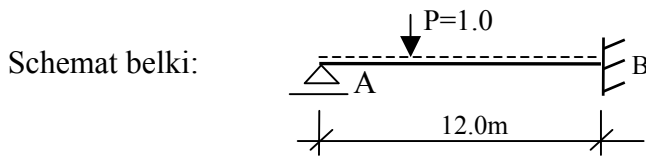
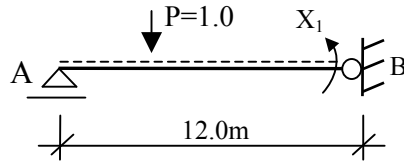


**Zad.2. Wyznaczyć linie wpływu momentu podporowego  $M_B$  dla belki ( $EI=const$ ):**



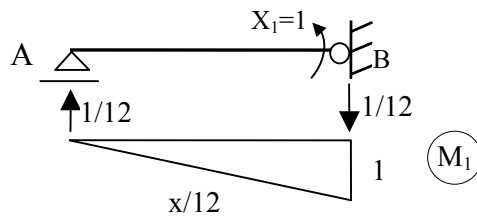
SSN=1

Układ podstawowy:



$$LwM_B = LwX_1$$

Stan  $X_1=1$



$$\delta_{11}LwX_1 + \delta_{1P}(x) = 0$$

$$\delta_{11} = \frac{1}{EI} \left( \frac{1}{2} \cdot 12 \cdot 1 \cdot \frac{2}{3} \cdot 1 \right) = \frac{4}{EI}$$

$$\delta_{1P}(x) = \delta_{P1}(x)$$

$$M(x) = \frac{x}{12}$$

$$EI \frac{d^2y}{dx^2} = -\frac{x}{12}$$

$$EI \frac{dy}{dx} = -\frac{x^2}{24} + C$$

$$EIy = -\frac{x^3}{72} + Cx + D$$

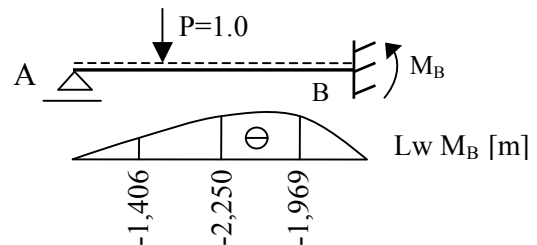
war.brzeg.:

$$x=0 \rightarrow y=0 \Rightarrow D=0$$

$$x=12 \rightarrow y=0 \Rightarrow C=2$$

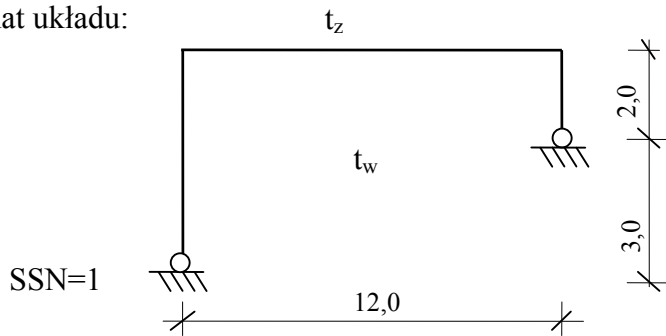
$$\delta_{P1}(x) = y = \frac{1}{EI} \left( -\frac{x^3}{72} + 2x \right)$$

$$LwM_B = LwX_1 = -\frac{\delta_{P1}(x)}{\delta_{11}} = \frac{x^3}{288} - \frac{x}{2}$$



**Zad.3. Korzystając z metody sił wyznaczyć wykres momentów zginających wywołanych równomiernym ogrzaniem ramy o  $t_0$  oraz nierównomiernym o  $\Delta t$ :**

Schemat układu:

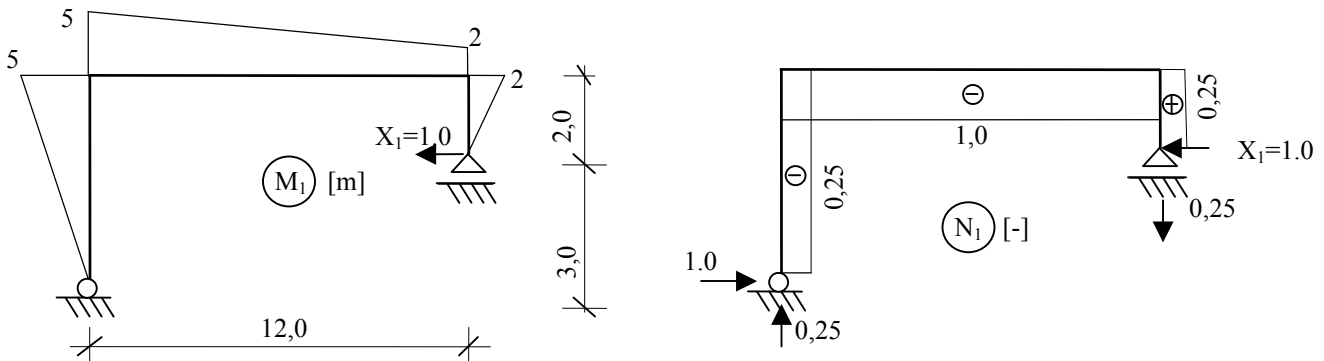


$EI = \text{const}$   
 $t_0 = 20^\circ\text{C}$   
 $\Delta t = t_w - t_z = 15^\circ\text{C} \Rightarrow t_w > t_z$   
 I260:  $I_x = 5740 \text{ cm}^4$   
 $E = 205 \text{ GPa}$   
 $\alpha_t = 1,2 \cdot 10^{-5} \text{ 1/}^\circ\text{C}$

$EI = 11767 \text{ kNm}$

Układ podstawowy:

Stan  $X_1 = 1$



$$\delta_{11} X_1 + \delta_{1t} = 0$$

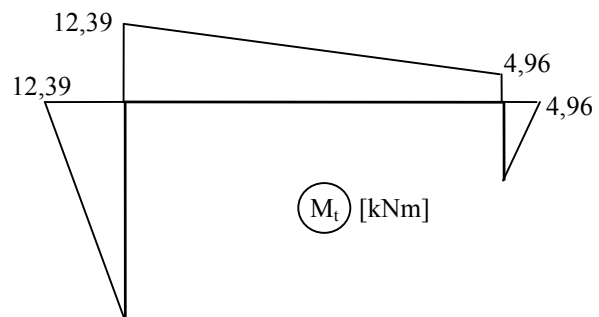
$$\delta_{11} = \sum \int \frac{M_1^2}{EI} ds = \frac{1}{EI} \left[ \frac{1}{2} \cdot 2 \cdot 2 \cdot \frac{2}{3} \cdot 2 + \frac{12}{6} (2 \cdot 5 \cdot 5 + 2 \cdot 2 \cdot 2 + 2 \cdot 5 \cdot 2) + \frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{2}{3} \cdot 5 \right] = \frac{200,3333}{EI}$$

$$\delta_{1t} = \sum \int M_1 \alpha_t \frac{\Delta t}{h} ds + \sum \int N_1 \alpha_t t_0 ds =$$

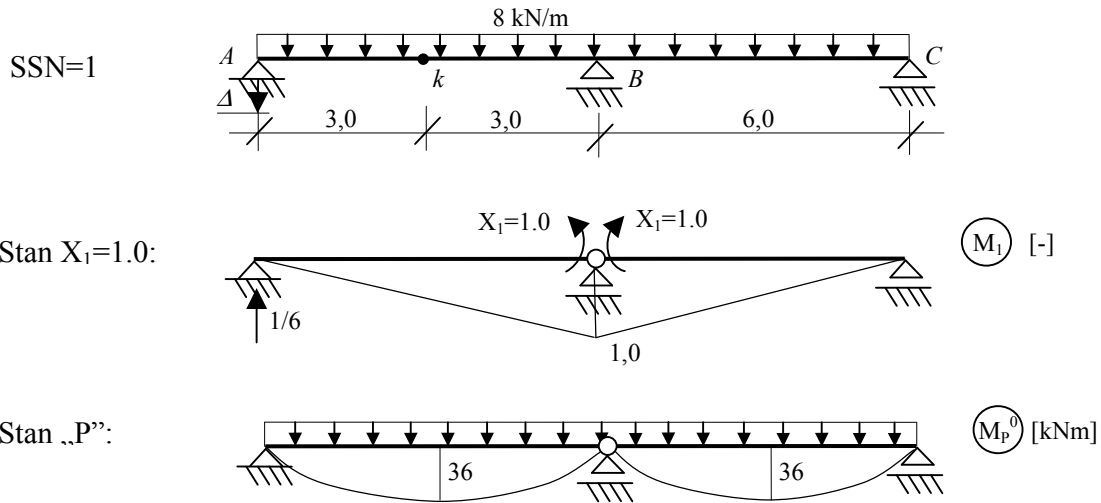
$$= -\alpha_t \frac{15}{0,26} \left( \frac{1}{2} \cdot 2 \cdot 2 + 7 \cdot 12 \cdot \frac{1}{2} + \frac{1}{2} \cdot 5 \cdot 5 \right) + \alpha_t \cdot 20 (2 \cdot 0,25 - 12 \cdot 1 - 5 \cdot 0,25) =$$

$$= \alpha_t (-3259,615 - 255) = -3514,615 \alpha_t = -0,04217$$

$$X_1 = -\frac{\delta_{1t}}{\delta_{11}} = \frac{0,04217 EI}{200,3333} = 2,48 \text{ kN}$$



**Zad.1. Korzystając z metody sił wyznaczyć przemieszczenie pionowe punktu  $k$  wywołane działaniem zadanego obciążenia zewnętrznego oraz osiadaniem podpory  $A$  o  $\Delta=0,06m$ :**



$$\delta_{11}X_1 + \delta_{1P} = 0$$

$$\delta_{11} = \int_s \frac{M_1 M_1}{EI} ds = \frac{1}{EI} \left[ \frac{1}{2} \cdot 6 \cdot 1 \cdot \frac{2}{3} \cdot 1 \cdot 2 \right] = \frac{4,00}{EI} \quad \left[ \frac{1}{kNm} \right]$$

$$\delta_{1P} = \int_s \frac{M_1 M_P}{EI} ds = \frac{1}{EI} \left[ \frac{2}{3} \cdot 6 \cdot \frac{8 \cdot 6^2}{8} \cdot \frac{1}{2} \cdot 1 \cdot 2 \right] = \frac{144}{EI} \quad [-]$$

$$X_1 = -\frac{\delta_{1P}}{\delta_{11}} = -\frac{144}{EI} \cdot \frac{EI}{4} = -36,0 \text{ kNm}$$

$$\delta_{11}X_1 + \delta_{1\Delta} = 0$$

$$\delta_{1\Delta} = -R_1 \cdot \Delta = -\left( -\frac{1}{6} \cdot 0,06 \right) = 0,01 \quad [-]$$

$$X_1 = -\frac{\delta_{1\Delta}}{\delta_{11}} = -0,01 \cdot \frac{EI}{4} = -25,0 \text{ kNm}$$

lub:

$$\delta_{1(P+\Delta)} = \delta_{1P} + \delta_{1\Delta} = \frac{144}{EI} + 0,01 = 0,0144 + 0,01 = 0,0244 \quad [-]$$

$$X_1 = -\frac{\delta_{1(P+\Delta)}}{\delta_{11}} = -0,0244 \cdot \frac{EI}{4} = -61,0 \text{ kNm}$$

$$v_k = \int_s \frac{\bar{M} \cdot M_{P+\Delta}^n}{EI} ds - \sum \bar{R} \cdot \Delta =$$

$$= \frac{1}{EI} \left[ \frac{1}{2} \cdot 3 \cdot \frac{3}{2} \cdot \frac{2}{3} \cdot 5,5 + \frac{2}{3} \cdot \frac{8 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot 2 + \frac{1}{2} \cdot 3 \cdot \frac{3}{2} \left( \frac{2}{3} \cdot 5,5 - \frac{1}{3} \cdot 61 \right) \right] - (-0,5 \cdot 0,06) =$$

$$= \frac{-2,25}{EI} + 0,03 = 0,02977m = 2,98cm$$

