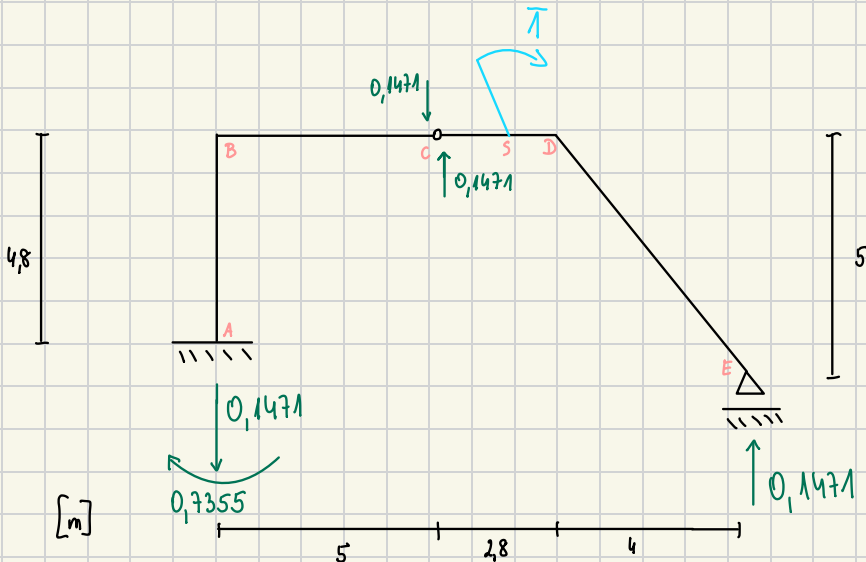
$$\sum Y^{1+1} = V_A = 0,1471 \frac{1}{m}$$

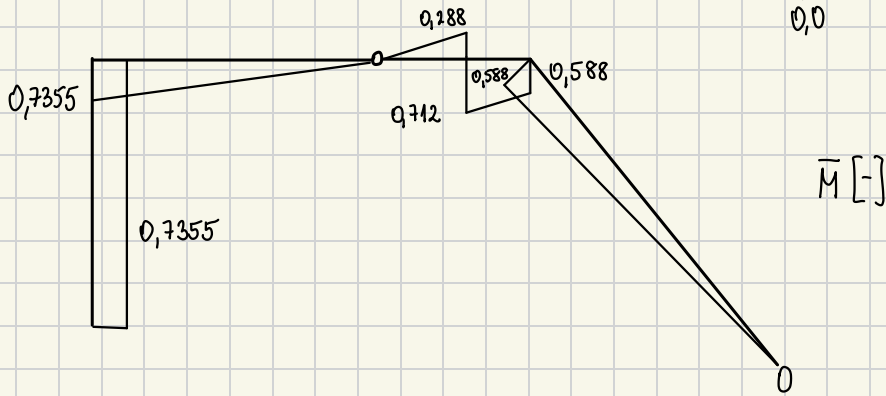
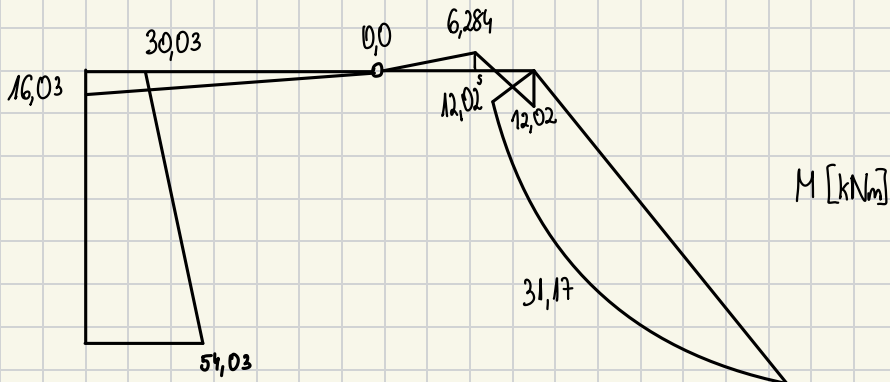
$$\sum Y'' = 0,1471 + V_C = 0$$

$$V_C = -0,1471 \frac{1}{m}$$

$$\sum M_C' = M_A - 5 \cdot 0,1471 = 0$$

$$M_A = 0,7355 [-]$$





$$\varphi_s = \sum \int \frac{M\bar{M}}{EJ} dx + \sum \frac{R\bar{R}}{k}$$

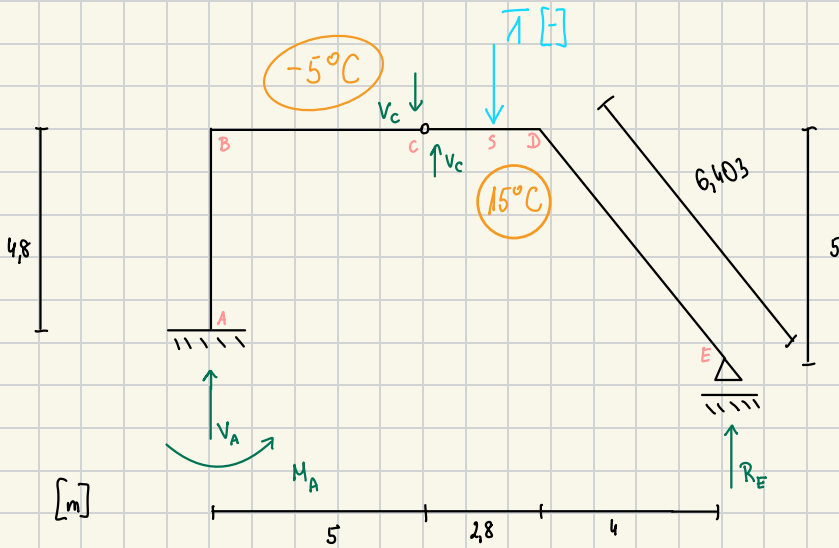
$$\begin{aligned} \varphi_s = \frac{1}{5229} & \left[\frac{1}{2} \cdot 54,03 \cdot 4,8 \cdot 0,7355 + \frac{1}{2} \cdot 30,03 \cdot 4,8 \cdot 0,7355 + \frac{1}{2} \cdot 16,03 \cdot 5 \cdot \frac{2}{3} \cdot 0,7355 \right. \\ & + \frac{1}{2} \cdot 1,96 \cdot 6,284 \cdot \frac{2}{3} \cdot 0,288 - \frac{1}{2} \cdot 0,84 \cdot 6,284 \left(\frac{2}{3} \cdot 0,712 + \frac{1}{3} \cdot 0,588 \right) \\ & + \frac{1}{2} \cdot 12,02 \cdot 0,84 \left(\frac{1}{3} \cdot 0,712 + \frac{2}{3} \cdot 0,588 \right) + \frac{1}{2} \cdot 12,02 \cdot 6,403 \cdot \frac{2}{3} \cdot 0,588 + \\ & \left. \frac{2}{3} \cdot \frac{12,4 \cdot 4^2}{8} \cdot 6,403 \cdot \frac{1}{2} \cdot 0,588 \right] + 0 \end{aligned}$$

$$\varphi_s = \frac{1}{5229} [148,4 + 19,65 + 1,182 + 1,407 + 46,21]$$

$$\varphi_s = \frac{216,8}{5229} \quad \checkmark$$

$$\varphi_s = 0,04146 \text{ rad} \cdot \frac{180}{\pi} = 2,375^\circ \quad \checkmark$$

przemieszczenie pionowe w punkcie S wywołane wpływami termicznymi oraz kinematycznymi (osiadaniem podpór)



$$t_m = -10^\circ\text{C}$$

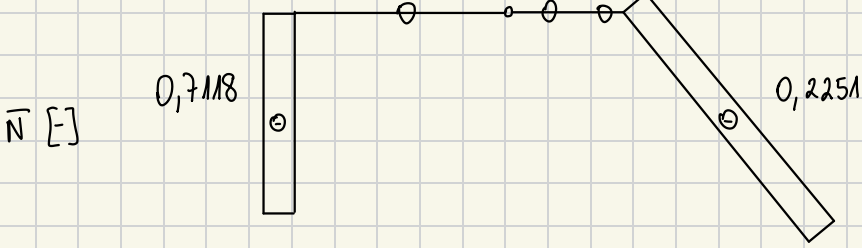
$$\begin{aligned} \sum M_c'' &= -6,8R_E + 1,96 \cdot 1 = 0 \\ 6,8R_E &= 1,96 \\ R_E &= 0,2882 [-] \end{aligned}$$

$$\begin{aligned} \sum Y'' &= R_E + V_c - 1 = 0 \\ V_c &= -0,2882 + 1 = 0,7118 [-] \end{aligned}$$

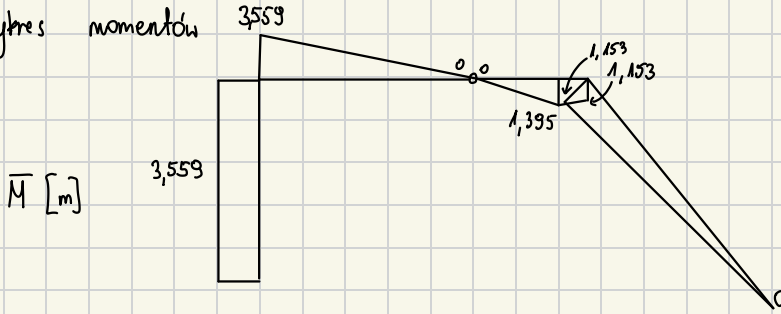
$$\sum Y' = V_A = 0,7118 [-]$$

$$\begin{aligned} \sum M_c' &= M_A = 5 \cdot 0,7118 \\ M_A &= 3,559 \text{ m} \end{aligned}$$

wykres normalnych



wykres momentów



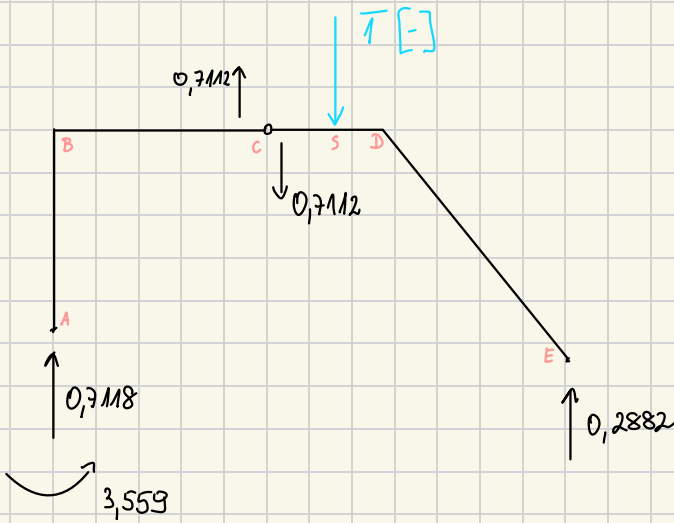
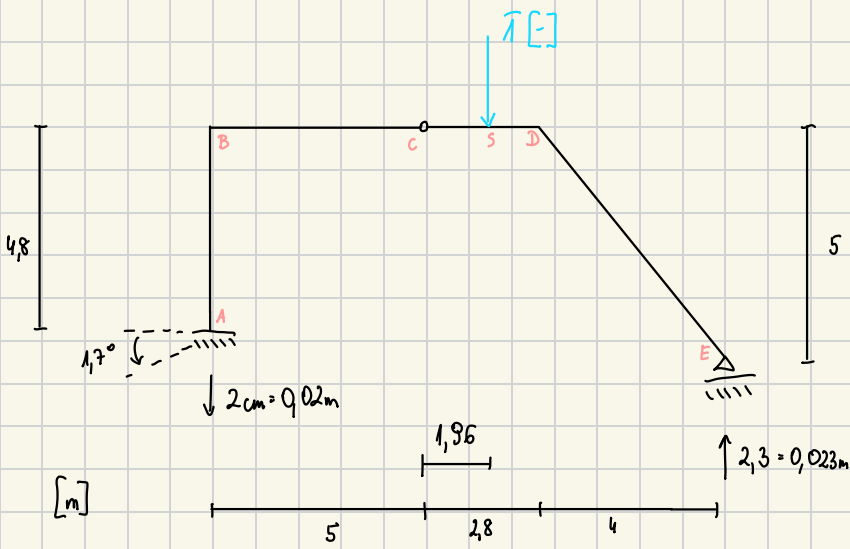
$$\delta_s^T = \sum \int \bar{M} \alpha_t \frac{\Delta t}{h} dx + \sum \int \bar{N} \alpha_t t_0 dx$$

$$|\Delta t| = 20^\circ\text{C}$$

$$t_0 = \frac{15 + (-5)}{2} - (-10) = 5 + 10 = 15^\circ\text{C}$$

$$\delta_s^T = 1,2 \cdot 10^{-5} \cdot \frac{20}{0,16} \left[-3,559 \cdot 4,8 - \frac{1}{2} \cdot 3,559 \cdot 5 + \frac{1}{2} \cdot 1,96 \cdot 1,395 + \frac{(1,395 + 1,153) \cdot 0,84}{2} + \frac{1}{2} \cdot 1,153 \cdot 6,403 \right] + 1,2 \cdot 10^{-5} \cdot 15 \left[-0,7118 \cdot 4,8 - 0,2251 \cdot 6,403 \right] = -0,03065 \text{ m}$$

✓



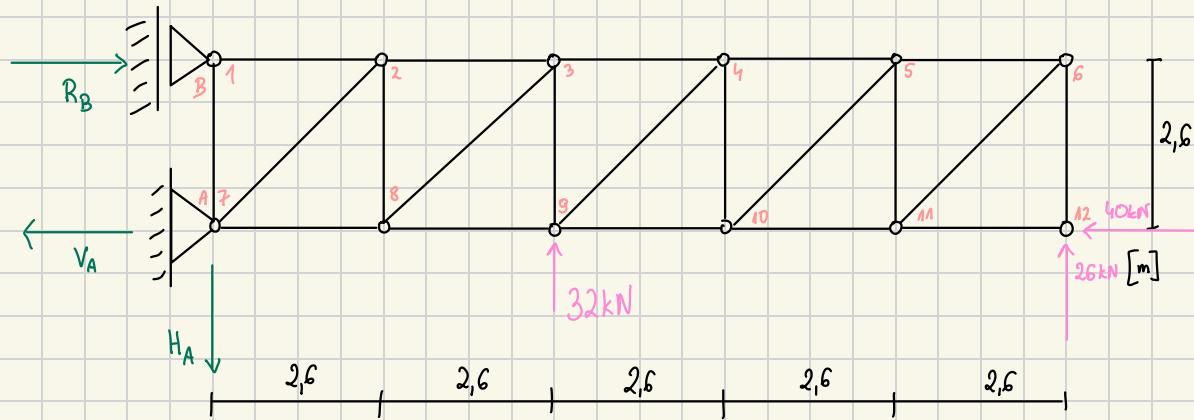
$$\varphi = 1,7^\circ = 1,7 \cdot \frac{\pi}{180} = 0,02967 \text{ rad}$$

$$\delta = -\sum \bar{R} \cdot \Delta$$

✓

$$\delta = - [-0,7118 \cdot 0,02 + 3,559 \cdot 0,02967 + 0,2882 \cdot 0,023] = -0,09799 \text{ m}$$

KRATOWNICA



$$\sum Y: H_A = 32 + 26$$

$$H_A = 58 \text{ kN}$$

$$\sum M_A: 2,6 R_B - 32 \cdot 2,6 \cdot 2 - 26 \cdot 5 \cdot 2,6 = 0$$

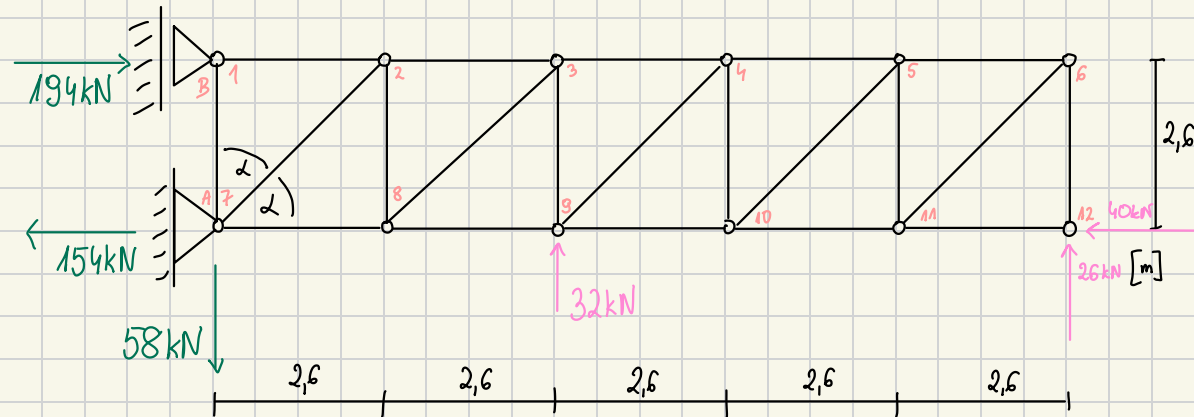
$$2,6 R_B = 504,4$$

$$R_B = 194 \text{ kN}$$

$$\sum M_B: V_A \cdot 2,6 - 32 \cdot 2,6 \cdot 2 - 26 \cdot 5 \cdot 2,6 + 40 \cdot 2,6 = 0$$

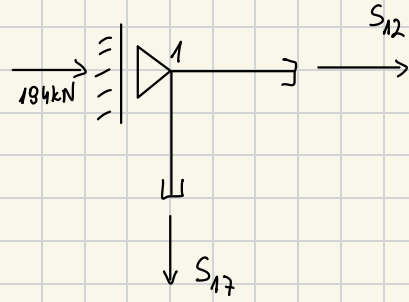
$$2,6 V_A = 400,4$$

$$V_A = 154 \text{ kN}$$



$$\alpha = 45^\circ \quad \sin \alpha = \frac{\sqrt{2}}{2} \quad \cos \alpha = \frac{\sqrt{2}}{2}$$

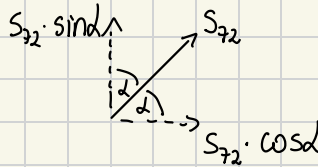
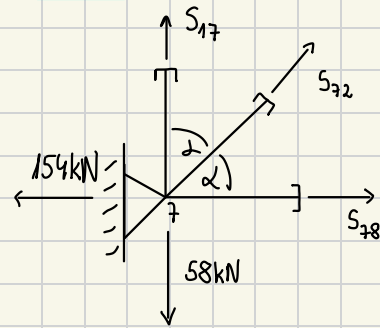
węzeł 1



$$\sum Y: S_{17} = 0 \text{ kN}$$

$$\sum X: S_{12} = -194 \text{ kN}$$

węzeł 7



$$\sum Y: S_{72} \cdot \sin \alpha - 58 = 0$$

$$S_{72} \cdot \frac{\sqrt{2}}{2} = 58$$

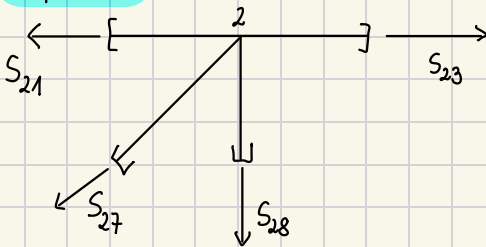
$$S_{72} = 82,02 \text{ kN}$$

$$\sum X: 82,02 \cdot \cos \alpha + S_{78} - 154 = 0$$

$$82,02 \cdot \frac{\sqrt{2}}{2} + S_{78} - 154 = 0$$

$$S_{78} = 96,00 \text{ kN}$$

węzeł 2



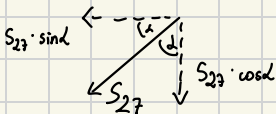
$$\sum X: -S_{21} + S_{23} - S_{27} \cdot \sin \alpha = 0$$

$$194 + S_{23} - 58 = 0$$

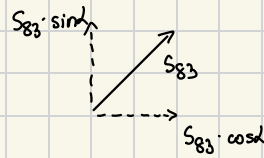
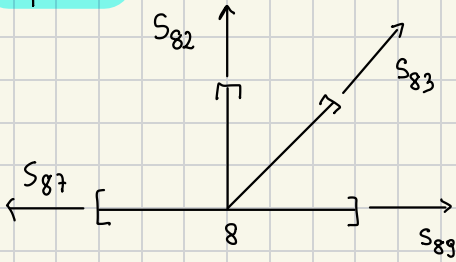
$$S_{23} = -136 \text{ kN}$$

$$\sum Y: -S_{27} \cdot \cos \alpha - S_{28} = 0$$

$$S_{28} = -58 \text{ kN}$$



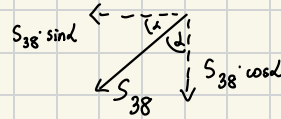
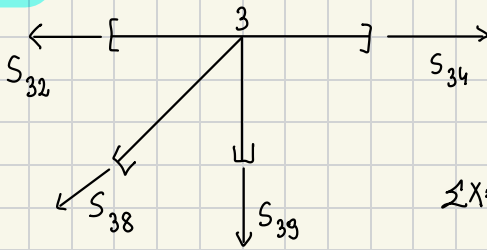
węzeł 8



$$\begin{aligned} \sum Y: S_{82} + S_{83} \cdot \sin \alpha &= 0 \\ S_{83} \cdot \sin \alpha &= 58 \\ S_{83} &= 82,02 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum X: -S_{87} + S_{83} \cdot \cos \alpha + S_{89} &= 0 \\ -96 + 82,02 \cdot \frac{\sqrt{2}}{2} + S_{89} &= 0 \\ S_{89} &= 38,00 \text{ kN} \end{aligned}$$

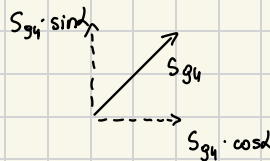
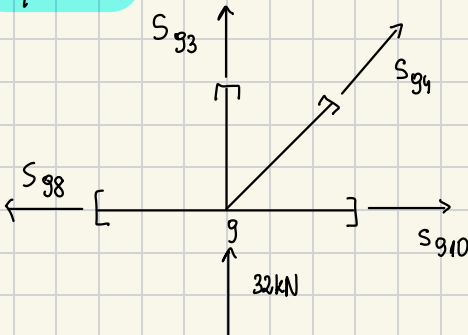
węzeł 3



$$\begin{aligned} \sum X: S_{34} - S_{32} - S_{38} \cdot \sin \alpha &= 0 \\ S_{34} + 136 - 58 &= 0 \\ S_{34} &= -78 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y: -S_{38} \cdot \cos \alpha - S_{39} &= 0 \\ S_{39} &= -58,00 \text{ kN} \end{aligned}$$

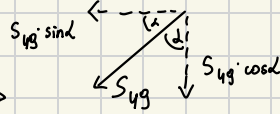
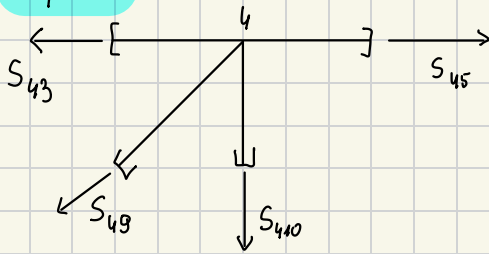
węzeł 9



$$\begin{aligned} \sum Y: S_{93} + 32 + S_{94} \cdot \sin \alpha &= 0 \\ S_{94} &= 36,77 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum X: -S_{98} + S_{910} + S_{94} \cdot \cos \alpha &= 0 \\ S_{910} &= 12 \text{ kN} \end{aligned}$$

węzeł 4



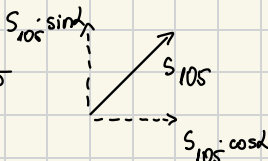
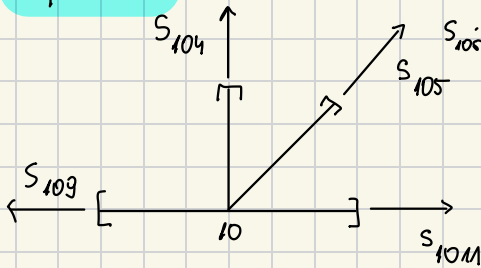
$$\sum X: -S_{43} + S_{45} - S_{49} \cdot \sin \alpha = 0$$

$$S_{45} = -52 \text{ kN}$$

$$\sum Y: -S_{49} \cdot \cos \alpha - S_{440} = 0$$

$$S_{440} = -26,00 \text{ kN}$$

węzeł 10



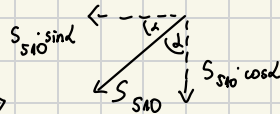
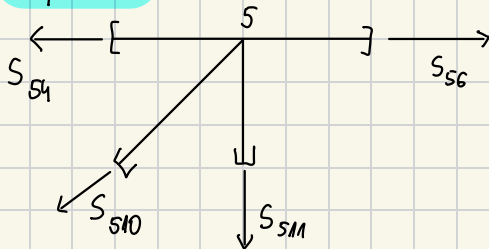
$$\sum X: -S_{109} + S_{10M} + S_{105} \cdot \cos \alpha = 0$$

$$S_{10M} = -14,00 \text{ kN}$$

$$\sum Y: S_{104} + S_{105} \cdot \sin \alpha = 0$$

$$S_{105} = 36,77 \text{ kN}$$

węzeł 5



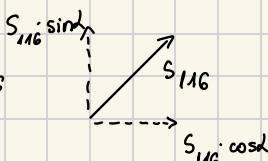
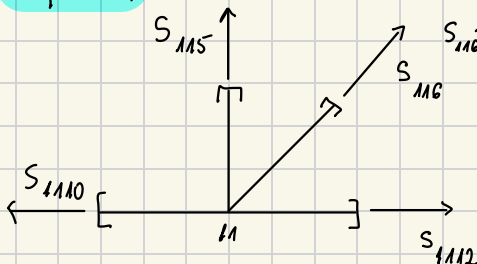
$$\sum X: S_{56} - S_{510} \cdot \sin \alpha - S_{54} = 0$$

$$S_{56} = -26,00 \text{ kN}$$

$$\sum Y: -S_{510} \cdot \cos \alpha - S_{5M} = 0$$

$$S_{5M} = 26,00 \text{ kN}$$

węzeł 11



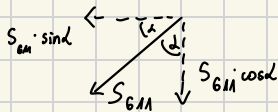
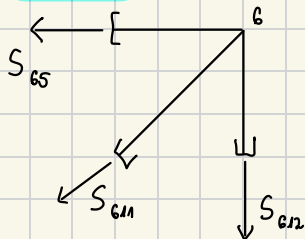
$$\sum X: -S_{1110} + S_{1112} + S_{116} \cdot \cos \alpha = 0$$

$$S_{1112} = -40,00 \text{ kN}$$

$$\sum Y: S_{115} + S_{116} \cdot \sin \alpha = 0$$

$$S_{116} = 36,77 \text{ kN}$$

węzeł 6



$$\sum X: -S_{65} - S_{611} \cdot \sin \alpha = 0$$

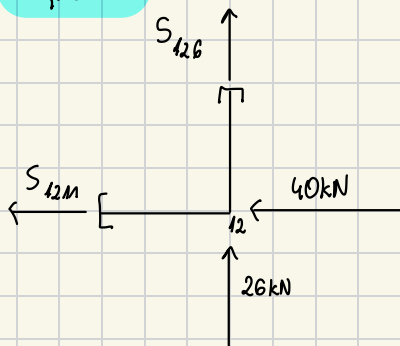
$$26,00 - 36,77 \cdot \frac{3}{5} = 0$$

$$-0,00003 = 0$$

$$\sum Y: -S_{611} \cos \alpha - S_{612} = 0$$

$$S_{612} = -26,00 \text{ kN}$$

węzeł 12



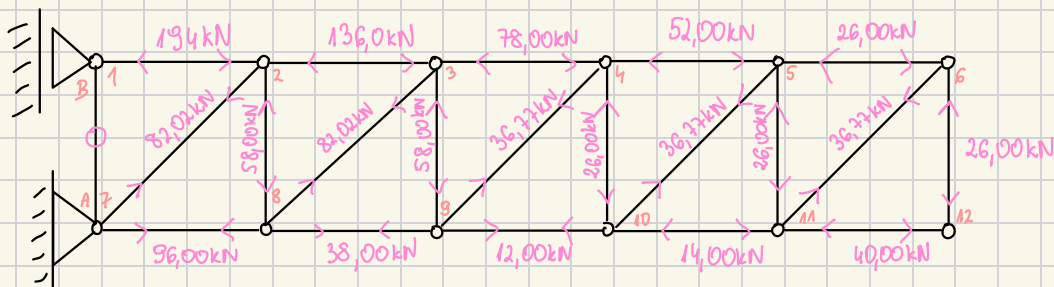
$$\sum X: -40 + 40 = 0$$

$$0 = 0$$

$$\sum Y: 26 - 26 = 0$$

$$0 = 0$$

wykres normalnych



projektowanie przekroju

$$\sigma_x = \frac{|N_{\max}|}{A} \Rightarrow A = \frac{|N_{\max}|}{\sigma_x}$$

$$N_{\max} = -194,0 \text{ kN}$$

$$f_y = 235 \text{ MPa}$$

$$\sigma_{\text{dop}} = 235 \text{ MPa} = 23,5 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma \leq \sigma_{\text{dop}}$$

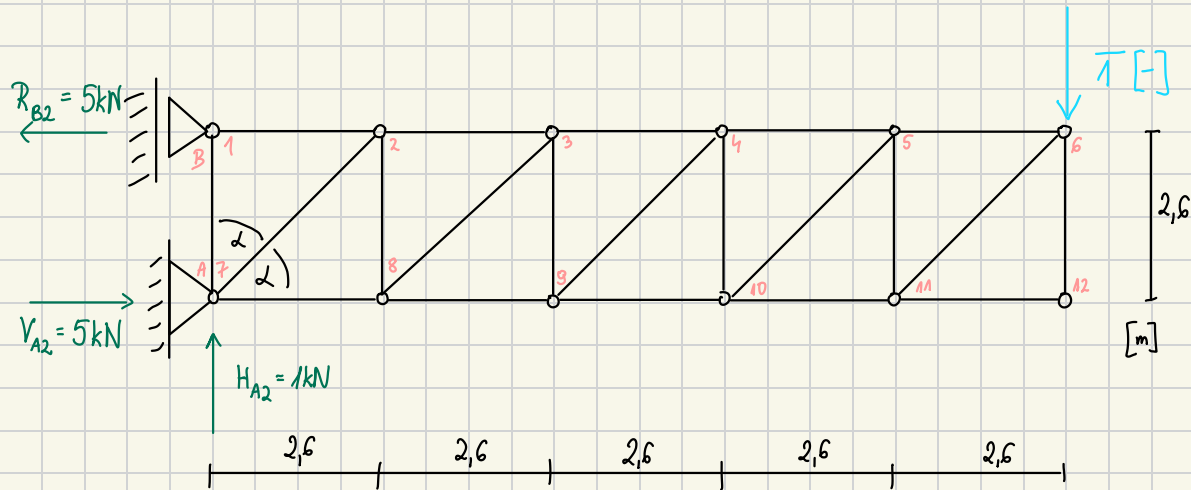
$$A \geq \frac{194}{23,5}$$

$$A \geq 8,255 \text{ cm}^2$$

		h	t	r _o	r _i	d	A	J _y
<input type="checkbox"/>	SHS 50x50x4	50.0	4.0	8.0	4.0	34.0	6.95	23.70
<input type="checkbox"/>	SHS 50x50x5	50.0	5.0	10.0	5.0	30.0	8.36	27.00
<input type="checkbox"/>	SHS 60x60x2	60.0	2.0	4.0	2.0	52.0	4.54	25.14
<input type="checkbox"/>	SHS 60x60x3	60.0	3.0	6.0	3.0	48.0	6.61	35.10
<input checked="" type="checkbox"/>	SHS 60x60x4	60.0	4.0	8.0	4.0	44.0	8.55	43.60
<input type="checkbox"/>	SHS 60x60x5	60.0	5.0	10.0	5.0	40.0	10.40	50.50

wybrano przekroj SHS 60x60x4 o polu 8,55 cm²

przemieszczenie pionowe węzła nr 6 wywołane zadanymi siłami zewnętrznymi



$$\sum M_A: 2,6 \cdot 5 - 2,6 R_{B2} = 0$$

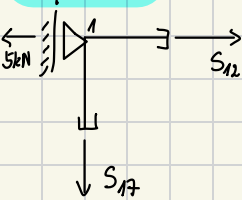
$$R_{B2} = 5 [-]$$

$$\sum X: V_{A2} = 5 [-]$$

$$\sum Y: H_{A2} = 1 [-]$$

$$\sin \alpha = \cos \alpha = \frac{\sqrt{2}}{2}$$

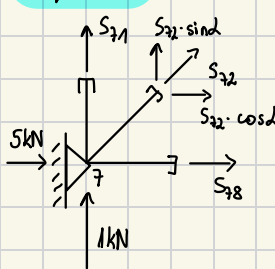
węzeł 1



$$\sum X: S_{12} = 5 \text{ kN}$$

$$\sum Y: S_{17} = 0 \text{ kN}$$

węzeł 7



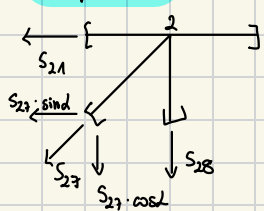
$$\sum Y: 1 + S_{72} \sin \alpha = 0$$

$$S_{72} = -1,414 \text{ kN}$$

$$\sum X: 5 + S_{72} \cos \alpha + S_{78} = 0$$

$$S_{78} = -4,000 \text{ kN}$$

węzeł 2



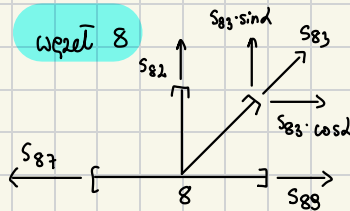
$$\sum Y: S_{27} \cos \alpha - S_{28} = 0$$

$$S_{28} = 1,000 \text{ kN}$$

$$\sum X: -S_{21} + S_{23} - S_{27} \sin \alpha = 0$$

$$S_{23} = 4,000 \text{ kN}$$

węzeł 8



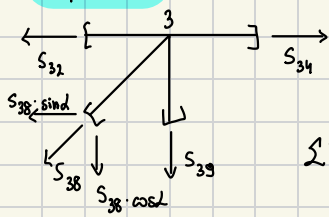
$$\sum Y: S_{82} + S_{83} \sin \alpha = 0$$

$$S_{83} = -1,414 \text{ kN}$$

$$\sum X: -S_{87} + S_{89} + S_{83} \cos \alpha = 0$$

$$S_{89} = -3,000 \text{ kN}$$

węzeł 3



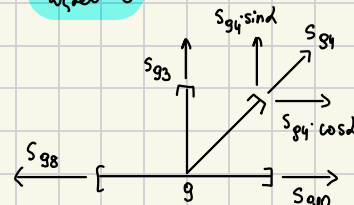
$$\sum Y: -S_{38} \cos \alpha - S_{39} = 0$$

$$S_{39} = 1,000 \text{ kN}$$

$$\sum X: -S_{32} + S_{34} - S_{38} \sin \alpha = 0$$

$$S_{34} = 3,000 \text{ kN}$$

węzeł 9



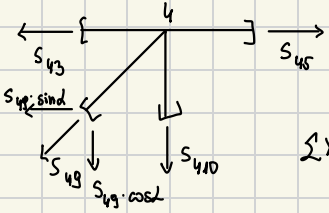
$$\sum Y: S_{93} + S_{94} \sin \alpha = 0$$

$$S_{94} = -1,414 \text{ kN}$$

$$\sum X: -S_{98} + S_{910} + S_{94} \cos \alpha = 0$$

$$S_{910} = -2,000 \text{ kN}$$

węzeł 4



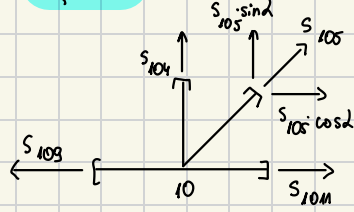
$$\sum Y: -S_{49} \cos \alpha - S_{410} = 0$$

$$S_{410} = 1,000 \text{ kN}$$

$$\sum X: -S_{43} + S_{45} - S_{49} \sin \alpha = 0$$

$$S_{45} = 2,000 \text{ kN}$$

węzeł 10



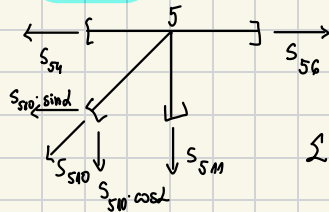
$$\sum Y: S_{104} + S_{105} \sin \alpha = 0$$

$$S_{105} = -1,414 \text{ kN}$$

$$\sum X: -S_{109} + S_{1011} + S_{105} \cos \alpha = 0$$

$$S_{1011} = -1,000 \text{ kN}$$

węzeł 5



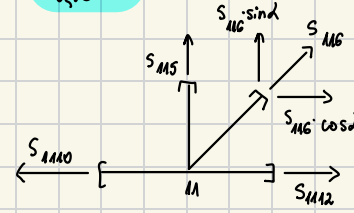
$$\sum Y: -S_{510} \cos \alpha - S_{511} = 0$$

$$S_{511} = 1,000 \text{ kN}$$

$$\sum X: -S_{54} + S_{56} - S_{510} \sin \alpha = 0$$

$$S_{56} = 1,000 \text{ kN}$$

węzeł 11



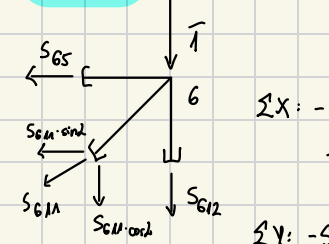
$$\sum Y: S_{115} + S_{116} \sin \alpha = 0$$

$$S_{116} = -1,414 \text{ kN}$$

$$\sum X: -S_{1110} + S_{1112} + S_{116} \cos \alpha = 0$$

$$S_{1112} = 0 \text{ kN}$$

węzeł 6



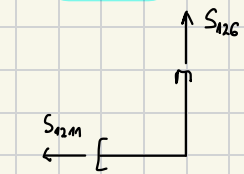
$$\sum X: -S_{65} - S_{611} \sin \alpha = 0$$

$$-0,00015 \approx 0$$

$$\sum Y: -S_{612} - 1 - S_{611} \cos \alpha = 0$$

$$S_{612} = 0 \text{ kN}$$

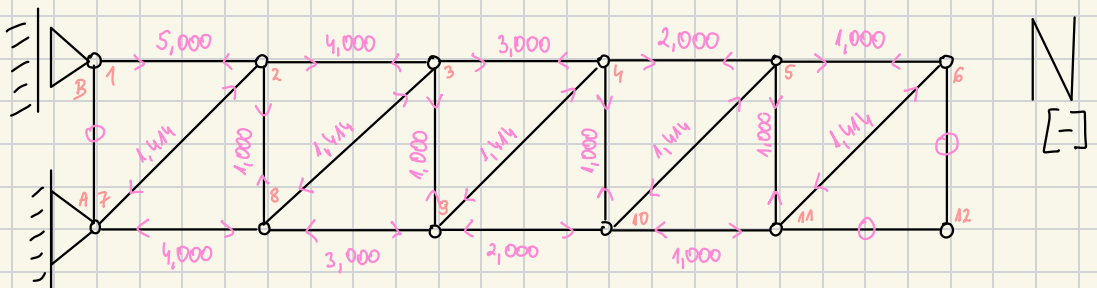
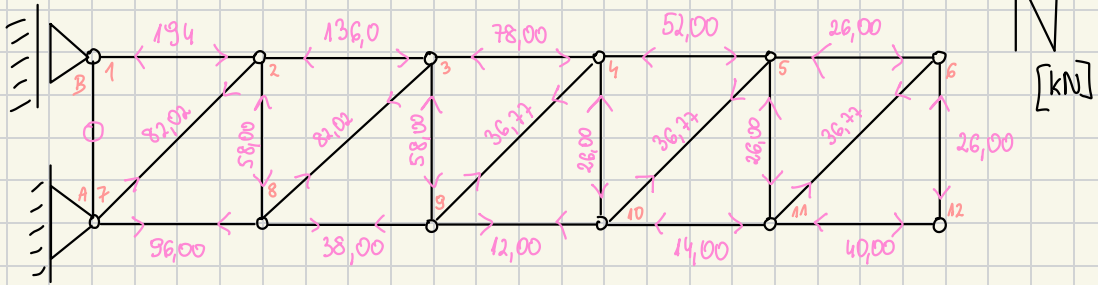
węzeł 12



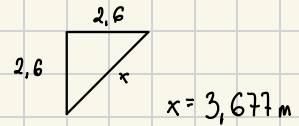
$$\sum X: S_{1211} = 0 \text{ kN}$$

$$\sum Y: S_{126} = 0 \text{ kN}$$

wykresy normalnych



$$\delta_x = \sum \int \frac{NN}{EA} dx + \sum \frac{RR}{k}$$



$$E = 210 \text{ GPa}$$

$$A = 8,55 \text{ cm}^2$$

$$EA = 210 \cdot 10^6 \cdot 8,55 \cdot 10^{-4} = 179550 \text{ kN}$$

$$\delta_6^P = \frac{1}{EA} \left[(-194) \cdot 5 \cdot 2,6 + 82,02 \cdot (-1,414) \cdot 3,677 + 96 \cdot (-4) \cdot 2,6 - 58 \cdot 1 \cdot 2,6 - 136 \cdot 4 \cdot 2,6 + \right. \\ \left. 82,02 \cdot (-1,414) \cdot 3,677 + 38 \cdot (-3) \cdot 2,6 - 58 \cdot 1 \cdot 2,6 - 78 \cdot 3 \cdot 2,6 + 36,77 \cdot (-1,414) \cdot 3,677 + \right. \\ \left. 12 \cdot (-2) \cdot 2,6 - 26 \cdot 1 \cdot 2,6 - 52 \cdot 2 \cdot 2,6 + 36,77 \cdot (-1,414) \cdot 3,677 - 14 \cdot (-1) \cdot 2,6 - \right. \\ \left. 26 \cdot 1 \cdot 2,6 - 26 \cdot 1 \cdot 2,6 + 36,77 \cdot (-1,414) \cdot 3,677 \right]$$

$$\delta_6^P = \frac{-8067}{179550} = -0,04493 \text{ m}$$