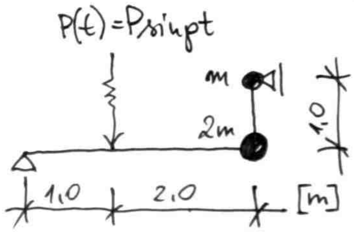
A

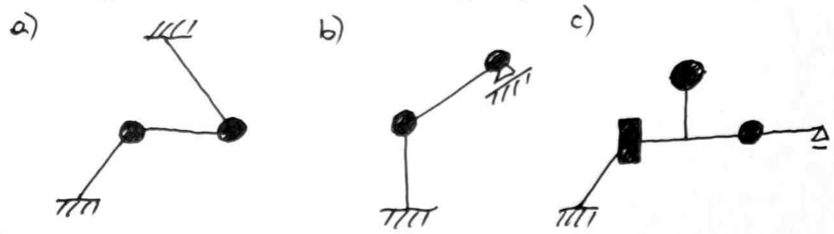
KOLOKWIUM NR 2 4.06.2008

ZAD. 1. Wyznaczyć częstość(ci) kołową(e) drgań własnych oraz amplitudę(y) drgań wymuszonych; równania ruchu zapisać (i rozwiązać :-))
a) przez współczynniki δ_{ik}
b) przez współczynniki r_{ik}

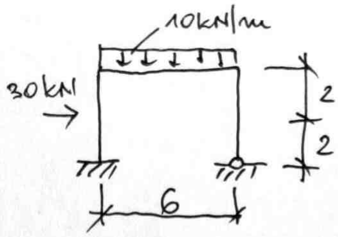


$m = 100$ kg
$EJ = const$
$E = 205$ GPa
$J = 2140$ cm^4
$P = 10$ kN
$p = 20$ Hz

ZAD. 2. Wyznaczyć SSD (zaznaczyć q_i na schemacie)



ZAD. 3. Wyznaczyć wektor $\underline{P} = \underline{P}_w - \underline{R}_o$. Zastosować redukcję statyczną na poziomie pręta, uwzględnić warunki podparcia układu.

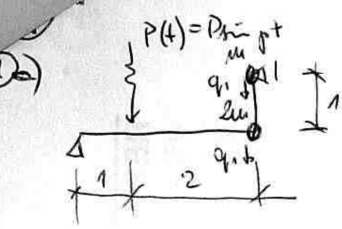


$$\underline{T} = \begin{bmatrix} \underline{c} & \underline{0} \\ \underline{0} & \underline{c} \end{bmatrix} \quad \underline{c} = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

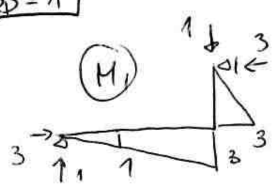
$$\underline{R}_o = \underline{T}^T \cdot \tilde{\underline{R}}_o$$

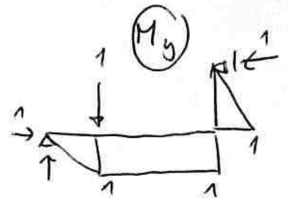
$\boxed{SSD = 1}$

1a)



$m = 100\ kg$
$E = 205\ GPa$
$J = 2140\ cm^4$
$P = 10\ kN$
$q = 20\ Hz$
$EI = 4387000\ Nm^2$





$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{2}\cdot 3\cdot 3\cdot \frac{2}{3}\cdot 3 + \frac{1}{2}\cdot 1\cdot 3\cdot \frac{2}{3}\cdot 3\right] = \frac{9+3}{EI} = \frac{12}{EI}$$

$$\delta_{1y} = \frac{1}{EI}\left[\frac{1}{2}\cdot 1\cdot 1\cdot \frac{2}{3}\cdot 1 + 2\cdot 1\cdot \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 1\cdot 3\cdot \frac{2}{3}\cdot 1\right] = \frac{1}{EI}\left[\frac{1}{3}+4+1\right] = \frac{5{,}(3)}{EI}$$

DRG. WŁASNE:

$q_1 = \delta_{11}(-3m\ddot{q}_1)$ $\quad q_1 = A_1 \sin\omega t \quad \ddot{q}_1 = -A_1\omega^2 \sin\omega t$

$A_1(1-\delta_{11}3m\omega^2) = 0$

$1-\delta_{11}3m\omega^2 = 0$

$$\omega = \sqrt{\frac{1}{\delta_{11}3m}} = \sqrt{\frac{4387000}{12\cdot 3\cdot 100}} = \underline{34{,}9086\ \frac{rad}{s}}$$

DRG. WYMUSZONE:

$q_1 = \delta_{11}(-3m\ddot{q}_1) + \delta_{1y}\cdot P(t)$

$q_1 = A_1 \sin pt \qquad p = 2\pi\cdot 20\ \left[\frac{rad}{s}\right]$

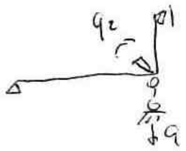
$\ddot{q}_1 = -A_1 p^2 \sin pt$

$A_1(1-\delta_{11}3mp^2) = \delta_{1y}\cdot P$

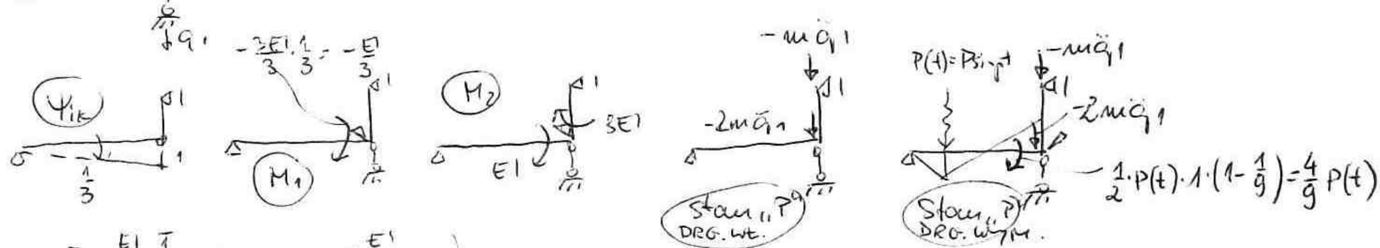
$$A_1\left(1 - \frac{36}{EI}\cdot 100\cdot (20\cdot 2\pi)^2\right) = \frac{5{,}(3)}{EI}\cdot 10000$$

$$\underline{A_1 = -0{,}0010166\ m}$$

b)



$\boxed{SGN = 2}$
$$\begin{cases} r_{11}q_1 + r_{12}q_2 + r_{1P} = 0 \\ r_{21}q_1 + r_{22}q_2 + r_{2P} = 0 \end{cases}$$



$r_{11}\cdot\bar{1} - \frac{EI}{3}\cdot\frac{1}{3} = 0 \rightarrow r_{11} = \frac{EI}{9}$

$r_{12}\cdot\bar{1} + EI\cdot\frac{1}{3} = 0 \rightarrow r_{12} = -\frac{EI}{3}$

$r_{21} = -\frac{EI}{3}$

$r_{22} = 4EI$

DRG. WŁASNE:

$r_{1P}\,\bar{1} - 3m\ddot{q}_1\,\bar{1} = 0 \rightarrow r_{1P} = 3m\ddot{q}_1$

$r_{2P} = 0$

DRG. WYMUSZONE:

$r_{1P}\cdot\bar{1} - 3m\ddot{q}_1\cdot\bar{1} + P(t)\cdot\frac{1}{3} + \frac{4}{9}\cdot P(t)\cdot\frac{1}{3} = 0 \rightarrow r_{1P} = 3m\ddot{q}_1 - \frac{13}{27}P(t)$

$r_{2P} = \frac{4}{9}P(t)$

DRG. WŁASNE:

$$\begin{cases} \frac{EI}{9}q_1 - \frac{EI}{3}q_2 + 3m\ddot{q}_1 = 0 \\ -\frac{EI}{3}q_1 + 4EIq_2 + 0 = 0 \end{cases}$$

$q_2 = \frac{EI}{3}q_1\cdot\frac{1}{4EI} = \frac{q_1}{12}$

$\frac{EI}{9}q_1 - \frac{EI}{3}\cdot\frac{q_1}{12} + 3m\ddot{q}_1 = 0$

$\frac{EI}{12}q_1 + 3m\ddot{q}_1 = 0 \quad \Big|\cdot\frac{12}{EI}$

$q_1 + \frac{12}{EI}\cdot 3m\ddot{q}_1 = 0$

$q_1 = \delta_{11}(-3m\ddot{q}_1)$ j/w

$\downarrow$

$$\underline{\omega = 34{,}9086\ \frac{rad}{s}}$$

DRG. WYMUSZONE

$$\begin{cases} EIq_1 - \frac{EI}{3}q_2 + 3m\ddot{q}_1 - \frac{13}{27}P(t) = 0 \\ -\frac{EI}{3}q_1 + 4EIq_2 + \frac{4}{9}P(t) = 0 \end{cases}$$

$q_2 = \frac{q_1}{12} - \frac{1}{9EI}\cdot P(t)$

$\frac{EI}{9}q_1 - \frac{EI}{36}q_1 + \frac{1}{27}P(t) + 3m\ddot{q}_1 = \frac{13}{27}P(t)$

$\frac{EI}{12}q_1 + 3m\ddot{q}_1 = \frac{12}{27}P(t) \quad /\cdot\frac{12}{EI}$

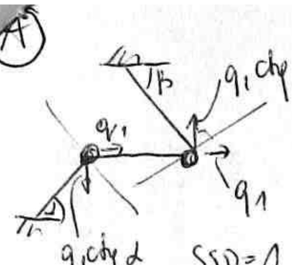
$q_1 + \underbrace{\frac{12}{EI}}_{=\delta_{11}}3m\ddot{q}_1 = \underbrace{\frac{144}{27}}_{=\delta_{1y}}P(t)$

$q_1 = \delta_{11}(-3m\ddot{q}_1) + \delta_{1y}P(t)$ j/w

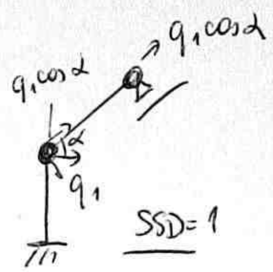
$\downarrow$

$$\underline{A_1 = -0{,}0010166\ m}$$

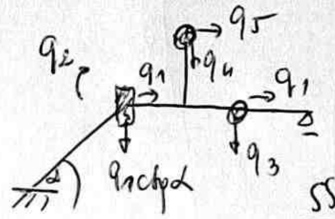
(A)



SSD = 1

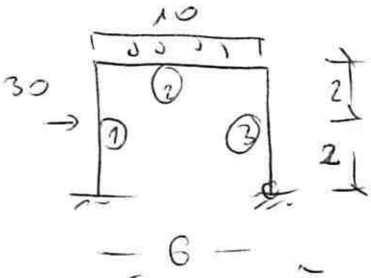


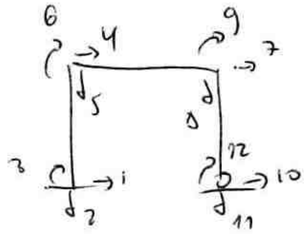
SSD = 1

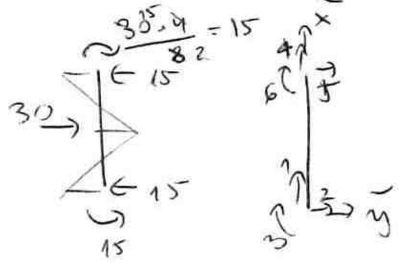


SSD = 5

(3)



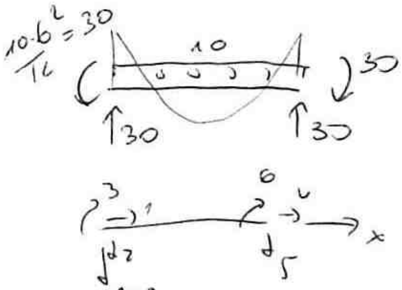




$\alpha = -90°$

$$\tilde{R}_0^1 = \begin{bmatrix} 0 \\ -15 \\ -15 \\ 0 \\ -15 \\ 15 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & -1 & 0 & & 0 & \\ 1 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \\ & & & 0 & -1 & 0 \\ & 0 & & 1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 0 & +1 & 0 & & 0 & \\ -1 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \\ & & & 0 & +1 & 0 \\ & 0 & & -1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -15 \\ -15 \\ 0 \\ -15 \\ 15 \end{bmatrix} = \begin{bmatrix} -15 \\ 0 \\ -15 \\ -15 \\ 0 \\ 15 \end{bmatrix}$$



$$\tilde{R}_0^2 = R_0^2 = \begin{bmatrix} 0 \\ -30 \\ -30 \\ 0 \\ -30 \\ 30 \end{bmatrix}$$

$$P = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -15 & & \\ 0 & & \\ -15 & & \\ -15 & 0 & \\ 0 & -30 & \\ 15 & -30 & \\ & 0 & 0 \\ & -30 & 0 \\ & 30 & 0 \\ & & 0 \\ & & 0 \\ & & 0 \end{bmatrix} = \begin{bmatrix} \cancel{15} \\ \cancel{0} \\ \cancel{15} \\ 15 \\ 30 \\ 15 \\ 0 \\ 30 \\ -30 \\ \cancel{0} \\ \cancel{0} \\ \cancel{0} \end{bmatrix}$$

$$\boxed{P = \begin{bmatrix} 15 \\ 30 \\ 15 \\ 0 \\ 30 \\ -30 \end{bmatrix}}$$