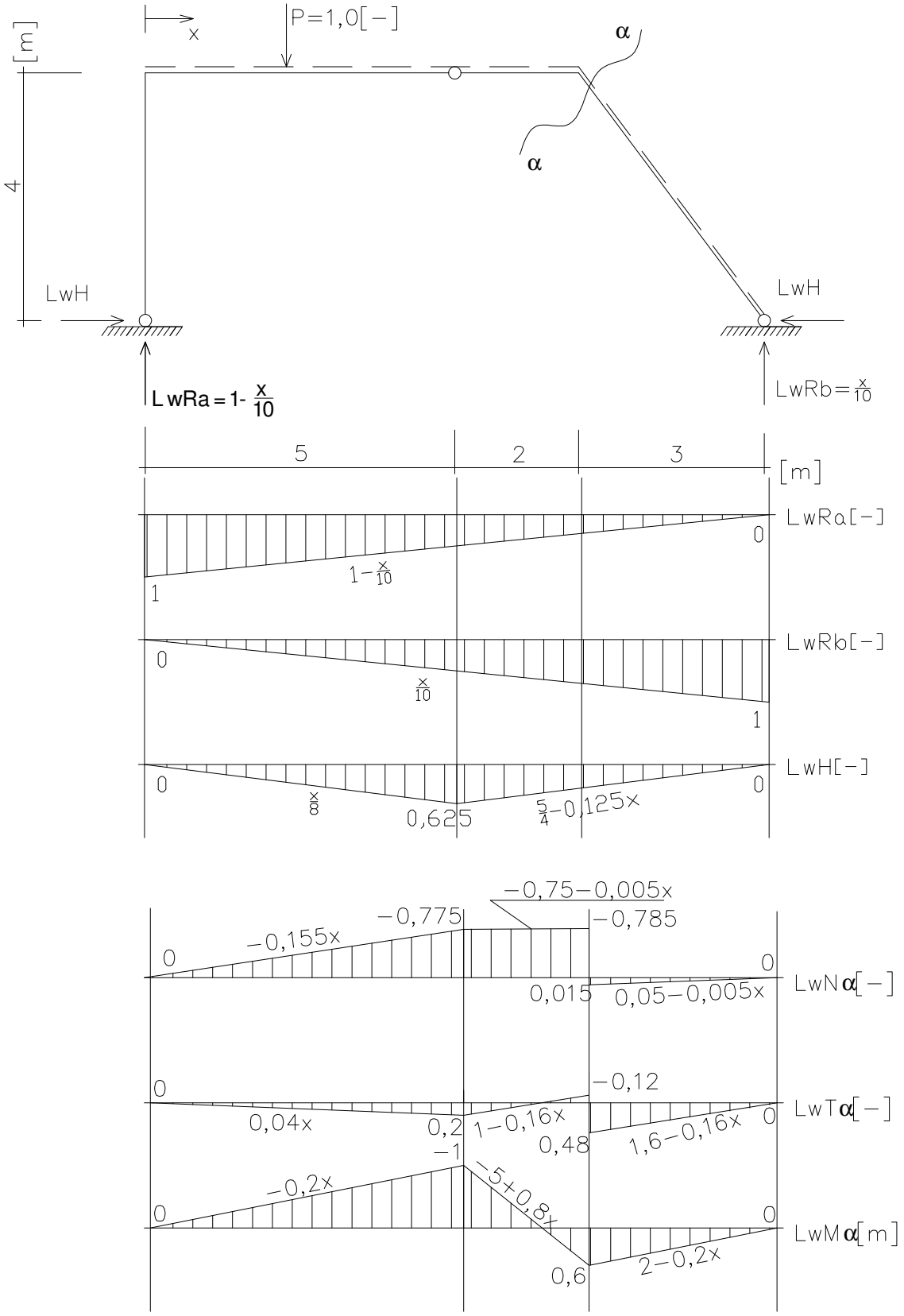
Zadanie 1.

Wyznaczenie linii wpływu Nα, Tα oraz Mα dla przedstawionej poniżej ramy.

a) Grupa A



$LwN\alpha$

Lewa część ramy

$x \in < 0,7 >$

$1) x \in < 0,5 >$

$$LwH = \frac{x}{8}$$

$$LwN\alpha = LwRa \cdot \sin\alpha - LwH \cdot \cos\alpha - 1 \cdot \sin\alpha$$

$$LwN\alpha = 0,8 - 0,08x - 0,075x - 0,8$$

$$LwN\alpha = -0,155x$$

$$x = 0 \rightarrow LwN\alpha = 0$$

$$x = 5 \rightarrow LwN\alpha = -0,775$$

$2) x \in < 5,7 >$

$$LwH = \frac{5}{4}(1 - \frac{x}{10})$$

$$LwN\alpha = LwRa \cdot \sin\alpha - LwH \cdot \cos\alpha - 1 \cdot \sin\alpha$$

$$LwN\alpha = 0,8 - 0,08x - \frac{5}{4} \cdot 0,6 - 0,8 + 0,125x \cdot 0,6$$

$$LwN\alpha = -0,75 - 0,005x$$

$$x = 5 \rightarrow LwN\alpha = -0,775$$

$$x = 7 \rightarrow LwN\alpha = -0,785$$

Lewa część ramy

$x \in < 7,10 >$

$$LwN\alpha = LwRa \cdot \sin\alpha - LwH \cdot \cos\alpha$$

$$LwN\alpha = 0,8 - 0,08x - 0,75 + 0,075x$$

$$LwN\alpha = 0,05 - 0,005x$$

$$x = 7 \rightarrow LwN\alpha = 0,015$$

$$x = 10 \rightarrow LwN\alpha = 0$$

$LwH = ?$

$$\sum M_C^L = LwRa \cdot 5 - LwH \cdot 4 = 0$$

$$LwH \cdot 4 = 5(1 - \frac{x}{10})$$

$$LwH = \frac{5}{4} - 0,125x$$

$$x = 0 \rightarrow LwH = 0$$

$$x = 5 \rightarrow LwH = 0,625$$

$$\sum M_C^P = LwRb \cdot 5 - LwH \cdot 4$$

$$LwH \cdot 4 = 5 \cdot \frac{x}{10}$$

$$LwH = \frac{x}{8}$$

$$x = 5 \rightarrow LwH = 0,625$$

$$x = 10 \rightarrow LwH = 0$$

$LwT\alpha$

Lewa część ramy

$x \in < 0,7 >$

$1) x \in < 0,5 >$

$$LwH = \frac{x}{8}$$

$$LwT\alpha = LwRa \cdot \cos\alpha + LwH \cdot \sin\alpha - 1 \cdot \cos\alpha$$

$$LwT\alpha = 0,6 - 0,06x + 0,1x - 0,6$$

$$LwT\alpha = 0,04x$$

$$x = 0 \rightarrow LwT\alpha = 0$$

$$x = 5 \rightarrow LwT\alpha = 0,2$$

Lewa część ramy

$x \in < 7,10 >$

$$LwT\alpha = LwRa \cdot \cos\alpha + LwH \cdot \sin\alpha$$

$$LwT\alpha = 0,6 - 0,06x + 1 - 0,1x$$

$$LwT\alpha = 1,6 - 0,16x$$

$$x = 7 \rightarrow LwT\alpha = 0,48$$

$$x = 10 \rightarrow LwT\alpha = 0$$

$2) x \in <5,7>$

$$LwH = \frac{5}{4}(1-\frac{x}{10})$$

$$LwT\alpha = LwRa \cdot \cos\alpha + LwH \cdot \sin\alpha - 1 \cdot \cos\alpha$$

$$LwT\alpha = 0,6 - 0,06x + 1 - 0,1x - 0,6$$

$$LwT\alpha = 1 - 0,16x$$

$$x = 5 \rightarrow LwT\alpha = 0,2$$

$$x = 7 \rightarrow LwT\alpha = -0,12$$

$LwM\alpha$

Prawa część ramy

$x \in <0,7>$

$1) x \in <0,5>$

$$LwM\alpha = LwRb \cdot 3 - LwH \cdot 4$$

$$LwM\alpha = 0,3x - 0,5x$$

$$LwM\alpha = -0,2x$$

$$x = 0 \rightarrow LwM\alpha = 0$$

$$x = 5 \rightarrow LwM\alpha = -1$$

$2) x \in <5,7>$

$$LwM\alpha = LwRb \cdot 3 - LwH \cdot 4$$

$$LwM\alpha = -5 + 0,3x + 0,5x$$

$$LwM\alpha = -5 + 0,8x$$

$$x = 5 \rightarrow LwM\alpha = -1$$

$$x = 7 \rightarrow LwM\alpha = 0,6$$

Prawa część ramy

$x \in <7,10>$

$$LwM\alpha = LwRb \cdot 3 - LwH \cdot 4 - 1(x-7)$$

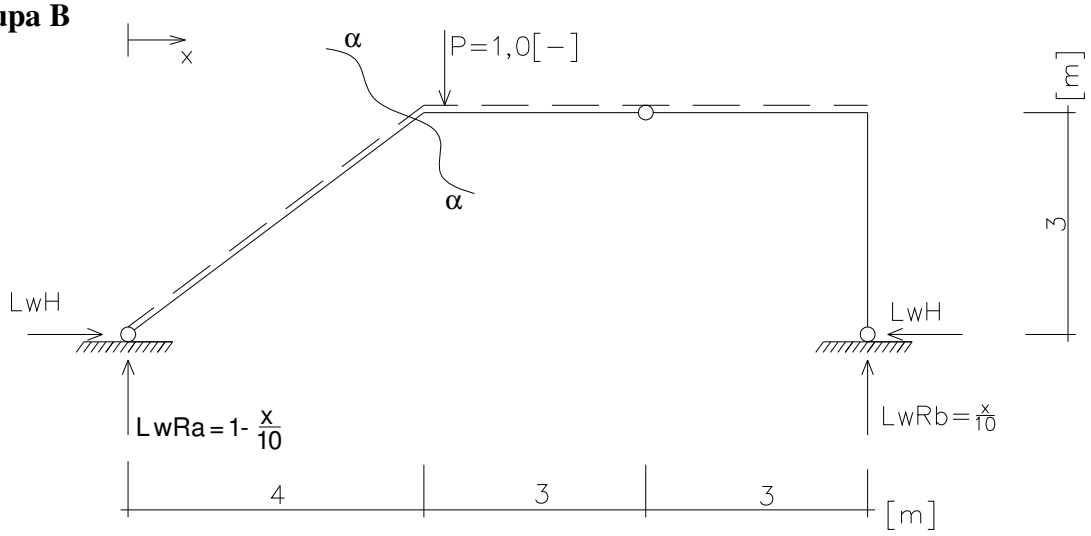
$$LwM\alpha = 0,3x - 5 + 0,5x - x + 0,7$$

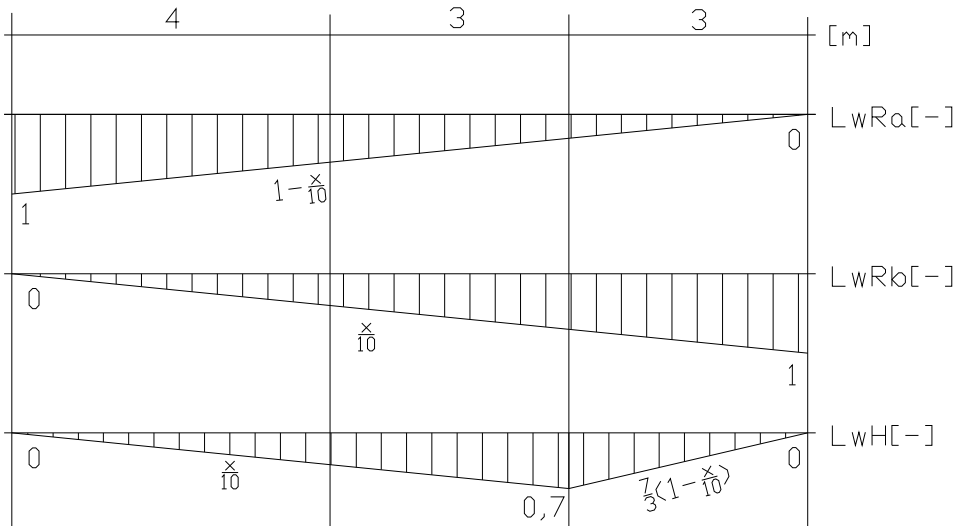
$$LwM\alpha = 2 - 0,2x$$

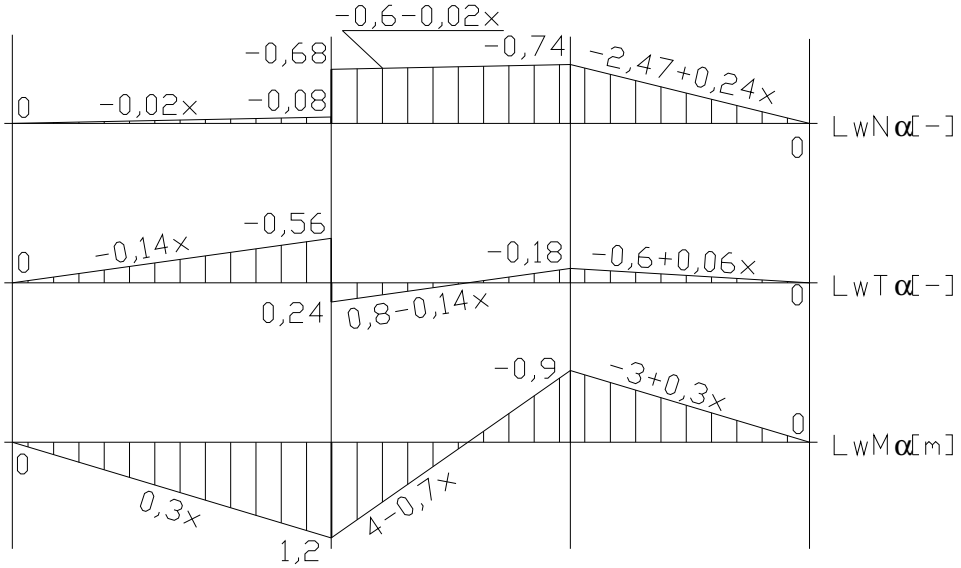
$$x = 7 \rightarrow LwM\alpha = 0,6$$

$$x = 10 \rightarrow LwM\alpha = 0$$

b) Grupa B


x
α
P=1,0[-]
α
[m]
3
LwH
LwH
LwRa = 1- x/10
LwRb = x/10
4
3
3
[m]


4
3
3
[m]
LwRa[-]
1
1-x/10
0
LwRb[-]
0
x/10
1
LwH[-]
0
x/10
0,7
7/3(1-x/10)
0


-0,6-0,02x
-0,68
-0,74
-2,47+0,24x
0
-0,02x
-0,08
0
LwNα[-]
-0,56
-0,18
-0,6+0,06x
0
-0,14x
0
0,24
0,8-0,14x
LwTα[-]
-0,9
-3+0,3x
0
0
0,3x
1,2
4-0,7x
LwMα[m]

$LwN\alpha$

Prawa część ramy

$x \in < 0,4 >$

$LwN\alpha = LwRb \cdot \sin\alpha - LwH \cdot \cos\alpha$

$LwN\alpha = 0,06x - 0,08x$

$LwN\alpha = -0,02x$

$x = 0 \rightarrow LwN\alpha = 0$

$x = 4 \rightarrow LwN\alpha = -0,08$

Prawa część ramy

$x \in < 4,10 >$

$1) x \in < 4,7 >$

$LwN\alpha = LwRb \cdot \sin\alpha - LwH \cdot \cos\alpha - 1 \cdot \sin\alpha$

$LwN\alpha = -0,6 - 0,02x$

$x = 4 \rightarrow LwN\alpha = -0,68$

$x = 7 \rightarrow LwN\alpha = -0,74$

$2) x \in < 7,10 >$

$LwN\alpha = LwRb \cdot \sin\alpha - LwH \cdot \cos\alpha - 1 \cdot \sin\alpha$

$LwN\alpha = 0,06x - 1,8667 + 0,1867x - 0,6$

$LwN\alpha = 0,2467x - 2,4667$

$x = 7 \rightarrow LwN\alpha = -0,74$

$x = 10 \rightarrow LwN\alpha = 0$

$LwT\alpha$

Lewa część ramy

$x \in < 0,4 >$

$LwT\alpha = LwRa \cdot \cos\alpha - LwH \cdot \sin\alpha - 1 \cdot \cos\alpha$

$LwT\alpha = 0,8 - 0,08x - 0,06x - 0,8$

$LwT\alpha = -0,14x$

$x = 0 \rightarrow LwT\alpha = 0$

$x = 4 \rightarrow LwT\alpha = -0,56$

Lewa część ramy

$x \in < 4,10 >$

$1) x \in < 4,7 >$

$LwT\alpha = LwRa \cdot \cos\alpha - LwH \cdot \sin\alpha$

$LwT\alpha = 0,8 - 0,08x - 0,06x$

$LwT\alpha = 0,8 - 0,14x$

$x = 4 \rightarrow LwT\alpha = 0,24$

$x = 7 \rightarrow LwT\alpha = -0,18$

$2) x \in < 7,10 >$

$LwT\alpha = LwRa \cdot \cos\alpha - LwH \cdot \sin\alpha$

$LwT\alpha = 0,8 - 0,08x - 1,4 + 0,14x$

$LwT\alpha = -0,6 + 0,06x$

$x = 7 \rightarrow LwT\alpha = -0,18$

$x = 10 \rightarrow LwT\alpha = 0$

$LwM\alpha$

Lewa część ramy

$x \in < 0,4 >$

$LwM\alpha = LwRa \cdot 4 - LwH \cdot 3 - 1(4 - x)$

$LwM\alpha = 4 - 0,4x - 0,3x - 4 + x$

$LwM\alpha = 0,3x$

$x = 0 \rightarrow LwM\alpha = 0$

$x = 4 \rightarrow LwM\alpha = 1,2$

Lewa część ramy

$x \in < 4,10 >$

1) $x \in < 4,7 >$

$LwM\alpha = LwRa \cdot 4 - LwH \cdot 3$

$LwM\alpha = 4 - 0,4x - 0,3x$

$LwM\alpha = 4 - 0,7x$

$x = 4 \rightarrow LwM\alpha = 1,2$

$x = 7 LwM\alpha = \rightarrow -0,9$

2) $x \in < 7,10 >$

$LwM\alpha = LwRa \cdot 4 - LwH \cdot 3$

$LwM\alpha = 4 - 0,4x - 7 + 0,7x$

$LwM\alpha = -3 + 0,3x$

$x = 7 \rightarrow LwM\alpha = -0,9$

$x = 10 \rightarrow LwM\alpha = 0$

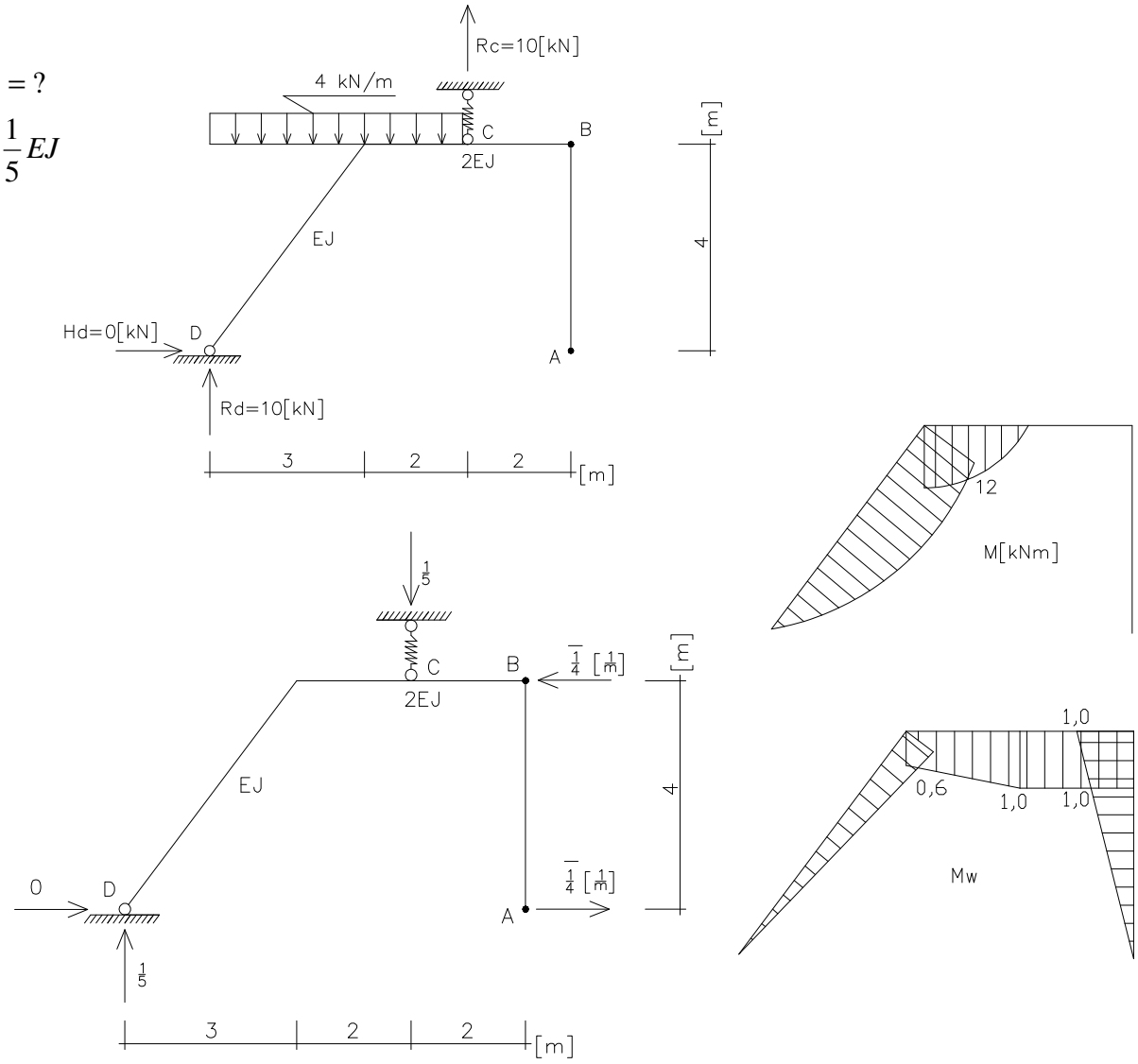
Zadanie 2.

Obliczania przemieszczeń metodą pracy wirtualnej.

a) Grupa A: obl. kąt obrotu cięciwy AB

$\varphi_{AB} = ?$

$k = \frac{1}{5} EJ$



$\overline{1,0} \cdot \varphi_{AB} = \sum \int_S \frac{\overline{M} M}{EJ} ds + \sum \overline{R} R \frac{1}{k}$

$$\overline{1,0}\cdot\varphi_{AB}=\frac{1}{EJ}[0,5\cdot 0,6\cdot 5\cdot\frac{2}{3}\cdot 12+\frac{2}{3}\cdot\frac{4\cdot 3^2}{8}\cdot 5(\frac{1}{2}\cdot 0,6)]+\frac{1}{2EJ}[\frac{1}{2}\cdot 12\cdot 2\cdot(\frac{2}{3}\cdot 0,6+\frac{1}{3}\cdot 1)+\frac{2}{3}\cdot\frac{4\cdot 2^2}{8}\cdot 2(\frac{1}{2}\cdot 0,6+\frac{1}{2}\cdot 1)]=$$

$$=\frac{1}{EJ}[12+4,5]+\frac{1}{2EJ}[8,8+2,133]=\frac{6,5}{EJ}+\frac{10,933}{2EJ}=\frac{21,967}{EJ}$$

$$k=\frac{1}{5}EJ$$

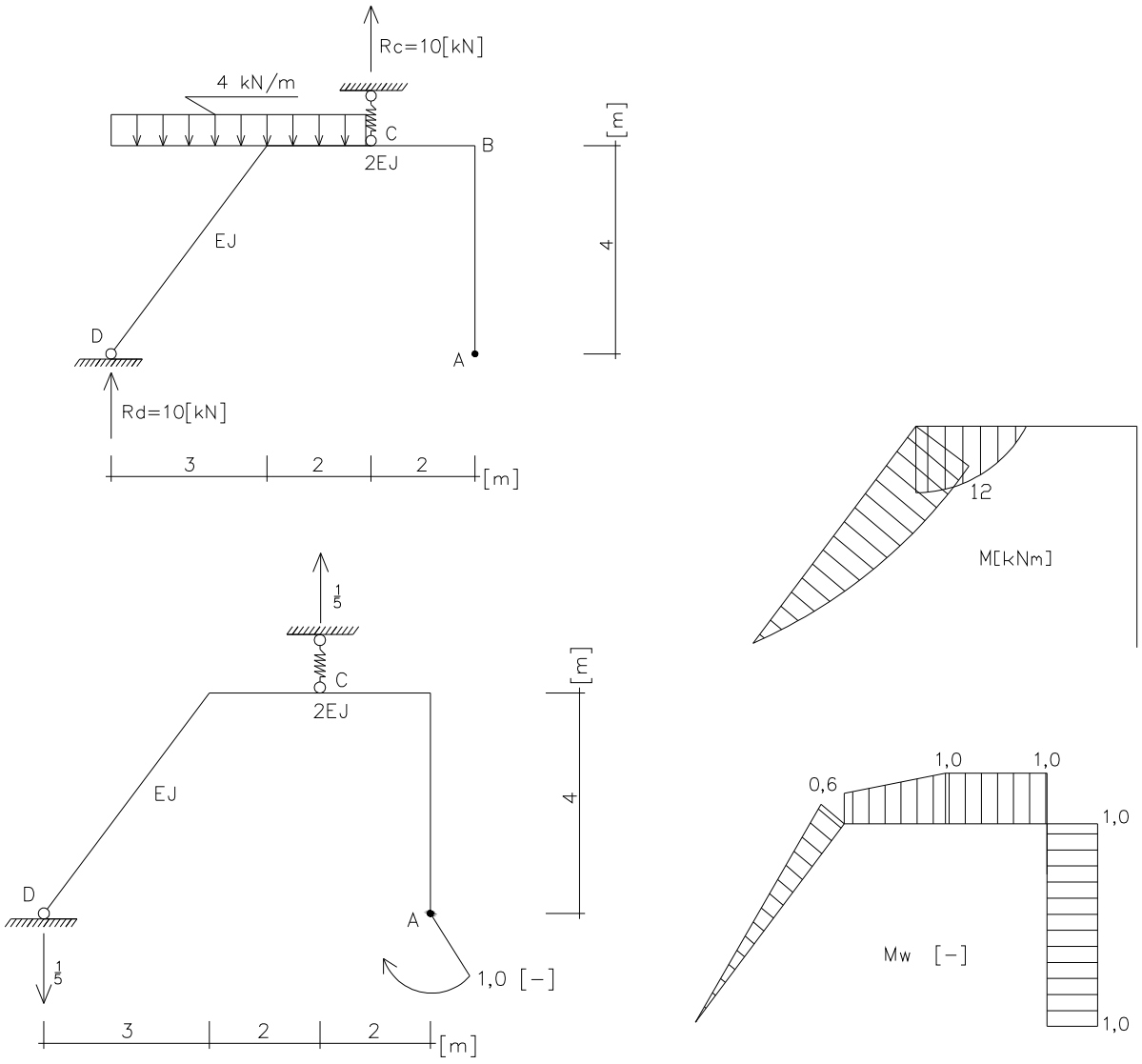
$$\overline{1,0}\cdot\varphi_{AB}^{(R)}=\sum\overline{R}R\frac{1}{k}$$

$$\overline{1,0}\cdot\varphi_{AB}^{(R)}=-10\cdot\frac{1}{5}\cdot\frac{1}{k}=-2\cdot\frac{5}{EJ}=-\frac{10}{EJ}$$

$$\overline{1,0}\cdot\varphi_{AB}=\frac{21,967}{EJ}-\frac{10}{EJ}=\frac{11,967}{EJ}$$

$$\boxed{\varphi_{AB}=\frac{11,967}{EJ}}$$

b) Grupa B: obl. kąt obrotu przekroju A



$$\overline{1,0}\cdot\varphi_A=\sum\int_S\frac{\overline{M}M}{EJ}ds+\sum\overline{R}R\frac{1}{k}$$

$$\overline{1,0}\cdot\varphi_A=\frac{1}{EJ}[-\frac{1}{2}\cdot0,6\cdot5\cdot\frac{2}{3}\cdot12-\frac{2}{3}\cdot\frac{4\cdot3^2}{8}\cdot5\cdot\frac{1}{2}\cdot0,6]+\frac{1}{2EJ}[-\frac{1}{2}\cdot12\cdot2(\frac{2}{3}\cdot0,6+\frac{1}{3}\cdot1)-\frac{2}{3}\cdot\frac{4\cdot2^2}{8}\cdot2(\frac{1}{2}\cdot0,6+\frac{1}{2}\cdot1)]=$$

$$=-\frac{16,5}{EJ}-\frac{10,933}{2EJ}=-\frac{21,967}{EJ}$$

$$k=\frac{1}{5}EJ$$

$$\overline{1,0}\cdot\varphi_A^{(R)}=\sum\overline{R}R\frac{1}{k}$$

$$\overline{1,0}\cdot\varphi_A^{(R)}=10\cdot\frac{1}{5}\cdot\frac{1}{k}=2\cdot\frac{5}{EJ}=\frac{10}{EJ}$$

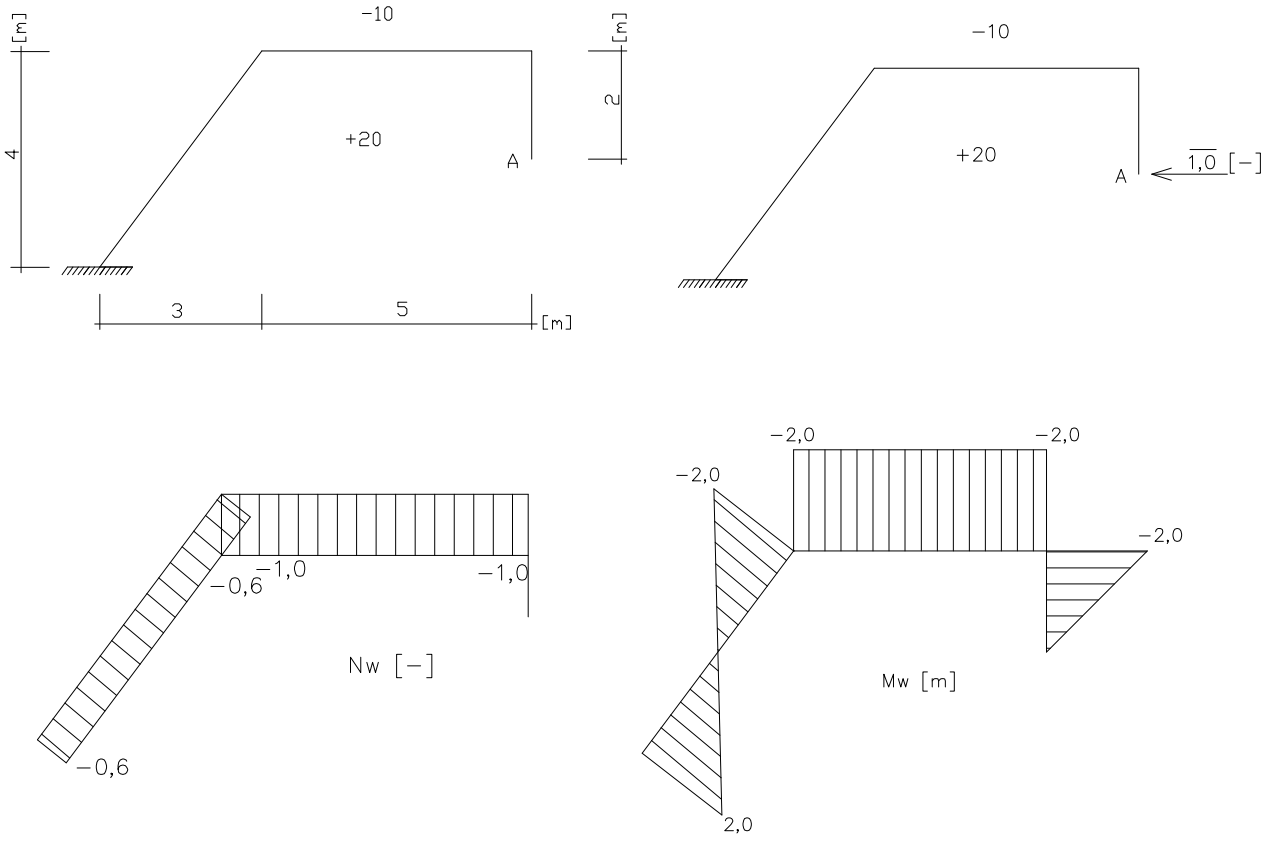
$$\overline{1,0}\cdot\varphi_A=\frac{10}{EJ}-\frac{21,967}{EJ}=-\frac{11,967}{EJ}$$

$$\boxed{\varphi_A=-\frac{11,967}{EJ}}$$

Zadanie 3.

Praca wirtualna (zadania z uwzględnieniem temperatury)

a) grupa A: obl. przemieszczenie poziome p.A



$I200$

$\alpha_t = 1{,}2 \cdot 10^{-5}[\frac{1}{^0C}]$

$t_m = 25^0 C$

$\Delta t = t_d - t_g = 20 - (-10) = 20 + 10 = 30^0 C$

$t_0 = t_{śr} - t_m = 5 - 25 = -20^0 C$

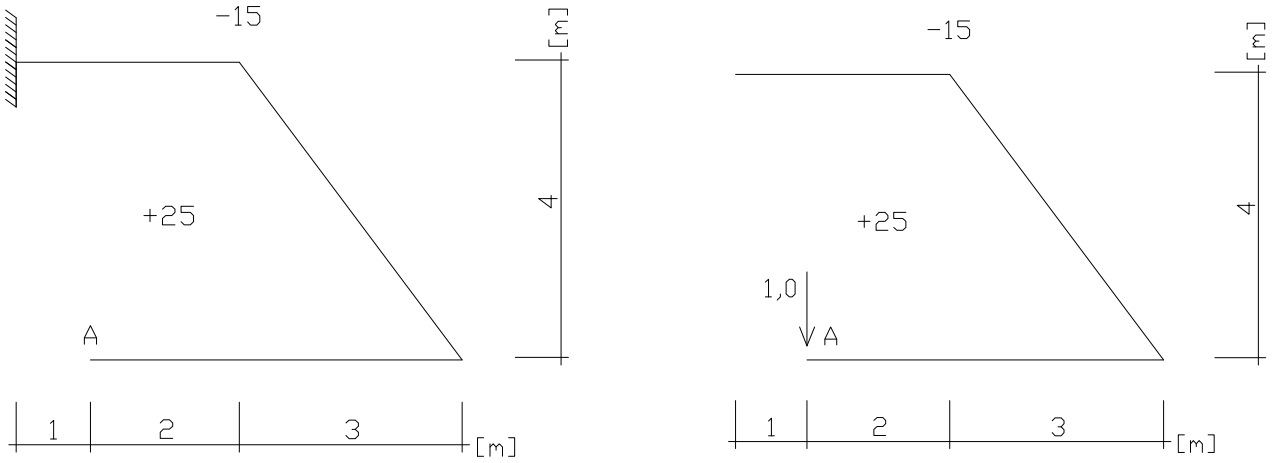
$$\overline{1{,}0} \cdot \delta_A^H = \sum \int \overline{M} \cdot \frac{\alpha_t \cdot \Delta t}{h} ds + \sum \int \overline{N} \cdot \alpha_t \cdot t_0 \cdot ds$$

$$\overline{1{,}0} \cdot \delta_A^H = \frac{30 \cdot \alpha_t}{0{,}2}[-\frac{1}{2} \cdot 2 \cdot 2 - 2 \cdot 5 - \frac{1}{2} \cdot 2 \cdot 5 + \frac{1}{2} \cdot 2 \cdot 5] + \alpha_t \cdot (-20)[5 \cdot (-1) + 5 \cdot (-0{,}6)] =$$

$$= 1{,}8 \cdot 10^{-3} \cdot (-12) - 2{,}4 \cdot 1\theta^{-4} \cdot (-8) = -0{,}0216 + 0{,}00192 = -0{,}01968[m]$$

$$\boxed{\delta_A^H = -0{,}01968m}$$

b) grupa B: obl. przemieszczenie pionowe p.A



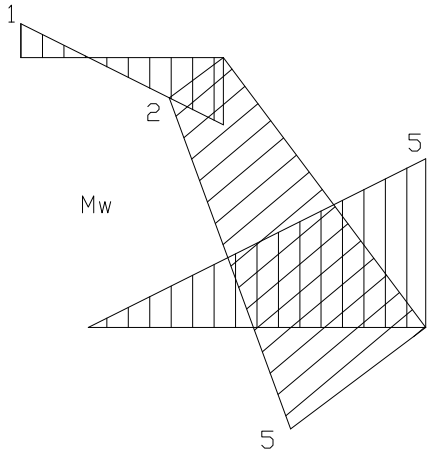
$I200$

$\alpha_t = 1{,}2 \cdot 10^{-5}[\frac{1}{^0C}]$

$t_m = 5^0 C$

$\Delta t = t_d - t_g = 25 - (-15) = 25 + 15 = 40^0 C$

$t_0 = t_{śr} - t_m = 5 - 5 = 0^0 C$



$$\overline{1{,}0} \cdot V_A = \sum \int \overline{M} \cdot \frac{\alpha_t \cdot \Delta t}{h} ds + \sum \int \overline{N} \cdot \alpha_t \cdot t_0 \cdot ds$$

$$\overline{1{,}0} \cdot V_A = \frac{\alpha_t \cdot 40}{0{,}2}(\frac{1}{2} \cdot 5 \cdot 5 + \frac{1}{2} \cdot 5 \cdot 5 + \frac{1}{2} \cdot 2 \cdot 5 + \frac{1}{2} \cdot 2 \cdot 3 - \frac{1}{2} \cdot 1 \cdot 3) =$$

$$= \frac{\alpha_t \cdot 40}{0{,}2}(25 + 5 + 3 - 1{,}5) = \frac{\alpha_t \cdot 40}{0{,}2} \cdot 31{,}5 = 0{,}0756[m]$$

$$\boxed{V_A = 0{,}0756m}$$

Zadanie 4.

Wyznaczyć linie wpływu:

a) **grupa A: Lw R_B; Lw M_α;**

b) **grupa B: Lw M_A; Lw T_α;**